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United States Department of Agriculture

A.R.S. & S.C.S.

Watershed Modeling Workshop

March 16, 17 & 18, 1970

Tucson, Arizona

Pt. I

AGENDA FOR ARS-SCS

WORKSHOP ON WATERSHED MODELING

Tucson, Arizona; March 16-19, 1970

March 16 (Monday)

1. Introduction

Remarks and Announcements	(8:15-8:30 a.m.)
Program Needs--Kent	(8:30-8:50 a.m.)
Response to Needs--Glymph	(8:50-9:10 a.m.)

2. What Is Watershed Modeling and Why?

From Operations View--Maclay	(9:10-9:40 a.m.)
From Research View--Holtan	(9:40-10:10 a.m.)
--B R E A K--	(10:15-10:30 a.m.)

3. Research Approaches to Watershed Modeling

Stochastic Approach--DeCoursey	(10:30-11:15 a.m.)
Deterministic Approach--Woolhiser	(11:15-12:00 noon)
--L U N C H--	(12:00-1:30 p.m.)
Parametric Approach--Snyder	(1:30-2:15 p.m.)
Objectives and Techniques of Watershed Modeling--Diskin	(2:15-3:00 p.m.)
--B R E A K--	(3:00-3:15 p.m.)

4. Operation Approaches to Watershed Modeling

Comparison of Methods for Watershed Modeling Dean Snider	(3:15-3:45 p.m.)
Water Balance Computations by a Modified Stanford Model Bob Rallison	(3:45-4:15 p.m.)
Regionalization of Stream Gage Data for Modification of a Flood Model Bill Owen	(4:15-4:45 p.m.)
Electronic Analog Model Approach for the Sevier River, Utah Erland Warnick	(4:45-5:15 p.m.)

March 17 (Tuesday)

ARS, SWC Research Program Presentation

1. Northeast Branch (8:00-9:00 a.m.)

   Engman--Simulation Models in the Northeast Branch

2. Corn Belt Branch (9:00-10:00 a.m.)

   Harrold--Watershed Research in the North Appalachian Watershed (Coshocton) Related to Watershed Modeling
   Amerman--Importance of Physical Laws of Water Flow Through Porous Media to Research in Watershed Modeling
   Saxton--Watershed Research in Iowa and Missouri Relating to Modeling

--B R E A K-- (10:00-10:30 a.m.)

3. Southern Branch (10:30-11:30 a.m.)

   Stephens--Watershed Program in Southern Branch
   Bowie--Watershed Program at Oxford, Mississippi, USDA Sedimentation Laboratory

--L U N C H-- (11:45-1:00 p.m.)

4. Southern Plains Branch (1:00-2:30 p.m.)

   Ree--Watershed Research in the Southern Plains
   Hartman--Prediction of Soil Moisture From Rainfall and Pan Evaporation Records (To be presented by Mr. Seely)
   McCool--Water Conservation Structures Laboratory, (Stillwater, Oklahoma) With Emphasis on Hydraulic Simulation and Testing of Watershed Model Components
   Knisel and Hartman--Riesel and Sonora, Texas Watershed Models
   Seely--Watershed Model for the Washita River Basin

--B R E A K-- ((2:30-3:00 p.m.)

5. Northern Plains Branch (3:00-4:00 p.m.)

Woolhiser--Research at Fort Collins, Colorado and Newell, South Dakota
Neff--Research at Sidney, Montana

6. Northwest Branch (4:00-5:00 p.m.)

Hamon--Precipitation, Snow Melt, Evapotranspiration, and Infiltration Components of the Reynolds Creek Watershed Model
Schreiber--Overland Flow, Groundwater and Channel Flow Components of the Reynolds Creek Watershed Model

March 18 (Wednesday)

ARS, SWC Research Program Presentation (Continued)

7. Southwest Branch (8:00-9:30 a.m.)

Renard--Analog Computer Model of Runoff in Semiarid Watershed
Osborn--Schematic Representation of Thunderstorm Rainfall
Diskin and Lane--Stochastic Model for Runoff Events in Semiarid Watershed in Southeast Arizona
Cooley--Rainfall-Runoff Relationship for Natural and Lined Channel Watersheds

8. USDA Hydrograph Laboratory (9:30-10:30 a.m.)

Holtan--Watershed Modeling From a Research Viewpoint
England--Framework for Comprehensive Mathematical Models of Watershed Hydrology
Overton--Development of Surface Water Component Models

--B R E A K-- (10:30-11:00 a.m.)

Open discussion and close, or early lunch (11:00-12:00 noon)

March 18 (Wednesday, p.m.)

SCS--Closed discussion
SWC--Closed discussion

March 19 (Thursday)

SCS--Closed discussion
SWC--Field Trip

March 20 (Friday, a.m.)

SCS--Closed discussion

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SCS PROGRAM NEEDS

ARS-SCS WORKSHOP
TUCSON, ARIZONA; MARCH 16-19, 1970

K. M. KENT
CHIEF, HYDROLOGY BRANCH
SOIL CONSERVATION SERVICE

INTRODUCTION

The Soil Conservation Service helped 98,000 cooperators last year with the usual problems involving soil, water and land use considerations. The SCS provided technical assistance to communities in resource development and developed measures to bring about a cleaner, healthier and more attractive environment. These cooperators were not all farmers or ranchers, but included local units of government and community organizations made up of many suburbanites and a few urban folks too. The scene is changing--SCS is getting more and more involved with the non-farmer on other than agricultural land use. The term "land use" has now taken on new dimensions. It now includes urban areas with residential, commercial and industrial developments; recreational areas; right-of-ways occupied by transportation facilities; and others, along with the agricultural lands to which it was formerly limited. Wherever land is involved, there are many problems that require the experience and skills of the soil and water resource conservationist. However, the inclusion of these new land uses has exposed the resource conservationist to new problems. The complexity of storm runoff related to the rural-urban transformation and the denuding or stripping of the natural cover and topsoil from large development areas are examples of those activities requiring new technological processes to be added to the engineer's old bag of conservation tricks. Aside from these new inclusions, an ever increasing demand remains for most of the many practices which the SCS has helped cooperators install over the past 33 years. A few practices involving the science of hydrology in their application are listed in Table 1. Shown there are the current annual rate of application

Table 1. - A few conservation measures applied*

Measure	Unit	Applied Fiscal 69	Applied to date
Pond	number	46,000	1,700,000
Grade stabilization structure	number	10,500	195,000
Terraces	miles	28,000	1,200,000
Watersheds approved	number	103	970
Floodwater retarding structure	number	420	9,750
Floodwater retarding storage	ac. ft.	245,000	4,590,000

*From Soil Conservation Magazine 35/6, January 1970

and the total amount applied to date. Note that the length of terraces built last year would more than circle the earth. The total length built to date would circle it almost 50 times.

The project measures, primarily floodwater retarding structures and channel improvement, included in the 937 approved watersheds are estimated to cost over two billion dollars. A wise and economical expenditure of two billion dollars must be supported by sound research.

Watersheds, their evaluation and the design of measures, and basin investigations demand over 80 percent of the SCS hydrologist's time. Every SCS hydrologist spends some, if not most, of his time on watershed planning and basin investigations. Therefore, it is only logical that research on each and all elements of the hydrologic cycle bearing upon natural basins ranging from a few acres up to 500 square miles, is of greatest importance to our SCS hydrologists. The SCS hydrologist is strictly a practitioner. He is not permitted to do research, even on items of his greatest need. He must rely on research agencies and primarily the Agricultural Research Service for these needs.

WATERSHED HYDROLOGY

The hydrologic processes by which a watershed is usually evaluated is shown in Figure 1. In some ways the flow chart is an oversimplification of the hydrologic model as a whole. The aim of course is to find the least costly system of structures that will attain local objectives and achieve benefits in excess of cost.

The prediction of flooding and effectiveness of measures is based on a sequence of processes commencing with a set of base maps and threading through a series of hydraulic computations; and a summarization of physical effects to which an economic evaluation can be applied.

Several arrangements of structures are usually tested to obtain the most efficient system of flood control for a given watershed. Improved channels for flood prevention, although not shown specifically in the model, are usually considered only after detention storage has been utilized to the greatest possible extent.

The processes are mostly computerized. Low level aerial photography, together with the Kelch process of identifying elevations at selected channel and flood plain sections are in common use replacing the data-gathering field surveys. Hydrologists may soon be using "digitizing" equipment that transfers distances and elevations directly from the Kelch process to data cards ready for further automated processing

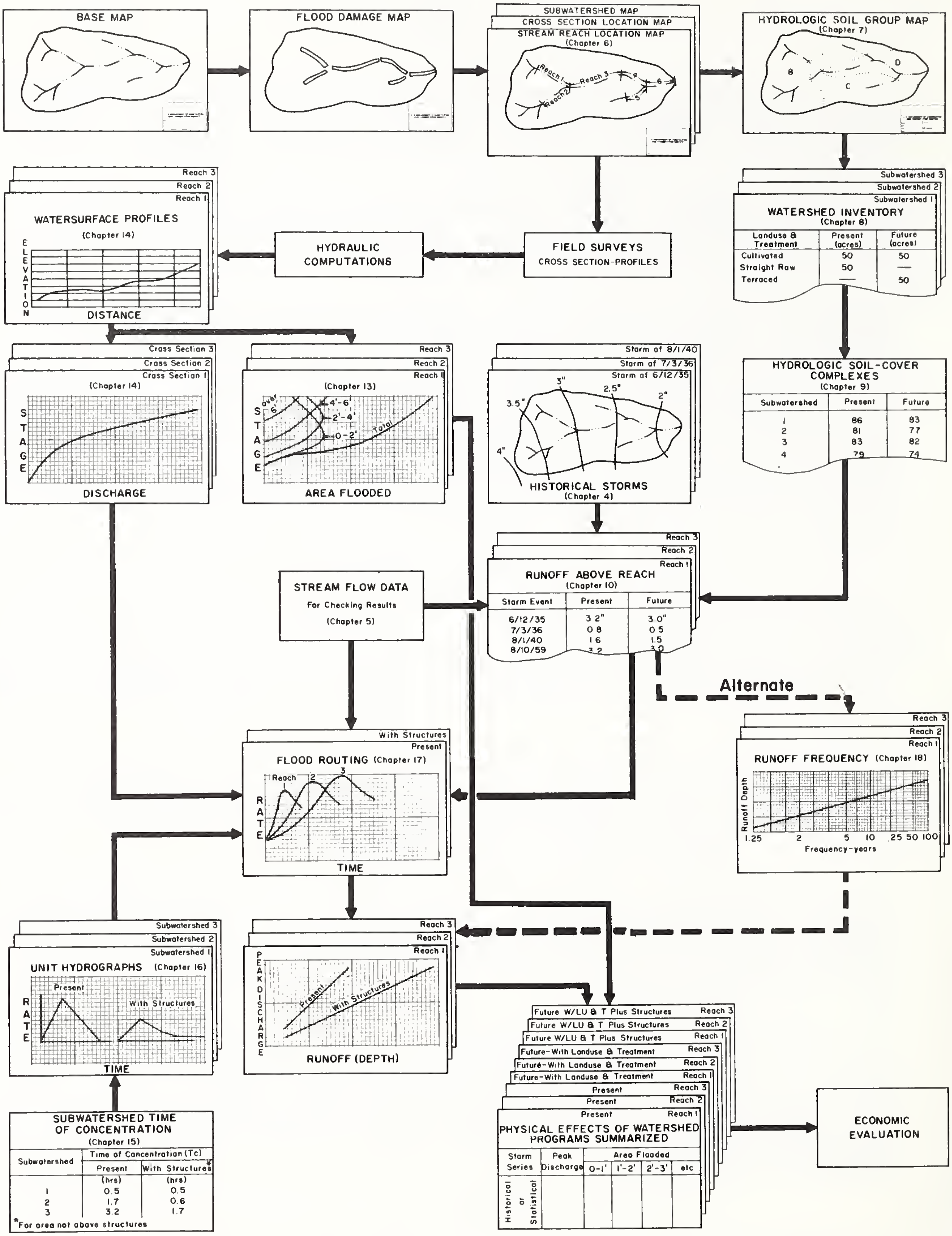

FIGURE 1 – General process hydrology of watershed project evaluation.

In spite of the more sophisticated solutions and the application of more advanced technology made possible by the computer, some weaknesses remain unsolved within the principal components of the overall model. The model can be divided into three main components (1) development of hydrographs which graphically predict the volume and rate of runoff caused by flood producing storms over subwatershed areas tributary to the main stream; (2) channel and flood plain hydraulics representing a reasonable estimate of discharge for various stages of flooding in order to establish a yardstick by which the flooded area can be related to storm magnitudes; and (3) routing of flood runoff from tributary areas through flood plain reaches of the main stream and thereby estimating the stage of flooding each storm would create.

The SCS continues to use the same runoff equation as originally developed by Mockus[1] for estimating the volume under the subwatershed hydrograph. It has been a practical and most useful tool to SCS hydrologists and to many others at home and in foreign countries for over 20 years. The equation is:

$$Q = \frac{(P - I_a)^2}{P - I_a + S} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where: Q = accumulated direct runoff.

P = accumulated rainfall (potential maximum runoff).

I_a = initial abstraction including surface storage, interception and infiltration prior to runoff.

S = potential maximum retention.

The "S" in the above equation relates the hydrologic characteristics of soil and its cover to runoff. The S-values that are assigned in the SCS hydrology handbook[1] to the cultivated and pasture lands of the humid and subhumid regions are for the most part based on adequate research data. Values for the range and forested lands in the arid and semiarid regions are more speculative, lacking adequate research data on which they should be based. Hydrologists for the Bureau of Land Management, Forest Service and SCS have been scraping together meager data from these regions for the past several years in an attempt to agree on better S-values for their wide range of hydrologic conditions.

Efforts along this line have pointed out deficiencies in the runoff equation and the need for additional research. The initial abstraction (I_a) has been roughly estimated as 0.2s for all situations. Researchers Russell Hamon and Kenneth Renard of ARS along with others working in the western areas have pointed out that this fixed relationship may be resulting in some very poor estimates of runoff. Further research is needed to define the portion of total rainfall which is intercepted above the surface of the ground by various types

of cover as apart from how much of the rainfall is absorbed into the soil before runoff begins. The latter suggests that rainfall intensity might need to be considered.

No one in research should feel himself cheated because so much has already been learned about infiltration rates and other hydrologic characteristics of the soil. "There is challenge enough, and to spare, in unraveling fresh regularities and in finding new ways to put together old building blocks." So too with our SCS standard runoff equation.

The unit hydrograph theory is employed for distribution of flood volume with respect to time as described in Chapter 16 (or Section 3.16 prior to 1970 revision) of the SCS Hydrology Handbook. However, some SCS hydrologists are advocating that the standard SCS unit hydrograph needs to be regionalized to fit the various physical characteristics of the country. Should not the flood hydrograph be defined by an equation as suggested by many? Research is needed not only for the improvement of the unit hydrograph but on whether the unit graph is the best procedure for developing the flood hydrograph.

The time of concentration is another parameter which is difficult to estimate in some regions. Further research may be necessary before adequate procedures can be developed for moderate to steep slopes in a mountainous terrain. Conditions of interflow that exist through a leafy mulch surface and flow over and around boulders in its path greatly affect the movement of runoff through small watersheds in this type of terrain, making it difficult to estimate the time of concentration.

Channel and overbank hydraulics in watershed development require a knowledge of several factors that are difficult to measure. One is the roughness of the flood plain. A poor assessment of this factor imposes the weakest link in the chain of factors leading up to the solution of water-surface profiles. A valley section may be divided into several segments, including the channel and two or more flood plain segments for computing a conveyance factor for the water-surface profile through a reach. The profile represents conditions longitudinally along the reach as well as across each segment of the valley section. The roughness often varies in the longitudinal direction as well as laterally. The conveyance factor (k_d) for each segment, given by the formula:

$$k_d = \frac{1.486}{n} AR^{2/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

must be combined before introducing length and slope into the solution of the profile. The reach length of each segment, for which the conveyance factor is computed, varies. In reaches that have meandering channels, the length of the channel segment may be twice that of a flood plain segment. However, as stated above the SCS water-surface profile solution uses one common length of reach for all segments since the conveyance factors are

added together before slope and reach length are introduced. The question is, what length should be used for defining the gradient of the water-surface--channellength, flood plain length, or some length in between?

Other factors that affect the accuracy of the water-surface profile solution are the quantitative effects of floating debris and ice-flow so often associated with flooding; and the change from stable to a semifluid state of some channel and overbank alluviums during high flood flows.

To have a single mean velocity represent the passing of a flood is an alarming thought for any hydrologist and yet the SCS's solution like most other solutions depends on just that.

Several textbook procedures have been used by the SCS for routing floods through natural reaches. Storage-type routing procedures have generally been used. Some account for the wedge-shaped storage factor while others such as the Storage-Indication Method[1] rely on making the routing reach short enough to ignore the wedge-shaped factor. However, in most storage routing procedures, the greatest shortcoming is the sensitivity to the routing interval. The SCS is using the Convex Routing Method described in Chapter 17 of the SCS Hanbook[1]. It was derived by Mockus mainly from the Muskingum Method. The routing equation is expressed as a convex-linear combination so that:

$$O_2 = (1 - C)\, O_1 + C I_1 \qquad (3)$$

where: O_2 = outflow at the end of the time interval.

O_1 = outflow at the beginning of the time interval.

I_1 = inflow at the beginning of the time interval.

C = a routing coefficient.

Research is needed to relate water velocity to wave velocity by some observable parameters. The routing coefficient (C) is defined as:

$$C = \frac{V}{U} \qquad (4)$$

where: V = discharge divided by the average cross sectional area in the reach.

U = wave velocity in the reach.

It has a fixed relationship with the "time interval" and "travel time" through the reach. Since the "travel time" depends on the wave velocity, this wave

velocity becomes an important factor in the fixed relationship between the coefficient "C" and the "time interval" for the routing equation. Hence a poor estimate of wave velocity for channel and flood plain flow results in poor estimates of discharge for the routed flood through a given reach. SCS fieldmen have often complained that their routed discharge does not check with the gage record; or that they obtain conflicting results from single reach routings as compared to combining two or more reaches for a single routing. This has been demonstrated by Holton and Overton.[2,3] This suggests but one aspect of the natural reach routing process that needs further attention by researchers. Williams[4] is investigating the practicability of and the improvement gained by varying the routing coefficient during the rise and fall periods of a flood for his reach routing method. This should improve the SCS routing procedure. However, all procedures must depend on the channel and overbank hydraulics while this remains the weakest link.

More needs to be known about transmission losses before adequate downstream estimates of routed floods can be made in some regions. Hartman, Ree, Schoof, and Blanchard[5] have demonstrated this great need in their work in the Washita. This need is expressed mostly by hydrologists engaged in river basin studies.

Since the SCS watershed model usually depends on a synthetic rainstorm for the primary input, there is much argument as to the characteristics which the synthetic storm should embody. First there is the short, intense summer thunderstorm that causes severe flooding on a small watershed but little or no flooding farther downstream. A low intensity spring or fall rain, extending over a much larger area, sometimes combined with snowmelt, and lasting for several days, may not cause flooding in the small watershed but will produce a major flood downstream. The river basin hydrologist must account for both conditions when evaluating the downstream effects of the combined upstream watershed projects. He needs to know more about the separate characteristics of these storm types and about the joint probability of their occurrences throughout the year.

DESIGN HYDROLOGY

The SCS hydrologists are more involved in the design of floodwater retarding structures than they are in the design of channels or other facilities. The typical processes involved in the hydrologic model for floodwater retarding structures is shown in the flow chart of Figure 2. Here again, stream gage records are not available for 90 percent or more of the structures which SCS builds. Therefore rainfall and physical watershed characteristics are the principal inputs for the development of design hydrographs. The SCS develops three design hydrographs for a floodwater retarding structure. One is the "principal spillway" hydrograph which is used to determine the detention storage needed to pass a specified flood through a principal spillway of a chosen capacity without flow through the emergency spillway. It

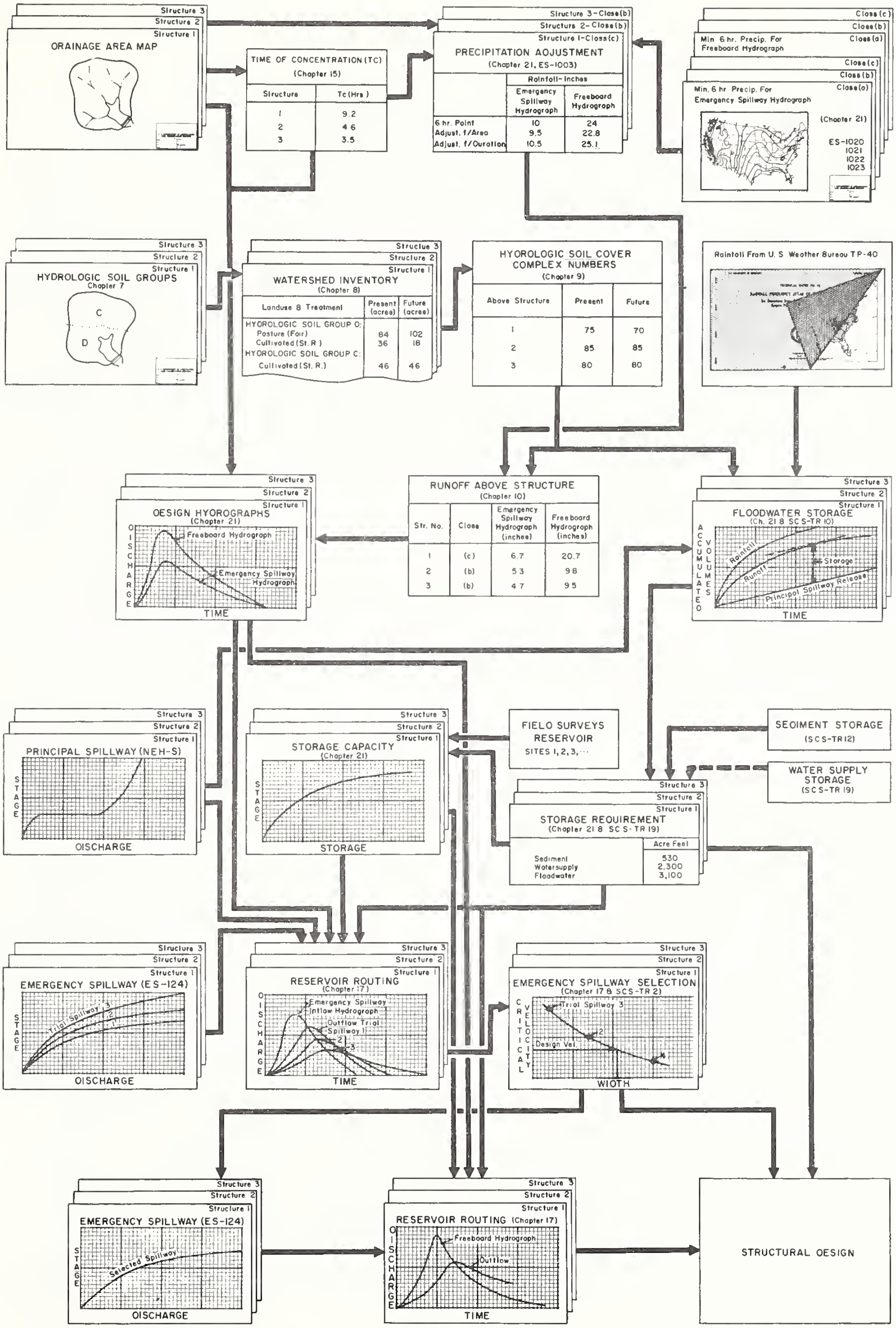

FIGURE 2 -Design hydrology for storage and spillways in floodwater retarding structures.

regulates the degree of flood regulation in downstream reaches and the frequency which the emergency spillway might be used.

The "freeboard" hydrograph is developed from runoff from probable maximum precipitation (PMP) and is used to determine the capacity of the emergency spillway to pass a very rare flood without overtopping the dam. The "emergency spillway" hydrograph is developed from a somewhat smaller storm than PMP but greater than the 100-year event. It is used to proportion the depth and width of the emergency spillway so that non-erosive velocities are not exceeded during the passage of the lesser floods.

A recent amendment to the small watershed act allows SCS to build floodwater retarding structures with detention storage capacities up to 12,500 acre feet. The previous limitation was 5,000 acre feet. The larger storage permits selection of sites with drainage areas more than twice as large as before. The larger drainage areas, several exceeding 100 square miles, have raised questions in regard to the present criteria for earthen and vegetated emergency spillways. Excessive velocities have been avoided on the smaller structures without specifying any limitation on the expected duration of flow. Since the larger structure sites introduce longer time of concentrations it must be recognized that some practical limit of velocity related to duration must be incorporated into the spillway criteria to prevent breaching. The velocity-duration relationship will vary with soil properties and kinds of vegetation that can be established to resist erosion. Present velocity limits are based on Ree and Palmer's[6] findings at the Stillwater and other outdoor hydraulic laboratories. However, further studies with duration included are needed for the design of adequate spillway facilities and a better estimate of expected maintenance requirements.

The larger drainage areas have cast doubt on the applicability of the 6-hour PMP rainfall durations and on assuming the depth of rainfall distributed uniformally over the drainage area as currently specified in the SCS criteria. More information is needed on the characteristics of a typical major storm in order that a model appropriate for the design of floodwater retarding structures could be employed. It is believed the model would vary regionally. The typical characteristics for a storm model for a given region should include an isohyetal pattern of areal distribution and storm depth with respect to time over a 24-hour or longer duration. This is similar to the need for improved rainfall models and better defined characteristics of the smaller storms used for evaluating project facilities in a watershed. Now that computers are used extensively, these hydrologic processes can be analyzed in much greater detail than was practical manually.

NEW HYDROLOGIC REQUIREMENTS

The Soil Conservation Service's involvement in relatively new activities such as community resource development, urban development and planning,

pollution control, and streambank erosion control have created a need for new hydrologic processes.

Community resource development is demanding greater knowledge of the potential water supply. The SCS hydrologists need to make estimates of surface water yield on a monthly, seasonal and annual basis. Adequate procedures for ungaged areas do not exist. However, the model reported by Holtan, England and Allen[7] for estimating surface runoff yield in ungaged watersheds is expected to fulfill this requirement. Ground water is a major source of water supply in many localities. We need to know more about modern techniques for determining the quantity and quality of this source and, as conservationists, how the rate of recharge can be made to replenish the rate of withdrawal.

A study of storm runoff measurements for a range of unit source areas in urban developments could provide a basis for adapting the SCS curve number and runoff equation to urban complexes.

Water quality ranks high among the current interests of the average citizen today. Water quality and pollution control is closely allied with water management. Augmentation of low or intermittent streamflow is an important consideration in the dilution of pollutants being discharged into small streams. The Water Pollution Control Administration (WPCA), Health, Education, and Welfare (HEW) and others are studying the dilution requirements mostly for industrial and community wastes, and to a lesser extent agricultural wastes. The studies being made at Coschocton on feedlot runoff and on pesticides, and fertilizers are very timely. Early reports on this work would be desirable.

Although streambank protection is an old practice to SCS, new and greater demands are being voiced by our society at large for the prevention of erosion along stream and artificial channel banks. The Environmental Quality Council, established by Executive Order 11472, is chaired by the President of the United States. Its members include the Vice-President and Secretaries of the Departments of Agriculture, Commerce, HEW, HUD, Interior, and Transportation. A standing committee on outdoor recreation is one of the five environmental committees in the Council. The first charge to this committee has been to initiate a study of stream channelization. This study group is evaluating the effects of channel improvement on the ecology of the area. The new demands are to reduce sediment which is considered a major pollutant, prevent damaging effects to fish and wildlife caused by bank sloughing, and preserve the stream's natural beauty. Hydraulic studies of how and why a stream meanders and erodes its banks need to be related to practical measures of control. What are the geometric limits in artificial channels for various soil and slope conditions? Car body riprap for stopping bank erosion no longer satisfies the general public's objectives.

A continuity of basic parameters between regions from which final research results are derived is desirable in order for them to be applicable for a national procedure. If soil and cover were principal parameters in the final results of research in the southeast but something other than soil and cover were basic parameters of similar research for the southwest, the SCS runoff equation may not have evolved as a national procedure.

Timing is important too in the fulfillment of national research needs. If research is completed for a national need in some one region before it is started or completed in all other regions, it may not be of any real value to a user agency until the final results from other regions are available.

It may be less desirable to the researcher if his agency were to pursue research objectives similar to his, on a national scale and direct it to be done simultaneously by numerous researchers having varied experiences and backgrounds, and above all, be restricted to a uniform approach with a specified selection of parameters. Since most scientists would argue that research under the above restrictions could not produce useful results or even be classified as research, there is an apparent conflict of interest between the researcher and the user. However, the Soil and Water Conservation Research Division of ARS has been most attentive to the hydrologic research needs of SCS. Most of the SCS hydrology procedures have originated from findings in the ARS research watersheds and are being modified and updated occasionally on new findings in these and new watersheds. Glymph, in his presentation at the Water Resources Symposium at the University of Texas at Austin in October 1968[11], stated that their research was primarily mission and problem oriented. He pointed out the importance of keeping research people knowledgeable of current research needs and that this was best accomplished through contacts with hydrologists and engineers responsible for designing water programs as is happening here this week. He explained further that hydrologic studies on experimental watersheds are now oriented to serve major physiographic or land resource areas and that a special effort was being made to extend the results of all experimental watersheds to other areas and to a broader range of conditions through satellite watersheds. We in SCS, as users of research results, can certainly ascribe to this philosophy and welcome the opportunity to participate in joint workshops between researchers and users and convened here this week.

Over 49,000 cooperators have established commercial recreation enterprises on their land according to latest reports.[8] SCS helped to establish over 1,600 last year. About 240 of these cooperators changed to the recreation enterprises last year as their primary source of income. The enterprises are mostly small lakes stocked for fishing. We need to know much about the regulation of water storage for propagation of desirable fish species. More needs to be learned too, on how regulation through storage effects downstream aquatic life. Dr. Wadleigh, Director, Soil and Water Conservation Research Division of ARS, at a recent meeting with SCS specialists, referred to studies[9] by Dr. Daniel F. Jackson of Syracuse University on the control of algae blooms by the use of viruses. Algae is commonly known as one of the first outward signs of deterioration on the ecology of lakes and rivers. Although Jackson's control methods focus on regulating the amounts of such inorganic nutrients as nitrates and phosphates entering the water, more should be known on how these nutrients are mixed or distributed within small bodies of water and how they mix or are attached to and transported with sediments in a stream.

A task committee serving under the ASCE Surface Water Hydrology Committee in 1966 reported on 83 individual research projects[10] having the highest priority of need in the field of surface water hydrology. Many of these are current SCS needs while others are closely allied with SCS functions. There are 12 research needs listed under the major heading "Runoff From Small Areas." SCS can ascribe to all 12 including such titles as "Time of Concentration of Flood Flows", "Development of Runoff Hydrograph From Small Areas", and "Laboratory Watershed Experimentation". Other major headings are "Channel Losses in Ephemeral Streams" and "Economic Flow Records of Desert Streamflow." These are not new needs but in some ways they have taken on a new meaning. ARS personnel are not only aware of these needs but have been doing research on many of them for some time. However, they are important enough to SCS that they should be restated periodically.

RESEARCH-USER CONSIDERATIONS

Several processes which the SCS hydrologist uses in his contribution to the soil and water programs in which the SCS is involved have been briefly described. Needed improvements and in some cases the need for new processes have been pointed out. They should be based on sound research. Adequate research may be available for improving an existing process or adding a new process within the confines of certain regional characteristics. Too often, however, the research has not been extensive enough to make the improvement applicable to all regions.

Total applicability is an essential requirement for national procedures and the widespread application of SCS programs. For example, the present SCS runoff equation would be of little value for a national application of soil and water practices if the curve numbers had been developed only for a small group of soils and cover conditions that exist in the southeast and nowhere else.

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[10] ______, Needed Research Projects in Surface Hydrology, ASCE Journal, Vol. 91, No. HY 1 (Proceedings of Paper 4201), January 1965.

[11] Glymph, L. M. and Holtan, H. N., published in "Effects of Watershed Changes on Streamflow," Water Resources Symposium No. 2, edited by Moore and Morgan. The University of Texas Press, Austin and London, pp. 44-68, 1969.

A continuity of basic parameters between regions from which final research results are derived is desirable in order for them to be applicable for a national procedure. If soil and cover were principal parameters in the final results of research in the Southeast but something other than soil and cover were basic parameters of similar research for the Southwest, the SCS runoff equation may not have evolved as a national procedure.

Timing is important too in the fulfillment of national research needs. If research is completed for a national need in some one region before it is started or completed in all other regions, it may not be of any real value to a user agency until the final results from other regions are available.

It may be less desirable to the researcher if his agency were to pursue research objectives similar to his, on a national scale and direct it to be done simultaneously by numerous researchers having varied experiences and backgrounds, and above all, be restricted to a uniform approach with a specified selection of parameters. Since most scientists would argue that research under these restrictions could not produce useful results or even be classified as research, there is an apparent conflict of interest between the researcher and the user. However, the Soil and Water Conservation Research Division of ARS has been most attentive to the hydrologic research needs of SCS. Most of the SCS hydrology procedures have originated from findings in the ARS research watersheds and are being modified and updated occasionally on new findings in these and new watersheds. Glymph, in his presentation at the Water Resources Symposium at the University of Texas at Austin in October 1968(11), stated that their research was primarily mission and problem oriented. He pointed out the importance of keeping research people knowledgeable of current research needs and that this was best accomplished through contacts with hydrologists and engineers responsible for designing water programs as is happening here this week. He explained further that hydrologic studies on experimental watersheds are now oriented to serve major physiographic or land resource areas and that a special effort was being made to extend the results of all experimental watersheds to other areas and to a broader range of conditions through satellite watersheds. We in SCS, as users of research results, can certainly ascribe to this philosophy and welcome the opportunity to participate in joint workshops between researchers and users as convened here this week.

RESPONSE TO NEEDS [1/]

L. M. Glymph [2/]

We are very fortunate in SWC-ARS to have clients for our research. In the main, these clients are the farmers and ranchers of the United States but we know that our research findings have increasing relevance for the urban dweller as well.

We are also fortunate to have a principal user of our research. Our broad mission is to enhance the art and science of the technology for USDA operations programs pertaining to the Nation's land and water resources, but we know that our research findings are applied primarily through operations programs in the Soil Conservation Service.

Mr. Kent has given us an overview of the variety of operations programs and responsibilities centered in SCS and a few statistics on the conservation measures applied and needed on the landscape. These are massive programs. Much has been accomplished but much remains to be done. These programs are big business. They represent large capital investments both by private and public interests; they are also of vital importance from the standpoint of their beneficial impacts upon natural resources and environmental quality. We frequently use SCS statistics on the workload of approved projects and conservation needs, such as Mr. Kent has given us, to show the relevance of our research and to justify it in the budgetary processes.

Our close working relationships with SCS are not accidental. They were established by Department policy which was first documented in Secretary's Memorandum No. 1318, October 14, 1952, and more completely specified by Secretary's Memorandum No. 1320, Supplement 4, November 2, 1953, which assigned to the Agricultural Research Service all soil conservation research except for the investigations required for the National Soil Survey conducted by SCS. Before these memoranda were issued, as many of you will recall, SCS had a research function and program. In fact, some of the old-timers among us here today have had many years of research experience as SCS employees.

1/ Contribution from the Soil and Water Conservation Research Division, Agricultural Research Service, USDA, Beltsville, Maryland.

2/ Assistant Director for Watershed Engineering, SWC-ARS-USDA, Beltsville, Maryland.

Through the years it has been the practice of SCS to periodically make a formal report of soil and water conservation research needs to ARS. ARS has responded in various ways. Some of our first responses were written comments or reports aimed at answering the specific items in the list of research needs, but response in this fashion generally was not satisfactory to either SCS or ARS. Response has taken various other forms including: detail of SWC personnel for special assignments in SCS offices, work conferences between individuals or small groups, visits by SCS personnel to SWC locations for program review conferences and research program planning sessions, and ARS-SCS local, regional and national subject matter conferences.

We call our gathering here this week a Workshop. It could be called something else just as well--perhaps subject matter conference would be an equally fitting term. What we call this gathering is not important, but the fact that we are meeting is very important. That we are meeting at a time when our Agencies are faced with stringent restrictions on all forms of expenditures gives added significance to the event. We are here with the confident expectation that the Workshop will be mutually beneficial for each Agency. This joint Workshop is a two-way proposition. It is an opportunity for SWC to report progress and research findings of benefit to operations programs, and for SCS to express its research needs more specifically and also present information on procedures and criteria followed in solving problems in the real world. The meeting will bring about a closer liaison between research and operations activities and further enhance USDA's service to the public.

Under the PPB system for program planning and budgeting, the current ARS research is classified into units called "activities." An activity is a unit of research effort directed toward the technical aspects of some economic problem, the solution of which would make a contribution to society that is definite, concrete, and (in most cases) measurable by a numerical analysis expressed in dollars. The orientation is toward problem areas, as opposed to scientific disciplines. Each activity is viewed as having a definite beginning and ending wherein specific technological changes are the objectives.

In defining and explaining an activity, one is called upon to describe the present technology (the current state of the art, how things are done now, improvements needed); the visualized technology (improvements in present procedures that will result from the research); research approaches (some of the questions

that must be answered for reaching the "technological fix"); character and magnitude of potential benefits; probability of success in the undertaking; and recommended research effort in scientific man-years.

SWC Division has 24 PPB activities which are listed below by title and with an identifying number. The titles indicate the subjects on which we are now working or the areas in which we say our research is now or will be making technological improvements:

011 - Predicting runoff and streamflow from agricultural watersheds.
012 - Recharging ground water for agricultural and urban uses.
013 - Compilation and reporting of hydrologic data.
014 - Prediction of sediment amounts and sources in watersheds and river basins.
015 - Improving control of reservoir sedimentation.
016 - Improving water erosion control and stream channel stabilization.
017 - Improved structures for water control and measurement.
018 - Increasing and conserving farm water supplies.
019 - Efficient irrigation and agricultural water use.
020 - Improved drainage systems for agricultural land.
021 - Reduction of salt damage to crops, soils and waters.
022 - Improving wind erosion control.
023 - Remote sensing for animal improvement, crop conditions, insect infestations, and soil-water-plant conditions.
024 - Control of pesticides in soil and water.
025 - Techniques for decontaminating soils polluted with radioactive materials or heavy metals.
026 - Handling and disposal of animal wastes to minimize soil, water, and air pollution.
027 - Control or prevention of soil and water pollution from fertilizers and agricultural processing wastes.
028 - Techniques for improving soil tillage and structure.
029 - Techniques for improving soil properties with amendments and conditioners.
030 - Techniques for efficient water use on nonirrigated crop and grass lands.
031 - Maintenance of soil fertility by use of fertilizers, crop residues, animal wastes, and better soil management practices.
032 - Methods to make beneficial changes in energy use in the soil-plant-atmosphere continuum (SPAC).

033 Biochemical control over the nutritional quality of plants.

034 Soil, plant, and animal factors affecting the concentration of essential and/or toxic trace elements in foods.

The above titles indicate our present lines of research in the Division. The work is carried on by some 985 full-time employees of which about 460 are professsional personnel (about 375 SMY) in a variety of disciplines and areas of specialization. Boundaries of the Division's seven regional Branches and the present location of research personnel are shown by Figure 1.

Our work on watershed modeling is encompassed in Research Activity 011, "Predicting runoff and streamflow from agricultural watersheds." The prime aim of this research activity is advancements of the technology for predicting the sequences of precipitation, runoff, and streamflow required for the design, evaluation, and allocation of costs and benefits of watershed projects. It is expected that several methods of varying degrees of complexity for predicting the required hydrologic sequences will result from this activity. The appropriate system model to be used will depend on the specific problem. The most detailed model is to provide a technology such that by using available information on the geology, soil, topography, and characteristics of the stream channel system one can predict the temporal and spatial sequences in the hydrology of a watershed under various land use patterns and treatments when subjected to a storm or series of precipitation events. The least detailed method will be a synthetic method or a statisticel prediction of flow sequences.

Several of the activities, in addition to 011, are potential sources of knowledge for attaining the objectives in watershed modeling, including: 012, 013, 014, 016, 018, 019, 020, 028, 029, 030, and 032. In addition, activities 015, 017, 024, 026, and 027 are also sources of information bearing upon some of the research needs mentioned by Mr. Kent. Activity 017, for instance, includes our research on spillways wherein Mr. Ree and his associates at Stillwater, Oklahoma, are working on earthen spillways, this need having also been recently emphasized by Messrs. Culp, Payne and Doubt during their participation in a meeting for review and planning of our structures research program.

One of the primary purposes of Activity 013, "Compilation and reporting of hydrologic data," is to enhance our capability for responding to an earlier specific request by SCS for publication of basic hydrologic data. Mr. Burford is moving forward in developing a Hydrologic Data Center at Beltsville, including an automated computerized system for preparing the data and associated information for publication in the "Black Book" series.

Let's take a look now at some of the questions for which we are seeking answers in our watershed modeling research as defined above. They are given below as they appear under the heading of "Research Approaches" in our writeup of Activity 011:

"4. Research Approaches:

A. Precipitation patterns and climatic factors influencing the hydrologic performance of watersheds.

(1) It is possible to formulate a probabilistic model for occurrence of rainfall or snow events of various magnitudes and intensities on watersheds of various sizes?

(2) Can such a model incorporate spatial as well as temporal variations in rainfall intensity?

(3) Can the parameters for such a model be estimated from data from rain gage networks?

(4) Can precipitation or snowfall distributions on watersheds be related to local topographic features?

(5) Can the applicability of models determined for experimental watersheds be extended by correlating parameters with more generally measured rainfall data?

B. Role of soils and vegetation in the hydrologic performance of upstream watersheds.

(1) What characteristics of soils have a significant effect on infiltration and seepage or lateral subsurface flow for continuous or intermittent rainfall?

(2) Can soils be grouped into hydrologic response units to permit the computation of rainfall excess as a distributed system in the watershed?

(3) What are the physical processes by which vegetation and land use affect infiltration and seepage or lateral subsurface flow?

(4) Can watershed evapotranspiration be predicted on a continuous basis to determine its effect on water balance and water yields?

C. <u>Aquifer-streamflow relationships in agricultural watersheds.</u>

(1) Under what geologic and climatic conditions will aquifer losses or accretions have on important effect on streamflow?

(2) Can land management practices be used to influence losses or accretions to streamflow?

(3) Can practical methods be developed for routing subsurface flows through geologic formations so that streamflow accretions or losses can be predicted?

(4) Can improved techniques be developed for estimating the storage and transmission characteristics of geologic formations?

D. <u>Hydrodynamics of channel systems in agricultural watersheds?</u>

(1) Can objective methods be developed for simplifying watershed geometry and estimating the parameters in the overland flow component of a watershed model?

(2) Can practical computational techniques be developed for solving unsteady, free-surface flow problems in channels of complex geometry with distributed or concentrated lateral inflow (or outflow)?

(3) Can reliable methods be developed to estimate conveyance parameters in natural streams?

E. Hydrologic system analyses.

(1) Can mathematical models representing hydrologic components be linked in the proper spatial and temporal configuration to predict surface and subsurface flows occurring as a response to historical or simulated precipitation inputs?

(2) How sensitive are streamflow parameters obtained from the general watershed model to variation of parameters of the individual components?

(3) Can regional watershed models be developed including those hydrologic components of greatest importance in that land resource area?

(4) What correlation techniques and simplified simulation models will be suitable for estimates of flood flows and water yields for use in design of low-cost, low-hazard projects?

(5) Can a watershed model be used as a tool in the design of agricultural practices to effect desired changes in runoff and streamflow?"

The questions as set forth above are not intended to be all-inclusive, but are indicative of the kinds of answers being sought, and a broad outline of a logical approach for evolution of the desired technology. Actually, of course, answers to specific questions such as those indicated above can become components of the new technology or can be inserted in the existing technology to the extent they result in a net improvement. Thus, some of these components of our research can be expected to provide specific answers to research needs as stated by SCS and, at the same time, satisfy new knowledge requirements essential for attaining the overall research objective.

Our current watershed hydrology research (Activity 011) is carried on by personnel at the following locations: Danville, Vermont; University Park, Pennsylvania; Beltsville, Maryland; Athens and Tifton, Georgia; Coshocton, Ohio;

Columbia, Missouri; Madison, Wisconsin; Sidney, Montana; Newell and Rapid City, South Dakota; Chickasha and Stillwater, Oklahoma; Temple and Riesel, Texas; Santa Rosa, New Mexico; Boise, Idaho; Lompoc, California; and Tucson, Arizona. The locations of research watersheds now being operated and those discontinued are shown on Figure 2. Though some of the current watersheds are operated for other purposes, records from them are also useful for watershed hydrology.

The size classification of watersheds where records of runoff are now being collected is as follows:

Class	Number
Less than 10 acres	71
10 to 100 acres	77
100 to 1,000 acres	95
1,000 to 10,000 acres	53
Greater than 10,000 acres	46
Total	342

In addition to records of runoff and streamflow, parameters of land use, soils, geology, topography, stream channel hydraulics, and climate are also obtained on many of these research watersheds. Thus, this mass of data constitutes the raw materials for research.

But the most important part of any research organization are the individual researchers who pose the specific questions, find the answers, and report their results. Not all of the relevant questions and answers require field data but they all require individuals with ideas and the determination to explore them.

ARS researchers have wide latitude for developing their individual research efforts. They are expected to know what needs to be done within the context of their assignment and to do it, within the limits of the resources available to them. They are rewarded according to their contributions of new knowledge, i.e., advancement of the technology.

In conclusion, I want to say again that we are fortunate in SWC-ARS to have SCS as a principal user of our research. Knowing that results of our research are put to use in solving

problems that need solving gives us a degree of personal satisfaction and a feeling that we have contributed something worthwhile. Furthermore, I am convinced that a mission-oriented identity may turn out to be a prime asset in these days of intense competition for dollars with which to do research. It behooves us then, as individual researchers, to sharpen our awareness of the real world and be sure there is some good reason for doing whatever it is that we are doing.

I am fully confident that the subsequent presentations will show that ARS is in the ball game and it may even turn out that someone will reveal an answer to a SCS research need.

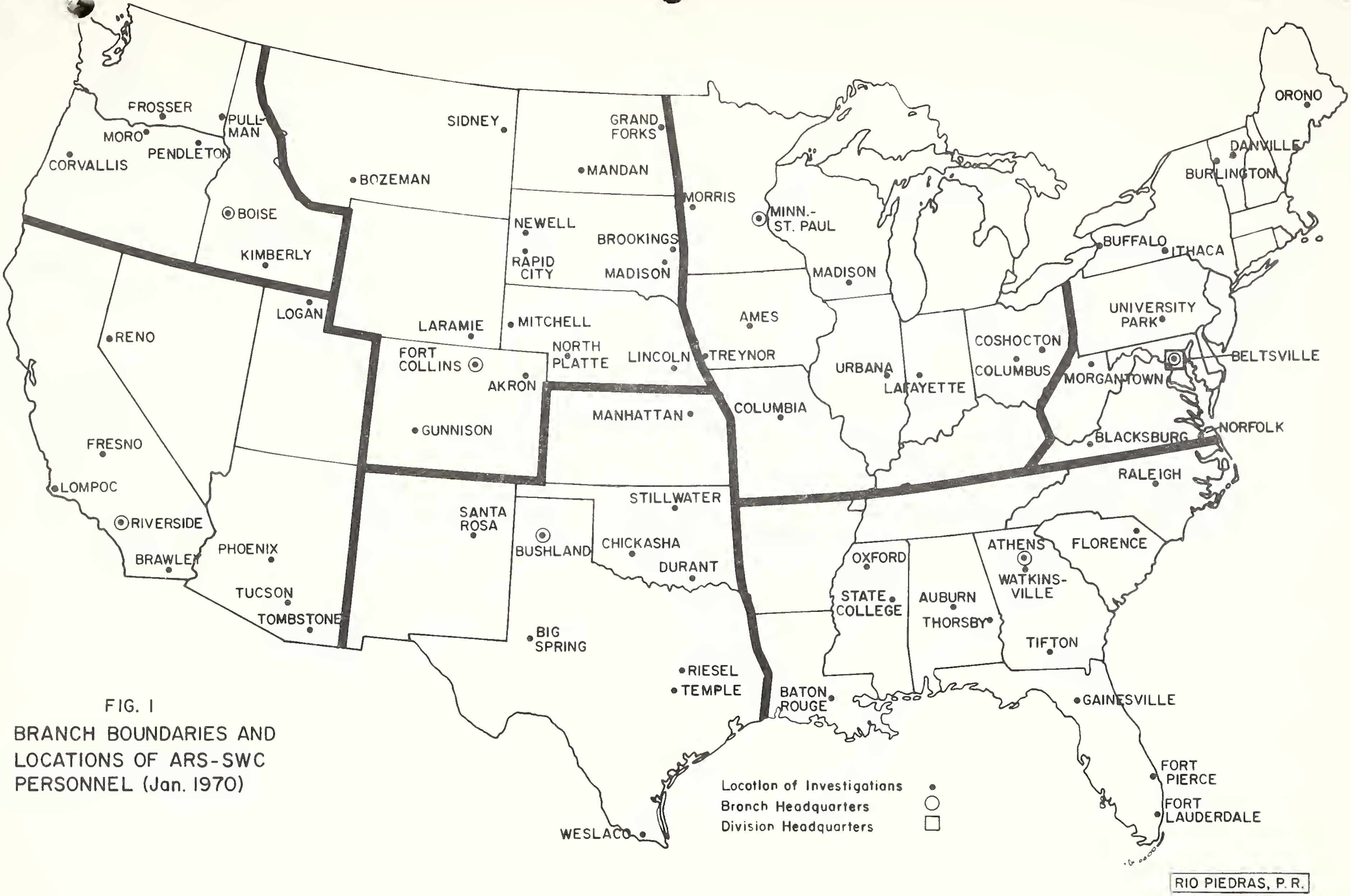

FIG. 1

BRANCH BOUNDARIES AND LOCATIONS OF ARS-SWC PERSONNEL (Jan. 1970)

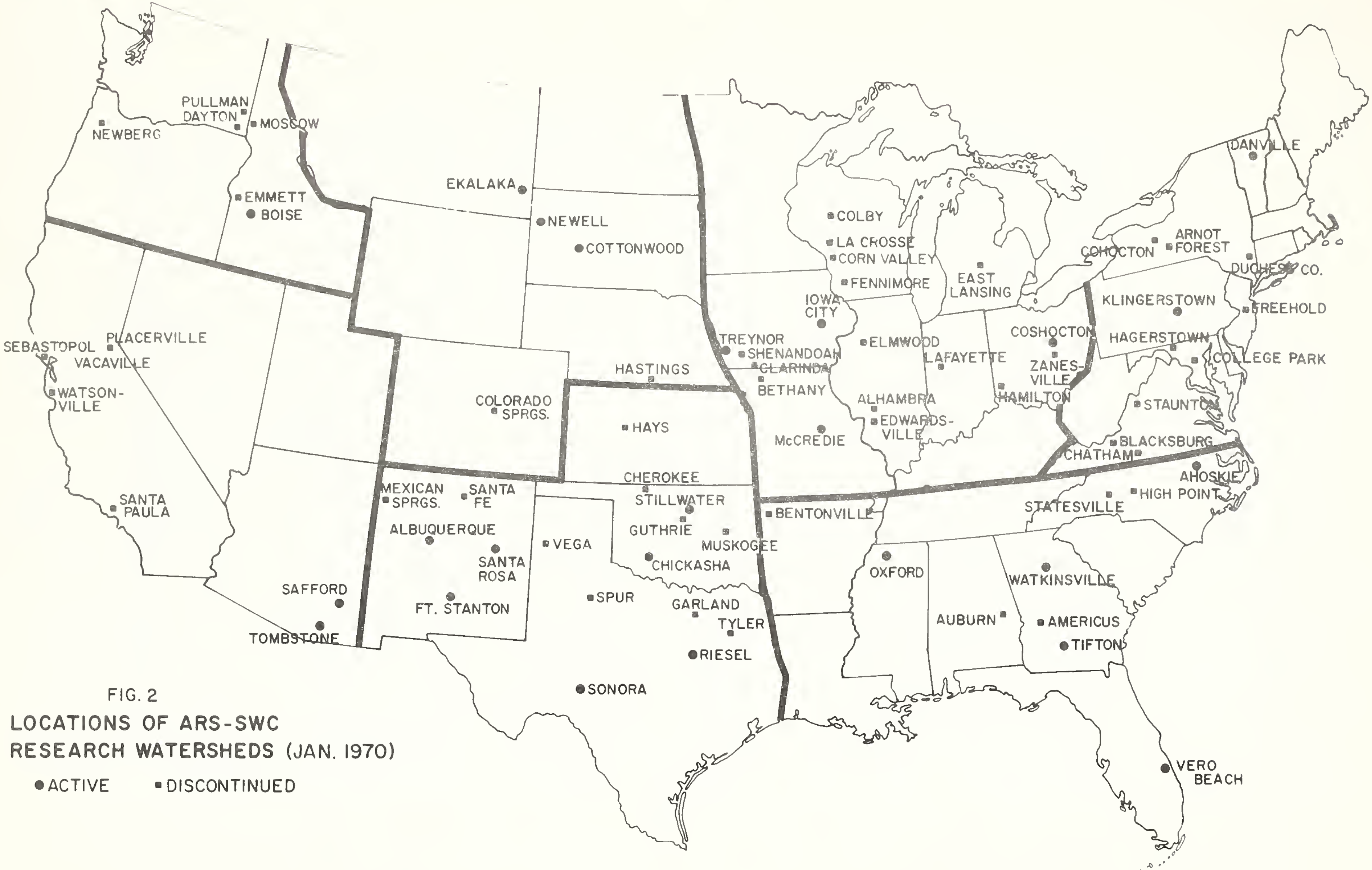

FIG. 2

LOCATIONS OF ARS-SWC RESEARCH WATERSHEDS (JAN. 1970)

● ACTIVE ▪ DISCONTINUED

WATERSHED MODELING FROM AN OPERATIONS VIEWPOINT[1]

R. E. Maclay[2]

It is a pleasure to have the opportunity to discuss my ideas of watershed modeling from the operations viewpoint with you folks from ARS. In spite of our efforts to the contrary, we in SCS probably tend to lag behind the state-of-the-art in modeling. Those of you who have been familiar with our model through the years realize it is still basically the same set of procedures we used for watershed planning before the advent of computers and it became fashionable to refer to it as modeling. Rather than extoll the virtues of our model, most of my comments will be addressed to some of the problems we experience with it, our ideas for improving it, and some suggestions about the direction we feel we should take on future efforts. Perhaps some of the later speakers already have the solutions to some of our problems.

A mathematical model can be described as a characterization of a process or concept, in terms of mathematics, which enables the relatively simple manipulation of variables to be accomplished in order to determine how the process or concept would behave in different situations.

The SCS runoff equation, with which you are all familiar, is an example of such a model. The variables affecting runoff can be altered to reflect the changes due to antecedent moisture condition, land use, soil type, etc. Although computing runoff from rainfall is an important part of our watershed planning process, it is only a part.

The need to determine streamflow throughout a watershed resulting from storms with both spatial and temporal variation means that we must consider models of an entire watershed or river system. We can think of a systems model as a body of information about a system gathered for the purpose of studying the system. Since the purpose of the study will will determine the nature of the information that is gathered, there is no unique model of a system.

1/ Contribution from the Engineering Division, Soil Conservation Service, Hyattsville, Maryland.

2/ Supervisory Hydraulic Engineer, Central Technical Unit, Hyattsville, Maryland.

The modeling can be of several types. One type that has found effective use in hydraulic problems is the physical model represented by an analog of a harbor, dam and reservoir, or an entire river system scaled to maintain static or dynamic similitude. The Corps of Engineers Waterways Experiment Station in Vicksburg, Mississippi has many such models. Bill Ree at Stillwater has modeled smaller structures including some of the ones the ARS personnel will be seeing later this week at Walnut Gulch. Closely related to these models are electric analogs which express the relationship and results in terms of appropriately scaled continuous electrical units. Erlend Warnick reports on one such application of this type later in the meeting. The third type, and the one having the most wisespread use today, is the digital model, requiring the use of a high-speed digital computer. It is conceivable that all three types could be used in one watershed.

Figure 1 in Mr. Kent's paper schematically represents the SCS model for watershed project evaluation. A large part of the computational activities represented in the figure are presently being done using four large-scale computer programs.

1) The hydraulic program computes water surface profiles for selected spatially varied discharges in natural channels and computes the floodplain area flooded by the discharges in the stream reach represented. The results are available as punched cards to be used as input data to other programs.

2) The hydrology program develops sub-area hydrographs of runoff from selected storms, floodroutes them through the appropriate stream reaches and reservoirs, and combines the routed flows with other hydrographs in the proper time sequence to form the composite hydrograph of flow. Output from this program also is used as input to another.

3) The dams program develops design inflow hydrographs and floodroutes them through reservoirs with various combinations of principal spillway and floodwater and surcharge storage. The planning engineer can then use this information for selecting the optimum combinations to meet the requirements for use at the site.

4) The economics program takes the output from the other programs, which is stated in physical terms, and computes the average annual monetary floodwater damages in each evaluation reach.

The model is deterministic in that the output is completely described in terms of its input. It contains the empiricisms and short cuts that were necessary when the work was done manually. The computer is being used as a high-speed calculator and we are not using it to its full potential. For example, we still use coefficient floodrouting techniques that will forever cause problems when trying to replicate the unsteady flow condition caused by passage of a flood hydrograph by using water surface profiles based on steady flow conditions. Also, the inclusion of some stochastic processes could make the model more representative of real-life conditions. It is well known that bridges tend to clog with ice and debris even though time and location may not be predictable. With the proper probability distribution, this random effect on flood damage computation could be considered. Many analytic techniques are available that require the use of a computer that we might use to good advantage.

Those of us who have been involved with the development of these programs have insisted that there be sufficient flexibility for the user to have detailed output options that can generate output in volumes unheard of with manual computations. However, since the purpose of the computation is to gain insight into the problem and not just a voluminous report, we tend to get discouraged when we see computer output that can be measured by the side foot and referred to, not so facetiously, as hernia reports.

To understand why the computing time and output can become excessive, it might be helpful to look briefly at how the planners use the programs.

The procedures as programmed are relatively straight-forward. Although certain updating of data and rerunning of water surface profiles is possible, it is basically a one-pass procedure. The exception would be changing a channel segment and rerunning for channel improvement. The output is then used in the hydrology program. Normal practice with the hydrology program is to develop hydrographs and do the floodrouting to determine the peak discharge at selected points in the watershed for without-project conditions. A selected system of reservoirs is then imposed on the watershed and the routing is repeated. The reduction in peak discharge, as translated to monetary terms in the economics program, is one measure of the benefits to be compared with the costs for that

particular system. This procedure is repeated for as many alternate systems as the planners wish to evaluate. Although there is no program limit on the number of alternates, each pass yields results only for that particular system. Considering 10 alternates produces 10 times as much output as one alternate. Even a small project with 10-20 structures can have many more possible combinations than it is practical to evaluate, even using a computer. Computing time rises spectacularly.

One partial solution to the problem of excessive computing time and output is to utilize a technique called holdout routing. The without-project discharges must be known or computed at desired points in the watershed. The holdout hydrograph of a structure is the difference between the inflow and outflow hydrograph. This holdout is then routed through the stream reaches like any other hydrograph except that no local-area hydrographs are developed and added to it. At any point the holdout can be subtracted from the without-project hydrograph to show the effect of that structure at that point. This procedure reduces the amount of floodrouting necessary to study alternate solutions.

The program is structured so that the input includes a standard control which is the sequential list of operations that develops hydrographs, floodroutes, combines hydrographs, etc. as well as the executive control that indicates what flood is to be routed through which portion of the standard control.

Based on his experience with the holdout routing method on the Wabash River while he was in Indiana and subsequent studies in the Central Technical Unit, Robert M. Pasley has concluded that there is about a 3 to 1 reduction in required computation steps through the standard control lists. This cuts down considerably on the computing time and volume of output. The holdout routing can be handled in the present operating program but the various holdouts still have to be subtracted afterwards. Indiana SCS personnel have written a program that selects and combines the stored holdouts as a separate step. Rather than incorporate that step in our program, we are presently investigating the possibilities for assigning dollar benefits to the structures as both a first and last increment and then optimizing the system by linear or non-linear programming techniques. Some of you may have some suggestions to offer.

Even with some improvements in the model as suggested, we must still remember that the model only permits evaluation of floodwater damage. With each change in our enabling legislation and the obvious climate in the country for considering

all phases of water resource development and its effect on our environment, benefits from reduction in floodwater damages become a smaller and smaller part of a watershed study.

I mentioned previously that there is no unique model of a system. Our model is a great time saver for its intended function, in spite of the aforementioned shortcomings. However, it is of no value for predicting watershed yield for low flow augmentation, municipal and industrial storage, etc. To use our runoff equation to estimate direct runoff from 24-hour duration rainfall has proved adequate but it is not suited to a model for computing yield made up of both subsurface and surface flows and where a complete water accounting is needed. The ARS Hydrograph Laboratory, under Heggie Holtan's direction, is working on one such model that we believe has considerable promise. We hope to explore its potential with Heggie in the near future.

However, in spite of the recognized need for sophisticated models for other phases of water resource development, we can't lose sight of the data requirements for calibration of the models. Stringent data requirements that might easily be met with plot data may be very difficult or impossible to attain on entire watersheds.

Ultimately the answer may be a shift from deterministic to stochastic models that derive their imputs from regionalized data. Using these regionalized parameters, the models can generate data and using simulation techniques can evaluate an ungaged watershed for a given evaluation period. The USGS is presently working on this problem and have reported on their findings in an open file report entitled "Generalization of Streamflow Characteristics from Drainage Basic Characteristics." I will leave it to the statisticians to pass judgment on their particular technique but their general approach may be something for us to consider.

We have on file throughout the states the basic data and evaluation on about 1500 planned watersheds. Most of this data is filed, never to be used again. It seems to me that we should be able to make some use of all this data in arriving at regionalized parameters. Our watersheds certainly run the gamut of changes in climate, land resource areas, etc. However, a critical review must be made of the data to determine its validity for future use. The cliche about garbage-in garbage-out still has merit.

I have mentioned some of the aspects of our digital model and Erlend Warnick will later discuss an analog model. Before closing it might be appropriate to discuss briefly the potential for combining the best of both types of modeling in our work using hybrid computers.

The digital computer is a sequential machine. Regardless of its size or speed, it must break even simple problems into small segments to be calculated one step at a time, the results stored, and recalled for later use. The analog computer elements are arranged on a panel much in the same pattern as the physical system they represent. Since the analog performs parallel computations, it solves the different parts of complex, multi-variable problems at the same time.

Modeling of dynamic physical systems is more difficult on digital computers but its data handling capabilities and output documentation are far superior to the analog. With the analog, there is complete freedom to modify the model and observe the effects on the entire system. This would be particularly useful for evaluating alternate structure locations.

The hybrid computers combine the best features of the digital and analog. Those of you who attended the ASCE Hydraulics Conference in Boston and Logan may recall that several applications were reported, each showing significant reduction in computing time and costs. It is difficult to generalize but apparently reduction of 100:1 in computing cost and 20 to 30:1 in project costs are commonplace. The hybrid computer may play an important role in our future efforts.

In closing I would say only that we in operations feel that there is a tremendous future for modeling and simulation in our expanding role of water resource development. We have only scratched the surface of utilizing its full potential. We hope we have learned some lessons by walking before we tried to run. And certainly we look forward to tapping the ARS reservoir of knowledge and experience in this field.

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Those of us who have been involved with the development of these programs have insisted that there be sufficient flexibility for the user to have detailed output options that can generate output in volumes unheard of with manual computations. However, since the purpose of the computation is to gain insight into the problem and not just a voluminous report, we tend to get discouraged when we see computer output that can be measured by the side foot and referred to, not so facetiously, as hernia reports.

To understand why the computing time and output can become excessive, it might be helpful to look briefly at how the planners use the programs.

The procedures as programmed are relatively straight-forward. Although certain updating of data and rerunning of water surface profiles is possible, it is basically a one-pass procedure. The exception would be changing a channel segment and rerunning for channel improvement. The output is then used in the hydrology program. Normal practice with the hydrology program is to develop hydrographs and do the floodrouting to determine the peak discharge at selected points in the watershed for without-project conditions. A selected system of reservoirs is then imposed on the watershed and the routing is repeated. The reduction in peak discharge, as translated to monetary terms in the economics program, is one measure of the benefits to be compared with the costs for that

particular system. This procedure is repeated for as many alternate systems as the planners wish to evaluate. Although there is no program limit on the number of alternates, each pass yields results only for that particular system. Considering 10 alternates produces 10 times as much output as one alternate. Even a small project with 10-20 structures can have many more possible combinations than it is practical to evaluate, even using a computer. Computing time rises spectacularly.

One partial solution to the problem of excessive computing time and output is to utilize a technique called holdout routing. The without-project discharges must be known or computed at desired points in the watershed. The holdout hydrograph of a structure is the difference between the inflow and outflow hydrograph. This holdout is then routed through the stream reaches like any other hydrograph except that no local-area hydrographs are developed and added to it. At any point the holdout can be subtracted from the without-project hydrograph to show the effect of that structure at that point. This procedure reduces the amount of floodrouting necessary to study alternate solutions.

The program is structured so that the input includes a standard control which is the sequential list of operations that develops hydrographs, floodroutes, combines hydrographs, etc. as well as the executive control that indicates what flood is to be routed through which portion of the standard control.

Based on his experience with the holdout routing method on the Wabash River while he was in Indiana and subsequent studies in the Central Technical Unit, Robert M. Pasley has concluded that there is about a 3 to 1 reduction in required computation steps through the standard control lists. This cuts down considerably on the computing time and volume of output. The holdout routing can be handled in the present operating program but the various holdouts still have to be subtracted afterwards. Indiana SCS personnel have written a program that selects and combines the stored holdouts as a separate step. Rather than incorporate that step in our program, we are presently investigating the possibilities for assigning dollar benefits to the structures as both a first and last increment and then optimizing the system by linear or non-linear programming techniques. Some of you may have some suggestions to offer.

Even with some improvements in the model as suggested, we must still remember that the model only permits evaluation of floodwater damage. With each change in our enabling legislation and the obvious climate in the country for considering

all phases of water resource development and its effect on our environment, benefits from reduction in floodwater damages become a smaller and smaller part of a watershed study.

I mentioned previously that there is no unique model of a system. Our model is a great time saver for its intended function, in spite of the aforementioned shortcomings. However, it is of no value for predicting watershed yield for low flow augmentation, municipal and industrial storage, etc. To use our runoff equation to estimate direct runoff from 24-hour duration rainfall has proved adequate but it is not suited to a model for computing yield made up of both subsurface and surface flows and where a complete water accounting is needed. The ARS Hydrograph Laboratory, under Heggie Holtan's direction, is working on one such model that we believe has considerable promise. We hope to explore its potential with Heggie in the near future.

However, in spite of the recognized need for sophisticated models for other phases of water resource development, we can't lose sight of the data requirements for calibration of the models. Stringent data requirements that might easily be met with plot data may be very difficult or impossible to attain on entire watersheds.

Ultimately the answer may be a shift from deterministic to stochastic models that derive their imputs from regionalized data. Using these regionalized parameters, the models can generate data and using simulation techniques can evaluate an ungaged watershed for a given evaluation period. The USGS is presently working on this problem and have reported on their findings in an open file report entitled "Generalization of Streamflow Characteristics from Drainage Basic Characteristics." I will leave it to the statisticians to pass judgment on their particular technique but their general approach may be something for us to consider.

We have on file throughout the states the basic data and evaluation on about 1500 planned watersheds. Most of this data is filed, never to be used again. It seems to me that we should be able to make some use of all this data in arriving at regionalized parameters. Our watersheds certainly run the gamut of changes in climate, land resource areas, etc. However, a critical review must be made of the data to determine its validity for future use. The cliche about garbage-in garbage-out still has merit.

I have mentioned some of the aspects of our digital model and Erlend Warnick will later discuss an analog model. Before closing it might be appropriate to discuss briefly the potential for combining the best of both types of modeling in our work using hybrid computers.

The digital computer is a sequential machine. Regardless of its size or speed, it must break even simple problems into small segments to be calculated one step at a time, the results stored, and recalled for later use. The analog computer elements are arranged on a panel much in the same pattern as the physical system they represent. Since the analog performs parallel computations, it solves the different parts of complex, multi-variable problems at the same time.

Modeling of dynamic physical systems is more difficult on digital computers but its data handling capabilities and output documentation are far superior to the analog. With the analog, there is complete freedom to modify the model and observe the effects on the entire system. This would be particularly useful for evaluating alternate structure locations.

The hybrid computers combine the best features of the digital and analog. Those of you who attended the ASCE Hydraulics Conference in Boston and Logan may recall that several applications were reported, each showing significant reduction in computing time and costs. It is difficult to generalize but apparently reduction of 100:1 in computing cost and 20 to 30:1 in project costs are commonplace. The hybrid computer may play an important role in our future efforts.

In closing I would say only that we in operations feel that there is a tremendous future for modeling and simulation in our expanding role of water resource development. We have only scratched the surface of utilizing its full potential. We hope we have learned some lessons by walking before we tried to run. And certainly we look forward to tapping the ARS reservoir of knowledge and experience in this field.

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No. 4

WATERSHED MODELING FROM A RESEARCH VIEWPOINT[1]

H. N. Holtan[2]

A model, if I may paraphrase Webster's definition, is a copy or imitation in miniature or it is a standard for emulation. Watershed hydrologists use models as vehicles for integrating the best of our knowledge into a useful system. In a multidisciplinary organization such as ARS, elements of models range from purely random probabilities and abstract reasoning to the rigidly deterministic procedures of dimensional analyses. In the action agencies we try to serve, limitations on input availability or the economics of the project at hand may preclude certain procedures and levels of sophistication. Our models must therefore provide the speed and ease of computation commensurate with project levels encountered in operations, but at the same time encompass alternatives for the insertion of basic knowledge essential to improvement.

We in research have been slow to respond to this challenge. In fact, since the inception of the SCS in the early 1930's research has provided the basic knowledge but the integration into a useful system has been a function of operations engineers. Consequently, we have more or less lagged behind the agencies we try to serve. From plots to small homogeneous watersheds, to larger complex tributaries and now to river basins, we have followed the progression of SCS from field planning to farm planning, to flood control, to comprehensive development of drainage basins for the best use of soil and water resources. By the time we had gotten geared to fill the needs of field planning, SCS had moved on to larger areas and we were left with the job of integrating our small plot information for complex watershed predictions. In flood control computations, research has provided good techniques for computing probability of flood peaks, but now water yields are equally important and operation agencies are improvising their own techniques for estimation. *Is it the role of research to follow or should we design our studies out ahead of current practice?*

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[1]Contribution from the Soil and Water Conservation Res. Div., Agricultural Research Service, Beltsville, Md.

[2]Res. Hydraulic Engineer and Director, USDA Hydrograph Laboratory, Beltsville, Md.

We in watershed engineering have a responsibility to keep an eye on advancements in related fields of endeavor to better anticipate the directions we might take in our research. I quote from an earlier report[3] ..... *Objectives in watershed engineering change in pace with population increases and with our desire for a higher, more controlled standard of living. We depend upon our watersheds for the necessities of life as well as for the enjoyment of our leisure. Our way of life demands a regulated environment protected against the extremes of nature; we want neither flood nor drought. Recently, we are becoming concerned with the safe disposition of sediments and waste materials from our watersheds[4]. Dispersed wastes such as fumes, smoke and other airborne pollutants from industry and inadvertent excesses of applied fertilizers, herbicides and pesticides in agriculture suggest the propriety of a dispersed system concept of watershed hydrology as a vehicle of disposal. Predictably then, our interest is extending to smaller and smaller increments of the watershed with complete accounting for dispositions of precipitation to evapotranspiration, moisture storage, ground water recharge, and surface and subsurface movements to streamflow.*

We have models such as the *rainfall-runoff relationship* concept which consider the watershed as a unit of hydrologic response; these are called *lumped systems*. We have other models referred to as *dispersed systems* which chronologically route flows vertically and horizontally through the watershed. Expedience may be on the side of lumped systems but the spatial and temporal sequencing usual to the dispersed system concept provide a better nesting place for new knowledge and a better insurance of continued progress in watershed engineering. Lumped systems are particularly prone to derivations of one set of parameter values for one purpose, such as water yields, and an entirely different set of values for the same parameters for a different purpose such as flood peak predictions. A stronger position would be to build a comprehensive dispersed model first and rely upon it for the development and checking of short-cut procedures.

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[3]Holtan, H. N. and Lopez, N. C. An engineering-oriented model of watershed hydrology. Proc. Internatl. Seminar for Hydrology Professors, Urbana. July 1969.

[4]Wadleigh, C. H. Wastes in relation to agriculture and forestry. U.S. Dept. Agr. Misc. Pub. 1065, 112 pp., 1968.

The builder of a dispersed system model is continually faced with the problem of advancing the model in spite of gaps in knowledge of sequential phenomenon. He has the choice of rationalizing an empiricism or of empiricizing a rationale. Empiricisms are expedient and do permit progress in the rest of the model, but sometimes they provide little knowledge other than that pertaining to the boundary conditions attending a particular phenomenon. On the other hand empiricizing a preconceived rationale sometimes leads to years of search for a condition or a media pure enough to provide the needed proof; and then we are still faced with the job of adapting to the real situation. The purity of glass beads or of laboratory packed homogeneous soils eludes the watershed hydrologist, but it is my belief that empiricisms can be used to isolate phenomena and their boundary conditions for rationalization. Both approaches - empirical and rationale - have a utility in modeling watershed hydrology since they are interdependent in their verification. Modeling without data is purely a mathematical gymnastic, but empiricisms unsupported by dimensional analyses give risky extrapolations beyond the experienced range of the data.

The modeler is also faced with the choice of stochastic versus deterministic concepts. Generally, hydrologists tend to apply probability to a phenomenon such as precipitation which is still beyond reliable prediction or management by man and to apply deterministic techniques to infiltration, evapotranspiration and hydraulics of surface flow which are highly responsive to agronomic and engineering practices on the watershed. Since action agencies are designing for the future, probability is essential to their computations; however, it would be a poor service for research to study the application of random theory to a manipulated process.

Watershed engineering in agriculture is closely allied with crop and soil sciences. Rainfall, infiltration, evapotranspiration, the hydraulics of surface runoff, erosion,. and flows through porous media are factors on agronomic and engineering decisions such as grazing management, cropping sequences, species, tillage practices, width of strips, terrace systems and land drainage. The soil survey of the SCS has contributed much that is useful in the dispersed system concept of watershed hydrology. The hydrologic grouping of soils in accordance with their infiltration capacities after prolonged wetting is well established in hydrologic modeling concepts in the USDA. It has been used empirically by SCS hydrologists in evaluating the effects of land use and treatment upon the hydrologic

response of a watershed. More recently, we in ARS have been studying land capability classes as hydrologic response units within the watershed.

Land capability classes incorporate many hydrologic considerations. They provide a best estimate of probable future use of watershed lands. Together with the already useful hydrologic grouping, land capability defines land areas having similar infiltration capacities, moisture retentions and evapotranspiration within the watershed. The usefulness of land capability classes in the dispersed concept of watershed hydrology will be discussed by England later in this Workshop. The point I wish to make here is that modeling, if attempted on a comprehensive basis, provides a nesting place for knowledge from many disciplines.

Comprehensive modeling is a fearful concept; we can never achieve it fully, or we will bog down in detail. It appears then that we can neither abide it in operations nor progress without it in research. The answer we have attempted in the USDA Hydrograph Laboratory is a structured model of subroutines which may be called or deleted by a mainline program in accordance with the level of the project at hand. Subroutines include infiltration, moisture accretion, evapotranspiration, subsurface return flows, overland flows and channel flows. Zones of land capability classes are treated in their elevation sequence with surface flows cascading or not cascading as indicated by adjacency, by terraces or other diversions of flow. Subroutines are called in chronological order within each zone.

In our experience with the IBM[5] 360-30 of ARS at Beltsville, execution time for comprehensive computations including soil moisture, ET, overland flow, and subsurface flow on each zone for an 8-year period required two to three hours computer time. But, computer time was considerably less for computations of surface yield volumes only, without hydrographs or subsurface return flows. If flood peaks are the only interest, the analyst can enter the program a month or so in advance of a major storm, thereby deriving antecedent conditions and the flood hydrograph in a matter of minutes.

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[5]Mention of a specific company is made solely for information and is not an endorsement or recommendation by the United States Government, U.S. Department of Agriculture, or the USDA Hydrograph Laboratory.

We hope to provide expedience for operations and segmentation for greater depth in research. As better subroutines are developed they will be adapted as alternatives. The user will choose a sequence of best estimates commensurate with available input and the level of his project needs; and research will have another channel for funneling results of specialized or abstract research into utility.

Research is heavily dependent upon replication and the experienced range of variation for the significance of its findings. It is seldom possible to study all phenomenon in the hydrologic sequence to the same depth of exhaustion at a given location. We all do our best work with those factors of greatest variation; But, if we want to put them together into a sequence for watershed hydrology, we are forced to draw upon less substantiated techniques for other segments in the sequence. A model composed of best predictors from over the Nation is an aid to the local researcher who wishes to test his specialty in a most beneficial context. Such a model also serves as a guide to research needs with some of the attending boundary conditions. Operations can see more clearly where we are headed in a comprehensive model and can anticipate possibilities for improvements in watershed engineering. It is the responsibility of operations agencies to develop shortcuts for expedience; Research has the responsibility of providing the edification required for security. This is a two-way exchange reliant upon media of communication. The model offers many possibilities in this respect.

Irwin D.J. Bross[6] expressed his sentiments on models very concisely.... *A big advantage of a model is that it provides a frame of reference for consideration of the problem. This is often an advantage even if the preliminary model does not lead to successful prediction. The model may suggest informational gaps which are not immediately apparent and consequently may suggest fruitful lines for action. When the model is tested the character of the failure may sometimes provide a clue to the deficiencies of the model. Some of the greatest scientific advances have been produced by <u>failure</u> of a model!*

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[6]Bross, Irwin D. J. Design for Decision. The Macmillan Company, N.Y. 1953.

Definition of Terms

Watershed modeling has become, in recent years, a subject that occupies the attention of hydrologists and engineers concerned with the behavior of watersheds and their response to human activities. The purpose of the joint workshop on watershed modeling is to study and explore the available knowledge and evaluate the needs in this branch of scientific hydrology. Models and modeling, as well as systems which the models represent, are terms which are extensively used and may have different meanings to different people. To provide a uniform basis for the discussions in the workshop, a few definitions are offered below.

System - A system may be considered to be an ordered assembly of interconnected elements that transform, in a given time reference, certain measurable inputs into measurable outputs. Inputs and outputs are usually represented as functions of time. These functions may be continuous or discrete.

Element - The elements of a system can be regarded as subsystems, with each element having its own inputs and outputs. The outputs of some elements may become inputs to other elements. The arrangement and relative positions of the various subsystems, showing the flow of inputs and outputs of all elements, constitute the structure of the system.

Notation - If the input to a system is denoted by $x_{(t)}$ and the output of the system is denoted by $y_{(t)}$, the transformation performed by the system is represented by:

$$y_{(t)} = \Phi\,[x_{(t)}]$$

where Φ denotes the operation or operations required to carry out the transformation. The operator Φ may involve simple mathematical manipulations or it may represent a flow chart containing both mathematical and logical operations.

Classification - Systems may be classified according to a number of properties. The most important classification pertains to whether the systems are linear or nonlinear, time invariant, or time variable, and

lumped or continuous. The above classification applies also to individual elements or subsystems so that it is possible to have a nonlinear system with linear elements in it, etc.

Linear Systems - Systems are said to be linear if they conform to the principle of superposition with regard to their input and output. The principle is demonstrated by considering the responses, $y_{1(t)}$ and $y_{2(t)}$, of the system to two distinct inputs, $x_{1(t)}$ and $s_{2(t)}$, where

$$y_{1(t)} = \Phi\ [x_{1(t)}] \text{ and } y_{2(t)} = \Phi\ [x_{2(t)}]$$

The system is said to be linear if its response $y_{3(t)}$ to an input $x_{3(t)}$, which is a weighted sum of the original inputs

$$y_{3(t)} = \Phi\ [x_{3(t)}] = \Phi\ [A\ x_{1(t)} + B\ x_{2(t)}]$$

is equal to the weighted sum of the original outputs obtained when the inputs were acting independently, i.e., the system is linear if

$$y_{3(t)} = A\ y_{1(t)} + B\ y_{2(t)}$$

where A and B are the weighting factors having constant magnitudes.

Time-Invariant - A system is said to be time-invariant if its response is independent of the time at which the input has occurred. The operator Φ of a time-invariant system and parameters that are used to describe the operator do not change with time. The input-output relationship for a time-invariant system is subject to the following statement. If the response system to an input starting at time $t = o$ is

$$y_{(t)} = \Phi\ [x_{(t)}]$$

then the output due to the same input starting at any other time t_o

$$z_{(t)} = \Phi\ [x_{(t-t_o)}]$$

is given by

$$z_{(t)} = y_{(t-t_o)}$$

regardless whether there were other inputs in the time interval from $t = o$ to $t = t_o$.

Lumped System - A system is said to be lumped if its inputs and outputs are concentrated at a finite number (usually a small number) of points. These points serve also as points of separation of some of the subsystems or elements of which the system is composed. The behavior and output of a lumped system depends on the relative locations of the points of input or output within the system, but not on their absolute location in space.

Models - Models are simplified systems that are used to represent real life systems and may be used as substitutes of the real systems for certain purposes. The models express formalized concepts of the real systems.

Use of Models - The main uses of models are: (a) for the understanding of the behavior of real life systems; and (b) for the prediction of outputs to be expected from these systems as a result of specified sets of inputs and of selected variations of inputs. Models may also be used to study the effects of proposed changes in the structure of real systems.

Understanding the behavior of the real systems is achieved by the comparison of outputs predicted by the model to observed outputs of the real system for specified inputs. Close agreement is taken as substantiation of the formalized concepts about the real system that were used in the construction of the model.

Model Usefulness - Since models are used to represent systems that are defined in relation to specified inputs and outputs, their usefulness does not usually extend beyond representing the relationship between these inputs and outputs. Models developed for one set of inputs and outputs could not be used to represent additional input-output relationships without altering their structure, usually making them more complicated.

Model Design - Models used mainly for prediction purposes should be constructed with a view of the objectives of these predictions. Normally they would be required to be the simplest models that will produce reliable estimates of the predicted variables, consistent with the quality of the original data available for the prototype.

Watershed Models - Watershed models are models that represent one or more of the processes that take place in natural watersheds. The simplest watershed model would represent the relationship between one type of input and one type of output. More complicated models would have more than one type of input or output, or both.

NOTE: The above definitions were developed in a preworkshop meeting held in Tucson, Arizona on December 19, 1969 in which the following persons took part:

D. L. Brakensiek - Chief Engineer, Beltsville, Md.
K. G. Renard - Research Center Director, Tucson, Arizona
D. L. Chery - Tucson, Arizona
D. G. DeCoursey - Chickasha, Okla.
M. H. Diskin - Tucson, Arizona
W. M. Snyder - Athens, Georgia
D. A. Woolhiser - Ft. Collins, Colorado

A draft of the definitions was circulated to the above individuals, and their comments were taken into consideration in the preparation of the final form presented herein.

Prepared by: M. H. Diskin
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USDA, ARS, SWC, Tucson, Arizona

STOCHASTIC MODELING 1/

Donn G. DeCoursey 2/

The word stochastic comes from the Greek word, stochastikos, meaning skillful in aiming but is defined as random. Specifically, it involves a variate at each moment of time. In application to the field of hydrology it generally refers to the random nature of hydrologic phenomena such as river flow, precipitation, wind velocity, etc.

In the discussion that follows, stochastic modeling means the sequential generation of hydrologic information. Monte Carlo methods, which refer to a process by which data are produced synthetically by a sampling technique or some form of a random-number generator, are used for this purpose.

Simple Monte Carlo Procedure

The simplest example of a stochastic process is the simple random walk shown in figure 1 where X_n is a random variable denoting the position at time n of a moving particle. The movement of the particle is either upward with a probability of .5, $Z_n = 1$, or downward with a probability of .5, $Z_n = -1$; i.e.,

$$\text{prob}(Z_n = 1) = \text{prob}\ (Z_n = -1) = .5 \tag{1}$$

The location of the particle is thus:

$$X_n = X_{n-1} + Z_n \tag{2}$$

where Z_n is the movement at each time interval.

The simple random walk briefly described above is a particular kind of Markov process. A Markov process is one in which the location of a point is dependent only upon the probability density function, p.d.f., of the variate at the previous point. Thus, p.d.f.'s conditional on values at two or more time points away need never be considered. As an example, consider the following two-state Markov chain. The Markov chain is the succession of p.d.f.'s occurring as a result of a Markov process.

1/ Contribution from the Soil and Water Conservation Research Division, Southern Plains Branch, Agricultural Research Service, USDA, in cooperation with the Oklahoma Agricultural Experiment Station.

2/ Research Hydraulic Engineer, USDA, Chickasha, Oklahoma

In studies of rainfall it has been found that the two-state Markov chain gives good results in certain areas; see Gabriel and Neumann (1) and DeCoursey and Seely (2). Consider a dry state, 0, and a wet state, 1, with α being the probability of a wet day following a dry day and $1-\alpha$ being the probability of a dry day following a dry day. The probability of a dry day following a wet day is β and $1-\beta$ is the probability of a wet day following a wet day. This is shown in the following matrix of transition probabilities.

$$P = \begin{array}{cc} & \begin{array}{cc} \multicolumn{2}{c}{\text{Future State}} \\ \text{dry} & \text{wet} \\ 0 & 1 \end{array} \\ \text{Present State} \begin{array}{c} \text{dry } 0 \\ \\ \text{wet } 1 \end{array} & \begin{bmatrix} 1-\alpha & \alpha \\ & \\ \beta & 1-\beta \end{bmatrix} \end{array} \qquad (3)$$

By using relative frequencies over a period of about 27 years, Gabriel and Neumann found in a study of Tel Aviv rainfall data the following transition probabilities

$$P = \begin{bmatrix} .750 & .250 \\ .338 & .662 \end{bmatrix}$$

Using these transition probabilities and a random sampling method, the occurrence or nonoccurrence of an event may be calculated.

Multistate Markov chains of higher order have been proposed and used in some instances. Pattison (3), for example, used a sixth order chain for distributing the total volume of rainfall over storm periods. However, the length of record needed to develop high order Markov chains is generally not available for hydrologic data. A good reference for such chains and for stochastic processes in general is the book <u>The Theory of Stochastic Processes</u> by Cox and Miller (4). Parzen (5) also has a good book on the subject of stochastic processes.

The time series can take several forms depending upon the hydrologic process being described and can vary from the very erratic form representative of daily rainfall to a fairly smooth form representative of monthly evaporation. Several generating methods are available for modeling these processes. The choice of the method is based upon how well the mathematical structure of the method conforms to the characteristics of the hydrologic process being modeled. The most common

methods are: (1) The moving average, (2) the sum of harmonics, and (3) the autoregression. Emphasis of this discussion will be placed on the last method although the other two methods will be discussed briefly first. A tool quite often helpful in these analyses is the correlogram. It is described following the discussion on the sum of harmonics method of modeling.

The Moving Average

The moving average method can be represented by the equation

$$y_i = b_o + b_1 x_i + b_2 x_{i-1} + \cdots + b_m x_{i-(m-1)} \tag{4}$$

where y_i is the value of the process being modeled at the i^{th} time period, x is a variable, and m is the extent of the moving average. In the above equation y could be annual runoff and x the effective precipitation. In this case, the coefficients, b, must be positive and sum to unity. A time series generated by such a scheme is not random; however, events separated by m or more time units are independent. Julian (6) used the moving average method in studying stream flow in the Colorado River Basin.

The Sum of Harmonics

The time series of many hydrologic processes are distinguished by obvious periodicities. This suggests the possibility of using Fourier series in the mathematical representation of the series. The basic form of the most general equation is given by

$$x_{(t)} = \bar{x} + \sum_{k=1}^{S} (A_k \cos \lambda_k t + B_k \sin \lambda_k t) + \varepsilon_t \tag{5}$$

where

$$A_k = 2/n \sum_{t=1}^{n} x_t \cos \lambda_k t, \tag{6}$$

and

$$B_k = 2/n \sum_{t=1}^{n} x_t \sin \lambda_k t. \tag{7}$$

In the above equations; x is the value of the process being modeled, k is the number of harmonics used in the representation, n is the sample size, λ_k are the frequency numbers used in the series and are functions of the cycle period, and ε_t is a stochastic component. The λ_k are restricted to the range betweein 0 and π and do not have to be true fractions of the period length.

Equation 5 has most generally been used in the analysis of hydrologic processes rather than in the generation of synthetic series. As such, it is referred to as spectral analysis or variance spectrum analysis. If

$$I_k^2 = A_k^2 + B_k^2 \tag{8}$$

is used to calculate

$$S_k = \frac{\sum\limits_{\lambda_k \leq \lambda} I_k^2(\lambda)}{\sum\limits_{\lambda_k = 1}^{S} I_k^2(\lambda)} \tag{9}$$

and S_k is in turn plotted against λ_k, the integrated periodogram is developed. In this plot, the ordinate S_k is the contribution to the variance of the system given by the harmonic of given frequency. A typical plot is shown on figure 2 for the monthly time series of the Elk River at Clark, Colorado, Roesner and Yevdjevich (7). The figure also shows the effect on the variance of the system of removing the various harmonic periods. A complete description of variance spectrum analysis is given by Blackman and Tukey (8), Wold (9), and Matalas (10). Its use is presented in Roesner and Yevdjevich (7), Quimpo (11), and Horn and Bryson (12). Quimpo (11) also has a good discussion and bibliography of spectrum analyses.

Correlograms

A correlogram is a graphical representation of the autocorrelation (the correlation of an element of a series with other elements in the same series lagged by integer amounts) coefficients as functions of the lag. The correlogram can at times help in deciding what type of generating process to use in the synthesis of data. The correlogram of a process governed by a moving average will show oscillation for lagged time periods less than the length of the moving average m,

but will be zero for lags greater than m. The correlogram for a harmonic process will oscillate with a fixed period and amplitude for an indefinite period. The correlogram of autoregression methods, described next, will decrease monotonically from a value of 1 at lag 0 to zero at lag infinity if the correlation coefficient is positive at lag 1. If the correlation coefficient is negative at lag 1, the correlogram will oscillate with a period of unity above and below the abscissa with a decreasing but nonvanishing amplitude. An example of a correlogram and the effects on the correlogram of removing periods of high correlation is shown in figure 3.

The correlogram is used primarily as a tool for the analysis of hydrologic processes rather than as a generating device. See Yevdjevich (13), Roesner and Yevdjevich (7), Yevdjevich (14), Caffey (15), Bhuiya and Yevdjevich (16), Yevdjevich and Jeng (17), Dumas and Moul-Seytoux (18), and Todorovic and Yevdjevich (19) for examples of its use.

Autoregressive Methods

Basic Recursion Relation - A time series can be represented by any one of several analytic functions as has been described above. Most of these functions are of a form

$$x_t = f(t) + \varepsilon_t \tag{10}$$

where x_t is the value of the series at time t, f(t) is a deterministic component, and ε_t is a random component. If the series shows a long term trend, polynomials or Fourier series may be used to represent the series. If, however, the trend component is constant, then the series is derived from a process that is stationary. In such a process, the lagged covariances between elements in the series are functions of the absolute differences between indices of elements in the series. As a result of this argument, the linear autoregressive relation may be developed.

$$x_t = \beta_o + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \cdots \beta_m x_{t-m} + w_t \tag{11}$$

where the β_i are autoregression coefficients and w_t is an independent random term.

Normal Distributions - The series described by equation 11 is said to be stationary in a wide sense or weakly stationary because its mean is equal to a constant and its autocovariance is a function of only the absolute differences between elements of the series. Thus the series is assumed to be

normally distributed. By taking the conditional expectation of equation 11 for a lag-one Markov process, m=1,

$$E(x_t|x_{t-1}) = \mu_x + \rho_x(1)(x_{t-1} - \mu_x) \qquad (12)$$

and

$$Var(q_t|q_{t-1}) = \sigma_x^2[1 - \rho_x^2(1)] \qquad (13)$$

the following equation may be developed

$$x_t = \mu_x + \rho_x(1)(x_{t-1} - \mu_x) + v_t\sigma_x\sqrt{1 - \rho_x^2(1)} \qquad (14)$$

where $\mu_x = \mu_{x,t} = \mu_{x,t-1}$ is the population mean, $\sigma_x = \sigma_{x,t} = \sigma_{x,t-1}$ is the population standard deviations, $\rho_x(1)$ is the lag 1 correlation coefficient, and v_t is a normal random deviate. Equation 14 therefore preserves both the first moment of the distribution and the second moment or variance by combining equations 12 and 13. If the mean, $\hat{\mu}_x$, standard deviation, $\hat{\sigma}_x$, and correlation coefficient $\hat{\rho}_x(1)$ of the historic series along with a random variate with zero mean and unit variance are substituted into equation 14, the following equation for the i^{th} element of the series is obtained.

$$x_i = \hat{\mu}_x + B(x_{i-1} + \hat{\mu}_x) + t_i\hat{\sigma}_x\sqrt{1 - \hat{\rho}_x^2(1)} \qquad (15)$$

where B is the least squares regression coefficient equal to the correlation coefficient because $\hat{\sigma}_x = \hat{\sigma}_{x,t} = \hat{\sigma}_{x,t-1}$.

If the time series shows seasonal trends such as those of stream flow, equation 15 may be used to represent the j^{th} season as follows:

$$x_{i,j} = \hat{\mu}_{x,j} + B_j(x_{i-1,j-1} - \hat{\mu}_{x,j-1}) + t_i\hat{\sigma}_{x,j}\sqrt{1 - \hat{\rho}_j^2(1)} \qquad (16)$$

where

$$B_j = \hat{\rho}_j(1)\frac{\hat{\sigma}_{x,j}}{\hat{\sigma}_{x,j-1}} \qquad (17)$$

In equation 16 the index i runs sequentially and index j runs cyclically across all seasons or periods in the cycle. The equation represents the time series where there is one element per season or period.

In a situation where there may be many elements per season; for example, estimating daily stream flow where the month is the season length, the equation becomes

$$x_{i,j} = \hat{\mu}_{x,j} + B_j(x_{i-1,j} - \hat{\mu}_{x,j}) + t_i\hat{\sigma}_{x,j}\sqrt{1 - \hat{\rho}_j^{\,2}(1)} \qquad (18)$$

where $B_j = \hat{\rho}_j(1)$.

The index i runs sequentially within season j. Equation 18 is equivalent to equation 15 if equation 15 were developed from only data in season j.

Synthetic data may be generated by either equations 15, 16, or 18 that will resemble the historic sequence in terms of $\hat{\mu}_x$, $\hat{\sigma}_x$, and $\hat{\rho}_x(1)$.

More detail on the derivation of these equations may be found in Thomas and Fiering (20, Fiering (21), Fiering (22), Matalas (23), and Yevdjevich (14).

Nonnormal Distributions - In the above discussion, the distribution of events was assumed to be weakly stationary or normally distributed. However, in many hydrologic processes obvious departures from this assumption are common. Many phenomena including stream flow and size of precipitation events are distributed like gamma or third-order stationary. Fiering (21) shows that the basic lag-one Markov process, equation 14, may be rewritten to incorporate the effect skewness, γ, by replacing x_t by its standardized form $(x_t - \mu_x)/\sigma_x$ and then cubing the equation to obtain γ_x. By taking expectations he shows that the skewness of the random component γ_ε required to maintain the skewness of the system $\hat{\gamma}_x$ is given by

$$\gamma_\varepsilon = \hat{\gamma}_x \frac{1 - \hat{\rho}_x^{\,3}(1)}{[1 - \hat{\rho}_x^{\,2}(1)]^{3/2}} \qquad (19)$$

The random normal deviates, t_i, must now be distributed like gamma such that $\mu_\varepsilon = 0$, $\sigma_\varepsilon^2 = 1$, and γ_ε is equal to equation 19. This is accomplished by the transform

$$\varepsilon_i = \frac{2}{\gamma_\varepsilon}\left[1 + \frac{\gamma_\varepsilon t_i}{6} - \frac{\gamma_\varepsilon^{\,3}}{36}\right]^3 - \frac{2}{\gamma_\varepsilon}\,. \qquad (20)$$

The equation, a third-order stationary lag-one Markov process, for generating data that will resemble the historic sequence in terms of $\hat{\mu}_x$, $\hat{\sigma}_x$, $\hat{\gamma}_x$, and $\hat{\rho}_x(1)$ is given by

$$x_i = \hat{\mu}_x + B(x_{i-1} - \hat{\mu}_x) + \varepsilon_i \hat{\sigma}_x \sqrt{1 - \hat{\rho}_x{}^2(1)} \qquad (21)$$

Both equations 16 and 18 which represent systems in which seasonal trends are present may be changed to incorporate the effect of skewness by calculating $\gamma_{\varepsilon,j}$ for each of the seasons or periods by equation 19. The appropriate value should then be used in calculating the random element ε_i which replaces t_i in each of the equations.

Both Thomas and Fiering (20) and Matalas (23) discuss in greater detail this method for making the generating process third-order stationary. Matalas (23) also shows a method in which the coefficient of skewness of $\hat{\gamma}_x$ of the generated series can be approximated but must be less that $2\sqrt{2}$. The procedure involves generating a series of random variates using a lag-one Markov process which is a function of $\hat{\rho}_x(1)$. The variates are then squared and m of them accumulated. The synthetic sequence is then a function of $\hat{\mu}_x$, $\hat{\sigma}_x$, m, and the accumulated variates.

Not only are many hydrologic time series skewed but they are often found to be log-normally distributed. This can be either a 2- or 3-parameter distribution. Such a system is represented by

$$y = \ln\,(x - a) \qquad (22)$$

where x is the random variate, a is a lower bound of the variate, and y is assumed to be a skewed normal distribution. Matalas (23) shows how the mean, $\hat{\mu}_x$; variance, $\hat{\sigma}_x{}^2$; the coefficient of skewness, $\hat{\gamma}_x$; and the lag-one correlation coefficient $\hat{\rho}_x(1)$ of the random variate x are related by a set of four equations to the lower bound, a; the mean, μ_y; the variance, $\sigma_y{}^2$; and the lag-one correlation coefficient $\rho_y(1)$ of the random variate y. By solving the four simultaneous equations, a, μ_y, σ_y, and $\rho_y(1)$ can be found and substituted in the equation

$$y_i = \mu_y + \rho_y(1)\,(y_{i-1} - \mu_y) + \sigma_y t_i \sqrt{1 - \rho_y{}^2(1)}\;. \qquad (23)$$

The value of x_i is obtained by adding a to the antilog of y_i. The sequence of values thus generated will resemble the historic sequence in terms of $\hat{\mu}_x$, $\hat{\sigma}_x$, $\hat{\gamma}_x$, and $\hat{\rho}_x(1)$.

Matalas also states that "If the values of a, μ_y, σ_y, and $\rho_y(1)$ had been obtained from logarithms of the historic events rather than in the manner outlined above, then equation 10 (equivalent to equation 23 above) would lead to a synthetic sequence that would not resemble the historic sequence in terms of $\hat{\mu}_x$, $\hat{\sigma}_x$, $\hat{\gamma}_x$, and $\hat{\rho}_x(1)$." This statement is theoretically true, but the magnitude of the error is not known.

The easiest method of obtaining a synthetic sequence of log-normally distributed events is to calculate the population parameters $\hat{\mu}_x$, $\hat{\sigma}_x$, $\hat{\gamma}_x$, and $\hat{\rho}_x(1)$ from the logarithms of the variate x and then use these parameters in equations 15, 16, 18, or 21. Fiering (21) states that he found the error in maintaining serial correlation incurred by this method was small.

Multiple-Lag Markov Process - Multiple-lag Markov processes have not been used extensively in hydrologic synthesis. This has generally been because the work involved in evaluating parameters of the model do not warrant the increase in accuracy over that of the lag-one process. Fiering (21), chapter 3, discusses in detail the multiple-lag problem. The following equation illustrates the method used to generate data with this system.

$$x_i = B_0 + B_1 x_{i-1} + \cdots B_m x_{i-m} + \sigma_x t_i \sqrt{1 - R^2} \qquad (24)$$

where B_i is the least squares partial regression coefficient of lag-i, x_i is the value of the variate at time period i, σ_x is the standard deviation of the x_i, t_i is a normal random sampling deviate, and R is the multiple correlation coefficient.

In the conclusion of the chapter Fiering makes the following statement: "On several important counts the model appears to provide significantly better results than the Markovian models used heretofore, failing mainly in its inability to verify emperical results for incomplete regulation." Fiering was using it primarily in the study of reservoir storage-yield functions.

Two-Variate Problems - Hydrologic analyses often have a need for two or more synthetic inputs which may be correlated with one another. In this section a technique for generating data for two interrelated variates is presented. The technique requires that the time interval be the same for both variates. Monthly streamflow on two forks of a river on which a reservoir is anticipated, or daily rainfall and evaporation for input to a water yield model are examples of synthetic data that could be generated by this technique.

Suppose that the two variates are x and y and that the means, standard deviations, skewnesses, and lag-one serial correlation coefficients for the variates are $\hat{\mu}_x$, $\hat{\mu}_y$; $\hat{\sigma}_x$, $\hat{\sigma}_y$; $\hat{\gamma}_x$, $\hat{\gamma}_y$; and $\hat{\rho}_x(1)$, $\hat{\rho}_y(1)$. The product-moment correlation coefficient between the data for variates x and y is $\hat{\rho}_{x,y}$. The technique requires that one of the two variates be selected as a key variate. The selection could be based on the length of record, quality of records, etc. Assume that variate x is selected. Variate y is therefore assumed to be subordinate to x. A synthetic sequence of events is generated for x by using either equation 15 or 21 depending upon whether or not the skew factor is assumed to be significant. A similar cross-correlation model is used to calculate the variate values for y.

$$y_i = \hat{\mu}_y + B(x_i - \hat{\mu}_x) + u_{i-1}\hat{\sigma}_y\sqrt{1 - \hat{\rho}_{x,y}^{\,2}} \tag{25}$$

where B is given by $\hat{\rho}_{x,y} \dfrac{\hat{\sigma}_y}{\hat{\sigma}_x}$ and u_i is a standardized random variate adjusted as follows to incorporate the serial correlation coefficient for y.

$$u_{i-1} = \frac{\pi}{\hat{\sigma}_y}(y_{i-1} - \hat{\mu}_y) + t_i(1 - \pi)^{1/2} \tag{26}$$

where

$$\pi = \frac{\hat{\rho}_y(1) - \hat{\rho}_x(1)\,\hat{\rho}_{xy}^2}{\sqrt{1 - \hat{\rho}_{xy}^2}} \tag{27}$$

In equation 26 t_i is a standardized random sampling number adjusted for skew if deemed necessary. The derivation π and equation 26 are described by Fiering (22). Prior to his presentation, the common approach was to consider u_{i-1} as a standardized variate. He shows that such a practice does not keep the serial correlation of y and introduces spurious correlation into the subordinate variate.

If the variates show seasonal trends, the statistical characteristics of the populations may be determined for each of the seasons and either equation 16 or 18 used to generate the synthetic sequence for the key variate. The choice of equations depends upon the number of observations per season. Equation 25 would then be used to generate the synthetic data for the subordinate variate being sure to use the appropriate set of statistical characteristics depending upon the season being considered.

Multivariate Problems - In systems analysis, for example the analysis of reservoir release rates in a river basin, there is quite often a need for multiple inputs. In the above example, these may be in the form of monthly discharges at several locations within the basin. Several procedures have been proposed for generating synthetic data for such systems. Multiple regression techniques are the most commonly used method. They may or may not be the most satisfactory depending upon the objectives of the study. They have, in general, three drawbacks: (1) The variates must be ranked or an ordering sequence established among them, (2) not all cross correlation coefficients are retained in the synthetic sequence, and (3) coefficients established by a regression analysis are valid only for the range of data used to evaluate them. If the period of record is short, it is quite possible that fairly common extremes may not have been present in the historic sequence and the coefficients may be biased to a greater extent than those developed using other techniques. If these drawbacks are not considered serious, the method can be used to good advantage because it is computationally much simpler than the other methods.

Assume there are m variates for which simultaneous synthetic sequences are desired. The variates are ordered using some preselected criteria. Regression equations are then calculated from the data. The equation for each variate is different from all others and is a function of all other variates preceding it in the sequence of variables. The basic equation for the j^{th} variate is of the form

$$x_{ij} = B_0 + B_1 x_{i-1,j} + B_2 x_{i,j-1} + \cdots B_j x_{i,1} + \hat{\sigma}_{x,j} t_i \sqrt{1 - R^2} \qquad (28)$$

where x is the variate value of the series, the B_i are regression coefficients, $\hat{\sigma}_{x,j}$ is the sample standard deviation of the j^{th} variate, t_i is a random, independent and normally distributed variate, R is the sample multiple correlation coefficient, the subscript i is the sequence of events, and j is the rank of the variate starting with 1 for the first variate selected. The first term following the constant is the effect of the preceding time period and all other terms except the last are the effects from other stations preceding it in the sequence of variates. The last term is the random component required to retain the variance of the j^{th} variate.

Several variations of equation 28 are used. Quite often the variates are reduced to standard normal deviates to reduce seasonal trends. In this case B_0 becomes zero and $\hat{\sigma}_{x,j}$ is 1. If the variates are skewed, they can be made approximately normal by solving equation 20 for t_i, and applying the resulting equation to the deviates. The transform is

$$x_{i,j} = \frac{6}{\hat{\gamma}_{x,j}} \left[\left[\frac{\hat{\gamma}_{x,j}}{2} \left[\frac{X_{i,j} - \hat{\mu}_{x,j}}{\hat{\sigma}_{x,j}} \right] + 1 \right]^{1/3} - 1 \right] + \frac{\hat{\gamma}_{x,j}}{6} \tag{29}$$

where $\hat{\mu}_{x,j}$, $\hat{\sigma}_{x,j}$, and $\hat{\gamma}_{x,j}$ have the usual meaning of mean, standard deviation, and skew for the j^{th} variate. $X_{i,j}$ is the raw value of the variate, and $x_{i,j}$ is the standard normal deviate. If equation 28 is used to generate synthetic data using $x_{i,j}$ defined as in equation 29, then the inverse transform of equation 29 must be used to calculate the synthetic series in raw form.

Beard (24) has gone one step farther in using the equation to generate synthetic sequences of stream flow. Most stream flow data can be reduced to standard normal deviates by using a three-parameter log-normal distribution. Beard added a small increment to all monthly flows, .001 times the mean annual, then took the logarithm of the result. He then reduced the logarithms to standard normal deviates using equation 29. In addition to using a log normal distribution, he added one term to the generating equation 28. The term he added was the average of all flows from the m stations for the previous six months.

Matalas (23) presents two methods of dealing with the problem of multiple variate data generation. The first method is designed to preserve the means, standard deviations, lag-one serial correlation coefficients, and lag zero cross correlations of the data matrix. For a set of m variables his model is of the form

$$x_i = A\, x_{i-1} + B\, \varepsilon_i \tag{30}$$

where x_i is an m by one matrix of variate deviates from the mean at time i, for the j^{th} variate the element is

$x_i(j) = X_i(j) - \hat{\mu}_x(j)$, x_{i-1} is the m by one matrix of deviates at time i-1, $x_{i-1}(p) = x_{i-1}(p) - \hat{\mu}_x(p)$, A and B are m by m matrices of coefficients, and the ε_i are random normal variates. The matrix A is defined by the equation

$$A = M_1 M_o^{-1} \tag{31}$$

where M_1 is the lag-one covariance matrix $x_i \, x_{i-1}^T$ where T indicates the transpose, M_o^{-1} is the inverse of M_o, and M_o is the lag-zero covariance matrix $x_{i-1} \, x_{i-1}^T$. The matrix B is obtained by solution of the equation

$$B B^T = M_o - M_1 M_o^{-1} M_1^T = C \tag{32}$$

where M_o and M_1 are as defined above. G. K. Young (25) shows that after evaluating the elements on the right hand side of equation 32, thus yielding the C matrix, the elements of B may be found by a set of recursive equations. Other solutions to B are available as there are an inumerable number of matrices which will, when post multiplied by their transform, yield the C matrix. Matalas (23) shows how the elements of B may be obtained by the techniques of principal component analysis.

The second method presented by Matalas (23) is the multivariate extension of equation 23. The technique preserves the means, standard deviations, skews, lag-one serial correlation coefficients, and the lag-zero cross correlation coefficients.

In addition, the data $x^{(p)}$ are presumed to follow a three-parameter log-normal distribution with a lower bound $a^{(p)}$. Thus for the p^{th} variate

$$y^{(p)} = \ln\left(x^{(p)} - a^{(p)}\right). \tag{33}$$

The equations referenced in the two-variate case for relating the mean, standard deviation, skew, and lag-one serial correlation coefficients of the variates $x^{(p)}$ to the log variates $y^{(p)}$ are used to find the parameters of $y^{(p)}$ for each of the p variates. The generating process

$$y_i = A' y_{i-1} + B' \varepsilon_i \tag{34}$$

where y_i is an m by one matrix of variate deviates from the mean of y at time i, y_{i-1} is the m by one matrix of deviates at time i-1, A' and B' are m by m matrices of coefficients and the ε_i are random normal variates. The matrices A' and B' are analogous to and evaluated by the methods used for matrices A and B in equation 30.

Fiering (22) introduced the use of multivariate analysis techniques into the synthesis of stream data. His technique adjusted to account for lag-one serial correlation is applicable to the synthesis of any type of data. It is predicated on finding all the truly orthogonal or independent variates in the system. This is accomplished by calculating the lag-zero cross correlation matrix, the elements of which are $\hat{\rho}_x(p)(q)(0)$, for all variates $p = 1, \cdots m$, $q = 1, \cdots m$. The diagonal elements, $p = q$, are equal to unity and the off diagonal elements are the cross correlation between variates. It is next assumed that all of the variance in the matrix can be represented by a different matrix λI in which λ is an m by one scalar and I is an m by m identity matrix. The relation between the correlation matrix and the new matrix is

$$R\,M = \lambda I\,M \tag{35}$$

in which R is the m by m correlation matrix, M is an m by m matrix of unknowns, and λI is a diagonal matrix. This can be rewritten

$$(R - \lambda I)\,M = 0 \tag{36}$$

The nontrivial solution for the scalar, λ, in this matrix is obtained only if the determinant of the correlation matrix is zero

$$|R - \lambda I| = 0 \tag{37}$$

Solution of this set of simultaneous equations yields the characteristic equation of R. The equation is a polynomial of degree m. The solutions of which, λ_i, are the m independent characteristic roots of the matrix. These roots may be substituted into equation 36 one at a time to find the value of the matrix M which relates the original correlation matrix to the new matrix of independent elements, λI. The roots λ_i and associated vectors M_i are commonly referred to as the eigenvalues and eigenvectors of the correlation matrix.

The advantage of determining the characteristic roots and vectors of the correlation matrix lies in the fact that it may be possible to use a reduced number of elements or components to represent the data. Consider, for example, a system in which all variates are linear transformations of one variate. Under these circumstances, all but one of the roots disappear and the matrix is reduced to a rank of one. If, however, all of the variates are independent, then all m roots are required to represent the data. Most real systems fall somewhere between these two extremes. If the

orthogonal components of the system are denoted as ζ, then they may be calculated by the equation

$$\zeta_i = \sum_{j=1}^{m} M_{i,j}\, x_j \tag{38}$$

The ζ_i are uncorrelated by definition, and have a variance λ_i. Since the matrix R is the correlation matrix, assumed to have been derived from a normal population, the total variance of the system is m and the percent explained by component i is

$$P_i = \frac{\lambda_i}{m}\, 100\% \tag{39}$$

Since the ζ_i are independent, they can be used as elements in a generating process. Fiering (22) used the following equation for a two-variate model.

$$\hat{x}_1 = \beta_1\, \zeta_1 + \beta_2\, \zeta_2 + t\, \hat{\sigma}_x(1)\sqrt{1 - R^2} \tag{40}$$

where $\hat{x}_1$ is the estimated value for variate 1, the ζ_i are the orthogonal components, $\hat{\sigma}_x(1)$ is the sample variance for variate 1, β_i are least squares regression coefficients which for the first coefficient is

$$\beta_1 = \frac{\hat{\rho}_x(1)}{\sqrt{\lambda_1}} \tag{41}$$

in which $\hat{\rho}_x(1)$ is the correlation between the elements of the first principal component, ζ_1, and the values of the first variate x_1

$$\hat{\rho}_x(1) = \frac{\sum \zeta_{i1}\, x_{i1}}{(n-1)\sqrt{\lambda_1}} \tag{42}$$

where n is the number of observations. R in equation 40 is the multiple correlation coefficient

$$R = \sqrt{\hat{\rho}_x^2(1) + \hat{\rho}_x^2(2)} \tag{43}$$

If equation 40 had been expanded to include all roots; i.e., ζ_i, i = 1 ··· m; then R = 1 and there would be no random component. In practice a reduced number of components is most generally used.

Matalas (23) showed that the form of generating equations proposed by Fiering would generate data which would resemble historic sequences in terms of $\hat{\mu}_x^{(p)}$, $\hat{\sigma}_x^{(p)}$, and $\hat{\rho}_x^{(p)(q)}(0)$ but not in terms of $\hat{\rho}_x^{(p)}(1)$, p, q = 1, ···· m. In his paper he indicated that the coefficients of the generating equation could be adjusted by introducing the matrix of lag-one serial correlation coefficients. (The mechanics of making the adjustment are not clear to the author at this time. They will be investigated however and described in the final report of this conference.)

The Stochastic Element as a Component of a Deterministic System

Thus far the discussion has centered on presenting the mechanics of stochastic models where the objectives have been the generation of synthetic data sequences. Stochastic elements also play an important role in deterministic systems. For example, consider a deterministic system that takes the form of a simple linear regression of a dependent variable, y, on a set of independent variates, x_i

$$y = f(x_1, x_2, \cdots x_n) \tag{44}$$

Let us further suppose that the multiple correlation coefficient between y and the set of x's is R. If the data set is multivariate normal or approximately so, the percent of the variance in y explained by the linear equation is $R^2 \leq 1.0$. If the equation is to be used subsequently for predicting new values of y from a different set of x's, and if the distribution of the new set of x's is the same as the sample set, the variance of the predicted y values will be

$$\hat{\sigma}_y^{\,2} = \hat{R}^2 \, \sigma_y^{\,2} \tag{45}$$

where $\hat{\sigma}_y^{\,2}$ and $\sigma_y^{\,2}$ are the variances of the predicted and observed dependent variates respectively. The variance of $\hat{y}$ can be retained by adding a stochastic element to equation 44

$$\hat{y} = f(x_1, x_2, \cdots x_n) + t \, \hat{\sigma}_y \sqrt{1 - \hat{R}^2} \tag{46}$$

where t is a standard normal variate.

If the data are not normally distributed, or the function of the dependent variables is nonlinear such that the distribution of the error terms is not normal, then it may be necessary to study the distribution and make its parameters a function of $\hat{y}$. If the data are normally distributed the mean of the error $\hat{\mu}_\varepsilon$ will be zero, the variance $\hat{\sigma}_\varepsilon = \hat{\sigma}_y\sqrt{1 - \hat{R}^2}$, and the skew of the error, $\hat{\gamma}_\varepsilon$, zero. For nonnormal distributions, both $\hat{\mu}_\varepsilon$ and $\hat{\gamma}_\varepsilon$ may not be zero. If $\hat{y}_d$ represents the predicted value of y including the stochastic element, then equation 46 may be rewritten

$$\hat{y}_d = f(x_1, x_2, \cdots\cdot x_n) + \hat{\mu}_\varepsilon + \hat{\sigma}_\varepsilon\ \varepsilon \tag{47}$$

where ε is the random normal variate distributed as gamma and calculated by the transform of equation 20.

If, in studying the distribution of the error, it is found that its parameters are a function of $\hat{y}$, it may be necessary to stratify $\hat{y}$ and calculate the values of $\hat{\mu}_\varepsilon$, $\hat{\sigma}_\varepsilon$, and $\hat{\gamma}_\varepsilon$ for each stratum. It might then be possible to make all three parameters functions of $\hat{y}$.

Statistical Tests Useful in the Analysis of Stochastic Models

The reliability of stochastic models is to a great extent a function of the statistical tests and assumptions made in the development of the models. For the most part assumptions should be backed by appropriate tests, many of which are well known to the hydrologist. Some of the better known tests which are well documented in the literature are: (1) The χ^2-test for normalcy, (2) the t-test used for comparing the means of two sets of numbers, and (3) the variance ratio or F-test for comparing the variance of two distributions. The significance of multiple correlation coefficients can also be tested by comparison with tabled values. See pages 159, 178, 179, and 241 of Crow, Davis, and Maxfield (26). The test for the significance of the multiple correlation coefficient is identical to the F test for the significance of regression as a whole.

Since many of the distributions dealt with in stochastic modeling are not normal and quite often of undetermined form, the usual tests based on the assumption of normalcy are not satisfactory. There are, however, a large number of non-parametric or distribution-free tests that can be applied to nonnormal distributions. One of the oldest and probably best known nonparametric test is the χ^2-test of fit developed by

Karl Pearson. These tests are as their title would indicate, distribution free. That is, no assumption is made as to the form of the distribution from which the data come. The book, Nonparametric Methods in Statistics by Fraser (27) is a good reference on the subject. Keeping (28) reviews in an illustrated form many of the nonparametric tests useful in the field of hydrology.

Perhaps the most useful nonparametric tests are the Kolmogorov-Smirnov one- and two-sample tests. The Kolmogorov-Smirnov one-sample test is a nonparametric goodness of fit test relating to the cumulative distribution function. It is in general more powerful than the chi-square test used for the same purpose because it is based on the maximum difference between the distributions rather than the cumulative difference. The test statistic is

$$D_n = \underset{-\infty < x < \infty}{\text{Max}} \left| F_n(X) - F_o(X) \right| \qquad (48)$$

where $F_o(X)$ is the cumulative population distribution being tested, and $F_n(X)$ is the cumulative empirical distribution being tested against. Equation 48 is the test statistic for a two-tailed test because D_n is the maximum value of the absolute difference between the distributions.

For very large samples; i.e., $n > 50$, the critical value of D_n, D_α, is inversely proportional to the square root of the sample size. For α equal to .05; $D_\alpha = 1.36/\sqrt{n}$ where n is the sample size.

The test is useful in hydrologic testing because the distribution of D_n is independent of the form of F(X). The distributions of D_n have been derived and are available in tables; Lindgren and McElrath (29) and Hoel (30).

The test is often used to test discrete population distributions although it is based on the continuous distribution. It has been found that it is on the "safe" side because the actual significance of the resulting test is no bigger than the one assumed in using the tables.

The Kolmogorov-Smirnov one-sample test may be extended to test whether two samples of the same or different sizes are from populations with the same distribution function. The test statistic is

$$D_n = \underset{-\infty < x < \infty}{\text{Max}} \left| G_o(X) - F_o(X) \right| \quad (49)$$

where $G_o(X)$ and $F_o(X)$ are independent distributions assumed to be from the same population. For large sample sizes, the critical value of D_n, D_α, is inversely proportioned to the square root of n/2 if the samples are both of size n. If the samples are not equal in size, but both are large, D_α is inversely proportional to the square root of $\frac{n_1 n_2}{n_1+n_2}$ where n_1 and n_2 are the two sample sizes. For α equal to 0.05, the statistic is

$$D_{.05} = \frac{1.36}{\sqrt{\frac{n_1 n_2}{n_1+n_2}}} \quad (50)$$

Another useful test is one for serial correlation. The lag-L serial correlation coefficient is given by

$$\gamma_L = \frac{\sum_{i=1}^{n} x_i x_{i-L} - n \hat{\mu}_{x_i} \hat{\mu}_{x_{i-L}}}{n \hat{\sigma}_{x_i} \hat{\sigma}_{x_{i-L}}} \quad (51)$$

where n is the sample size and the other terms have their usual meaning. Since the $\hat{\mu}_{x_i} \doteq \hat{\mu}_{x_{i-L}}$ and $\hat{\sigma}_{x_i} = \hat{\sigma}_{x_{i-L}}$ the only term that changes with L is the sum

$$\gamma_{L*} = \sum_{i=1}^{n} x_i x_{i-L}. \quad (52)$$

Wold and Wolfowitz (31) show that the statistic, γ_L*, may be tested for significance by the following method if n is greater than about 20. At the 5 percent level of significance, the null hypothesis which states that there is no correlation between successive observations of the variate is rejected if

$$\frac{\gamma_L* - \hat{\mu}_{\gamma_L*}}{\hat{\sigma}_{\gamma_L*}} > 1.64 \quad (53)$$

where

$$\hat{\mu}_{\gamma_L *} = \frac{S_1^{\,2} - S_2}{n - 1} \tag{54}$$

and

$$\hat{\sigma}^2_{\gamma_L *} = \frac{S_2^{\,2} - S_4}{n - 1} + \frac{S_1^{\,4} - 4S_1^{\,2}S_2 + 4S_1S_3 + S_2^{\,2} - 2S_4}{(n - 1)\ (n-2)} - \hat{\mu}^2_{\gamma_L *} \tag{55}$$

in which

$$S_k = \sum_{t=1}^{n} x_t^{K} \tag{56}$$

Other nonparametric tests which are not as often used such as run and sign tests used in checking for randomness, etc. are adequately presented in Keeping (28). Grace and Eagleson (32) also present a brief discussion of some of the more common statistical tests in appendix chapter A of their report.

The Random Number Generator

The random number generator used in stochastic models is one of the most important elements of the process. If the system is to perform as designed, then the "random numbers" used in the model must be random numbers. Many of the standard random number generators that are available as computer packages are not adequate. For example, the random number generator available in the I.B.M.1/ scientific subroutine package was tested by the author for length of sequence and found to repeat after about 7,000 numbers. χ^2-tests of the distribution showed that it was not uniform. These results are in accordance with results of tests by MacLoren and Marsaglia (33) and Von Gelder (34). They found that of all the random number generators using standard methods that were tested, none gave satisfactory results.

Von Gelder found several power residue (sometimes called congruential or multiplicative) generators which performed well in his tests. The tests which he used differed somewhat

1/ Brand names are used for simplicity and easy reference and do not constitute endorsements.

from those of MacLoren and Marsaglia but were found to be compatible with theirs. Of the several methods found to be satisfactory, one, a power residue method with a recycle period of over one-half billion numbers is highly satisfactory. Random numbers are generated by the equation

$$U_{n+1} = X\ U_n\ (\text{mod } 10^d) \tag{57}$$

where d is the significant number of digits in the computer, generally set equal to 10; X is a constant equal to 100003; and U_n and U_{n+1} are consecutive random numbers.

Both chi-square tests for distribution and tests for serial correlation were performed on numbers from the generator. Results indicated that the random numbers were independent and uniformly distributed at the 95 percent confidence limits.

Random normal numbers can be calculated from the uniform numbers by using the direct method.

$$X_1 = (-2\ \ln U_1)^{1/2} \cos 2\ \pi\ U_2 \tag{58}$$

where U_1 and U_2 are random numbers and X_1 is a random normal number.

Tests comparing this method with the "sum of uniform deviates" method of obtaining normal numbers indicated that the direct method is superior.

Examples of Stochastic Models

The use of stochastic models in the field of hydrology has been related in the past primarily to the generation of rainfall and stream flow, although it has been used recently for the generation of data in the study of pollution and economics.

Rainfall - The stochastic models used in the generation of rainfall data have varied considerably depending primarily on the type data generated. Annual and in some cases monthly rainfall amounts have been found to be independent; i.e., the persistence or serial correlation were not statistically significant. (See references, Yule (35), Kotz and Neumann (36), Brittan, et al (37), Hoel (38), Pattison (3), and Beer (39).) Under these conditions, the synthesis of data is very simple and consists only of generating a series of random numbers having the distribution of the observed record. See Slade (40), Thom (41), Merriam (42), Markovic (43), Whitcomb (44), Stidd (45), and Beals (46).

It has been found that, for the most part, daily and in some cases monthly rainfall amounts exhibit a statistically significant degree of persistence. See Besson (47,48), Namias (49), Uttinger (50), Hannan (51), Sellers (52), Williams (53), Longley (54), and Cooke (55). DeCoursey (56) however, in analyzing Texas rainfall found no significant persistence in daily rainfall. In generating daily sequences, a problem, not normally encountered in the generation of monthly sequences, is that of generating wet and dry periods. The methods proposed for handling this problem have been either a Markov process based on transition probabilities or a probability distribution fitted to lengths of wet and dry spells. Gabriel and Neumann (1), DeCoursey and Seely (2) used a two-state Markov chain as exemplified by equation 3. Other authors such as Caskey (57) and Weiss (58) found the same system to be satisfactory whereas Newnham (59), Jorgensen (60), and Cooke (55) were not satisfied with the method. Feyerherm and Bark (61) used a second-order Markov chain and suggested that the two-state Markov chain model be made a function of the time of year. Green (62) used exponential distributions for the lengths of wet and dry spells and sampled alternately from them. Grace and Eagleson (32) represented the distributions of the wet and dry periods by a Weibull distribution from which they sampled alternately.

Depths of daily rainfall have been calculated for the most part by serial regression type techniques. See Enger (63), Hammerle (64), Sellers (52), and Pattison (3). Grace and Eagleson (32), however, introduced storm duration into the regression model. Pattison (3) also tried using transition probabilities to select rainfall amounts from probability distributions.

The distribution of rainfall amounts for short time intervals was studied by Pattison (3), Grace and Eagleson (32), and Ramaseshan (65). Ramaseshan's work is also presented briefly in Chow and Ramaseshan (66). Pattison (3) tried two different models for daily rainfall. One method used multistate transition probabilities for both the distribution of wet and dry periods and for the selection of rainfall depths. The second method was a linear regression model which distributed total daily rainfall. Different regression equations were used for different size classes of total rainfall. He concluded that the first model was the best for his purposes.

Ramaseshan (65) studied only annual maximum storms and tried five different regression models. He found that the simple first-order Markov model performed as well as the other more complex ones.

Grace and Eagleson (32) used the Weibull distribution to describe the lengths of wet and dry periods. They then found the storm depths to be a function of the storm duration. The distribution of rainfall throughout the storm was made by using a type of urn model, the parameters of which were selected by a trial and error procedure.

Runoff - The stochastic models used in the generation of runoff do not have the wide variation of type developed for rainfall. This is because periods of no flow are not as pronounced as are dry periods in the synthesis of rainfall sequences. In almost all cases, serial correlation is statistically significant and must be considered in the models selected for the generating process. All of the models previously described in the section on autoregression methods have been used at one time or another. The trend in most recent articles is towards the use of synthetic data sequences in the study of water resources systems. As such, most of the data generation described has been of a multivariate nature. References 67 through 73 are primarily devoted to discussions of the use of stochastic processes for the generation of synthetic stream flow data. In general, there is very little difference between one used for annual flows and one used for daily or monthly flows. The major difference being the use of techniques, such as those described previously, to account for seasonal trends.

Synthetic stream flow sequences have also been used to good advantage in studying the operation of reservoirs both from the standpoint of flood control and water supply. The application of stochastic processes to water supply has been described by Yevdjevich (74) and Fiering (21). Other references on this subject are Young, et al (71), and Jeng and Yevdjevich (75).

A group of general references which might be of interest to someone studying the stochastic generation of runoff in detail are: Roesner and Yevdjevich (7), Yevdjevich (14), Bhuiya and Yevdjevich (16), Yevdjevich and Jeng (17), Fiering (21,22), Matalas (23), Beard (24), Kneese and Smith (76), Hufschmidt and Fiering (77), Fiering (78), and Loucks (79). Two other excellent references which have several individual papers on application are: The Proceedings of The International Hydrology Symposium, Sept. 6-8, 1967, Fort Collins, Colorado (80), and The Proceedings of the Thirteenth Congress of The International Association for Hydraulic Research, Vol. 1, Aug. 31-Sept. 5, 1969, Kyoto, Japan (81).

In the last few years stochastic processes have come into use in other fields. Smart (82), Surkan (83), and Seginer (84) have been using them in studying drainage networks; Bhavagri and Bugliorello (85) in studying flood proofing; Benson and Matalas (86) and Matalas and Gilroy (87) in regional analysis; and Berthouex (88) in the simulation of industrial wastes.

Other Considerations

The parameters upon which all synthetic sequences are based $\hat{\mu}_x^{(p)}$, $\hat{\sigma}_x^{(p)}$, $\hat{\gamma}_x^{(p)}$, $\hat{\rho}_x^{(p)}(1)$, $\hat{\rho}_x^{(p)(q)}(0)$, and $\hat{\rho}_x^{(p)(q)}(1)$, p, q = 1, ... m, are unbiased estimates of the corresponding population characteristics. However, they are biased estimates with respect to the synthetic sequences as shown by Matalas (23). The mean $\hat{\mu}_x^{(p)}$, for example, is an unbiased estimate of $\mu_x^{(p)}$ because its expected value is $\mu_x^{(p)}$. However, it is a consistent estimator because its standard error, $\sigma_x^{(p)}/\sqrt{n}$, approaches zero as n approaches ∞. Very seldom if ever is the estimate $\hat{\mu}_x^{(p)}$ actually equal to the true population value $\mu_x^{(p)}$. Yet it acts as the true population value in the synthetic sequences. If several synthetic sequences of length n are generated, the means of the synthetic sequences will be distributed about $\hat{\mu}_x^{(p)}$ and will approach $\hat{\mu}_x^{(p)}$ as n tends to infinity rather than to $\mu_x^{(p)}$.

The same argument holds for all of the distribution parameters. However, parameters from higher order moments experience larger standard errors. They are therefore more highly biased with respect to the synthetic sequences.

As an extreme example, suppose the parameters to be used in generating a synthetic stream flow sequence are taken from a 25-year period of record with a mean of 30 units and standard deviation of 6 units. Then the estimate of the standard deviation of the sample mean about the true mean would be

$$\sigma_x^{(p)}/\sqrt{n} \simeq \hat{\sigma}_x^{(p)}/\sqrt{n} = 6/\sqrt{25} = 1.2 \text{ units} \tag{59}$$

If this 25-year period were in a dry cycle, then it would not be unlikely for the mean to be as much as 2 or 3 units from the true mean; i.e., $\mu_x^{(p)} = 32$. Thus if the sample estimate were used to generate synthetic 25-year sequences, only about 1 out of 20 would be as high as the true mean. This is because the true mean is about two standard deviations greater than the sample mean.

Another thing that must be kept in mind when working with synthetic sequences is that the parameters upon which the synthetic sequences are based were developed from a truncated distribution; i.e., the sample was of finite length. Therefore, the synthetic series cannot be used to estimate extreme events. It should be used only to generate synthetic sequences of approximately the same length as the sample period of record. Many such sequences can, however, be generated for the purpose of observing the different patterns and sequences that are likely to occur.

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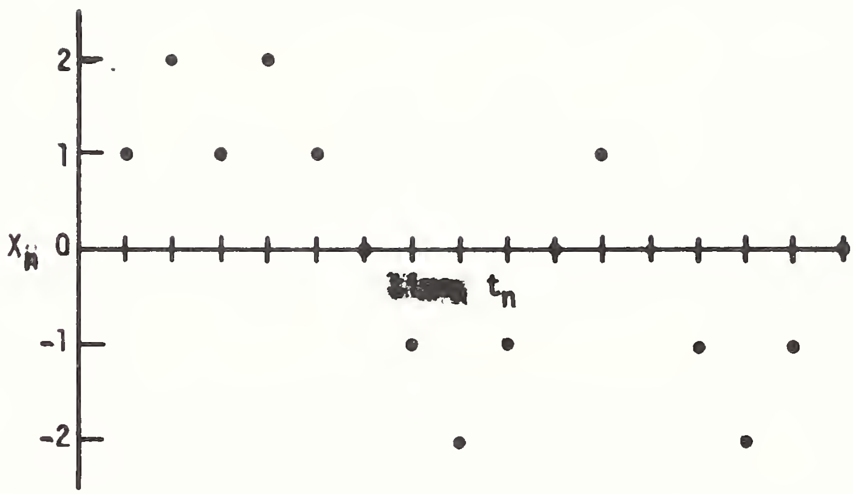

Figure 1. Example of a simple random walk

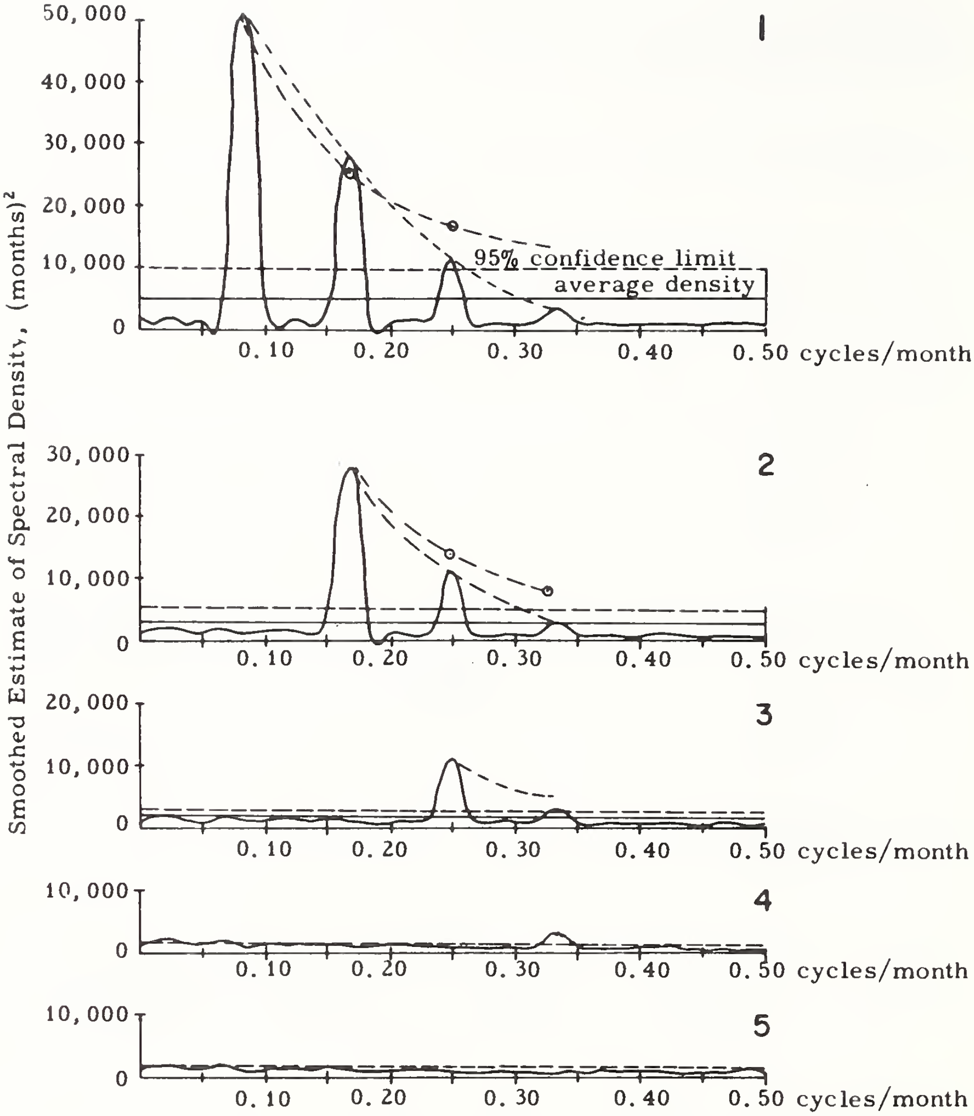

Figure 2. Effect of removing periods from the time series on the variance spectrum for station 9.378, Elk River at Clark, Colorado: (1) Variance spectrum with 12-, 6-, 4-, and 3-month periods present; (2) Variance spectrum with 6-, 4-, and 3-month periods present; (3) Variance spectrum with 4- and 3-month periods present; (4) Variance spectrum with 3-month period present; and (5) Variance spectrum with all periods removed.

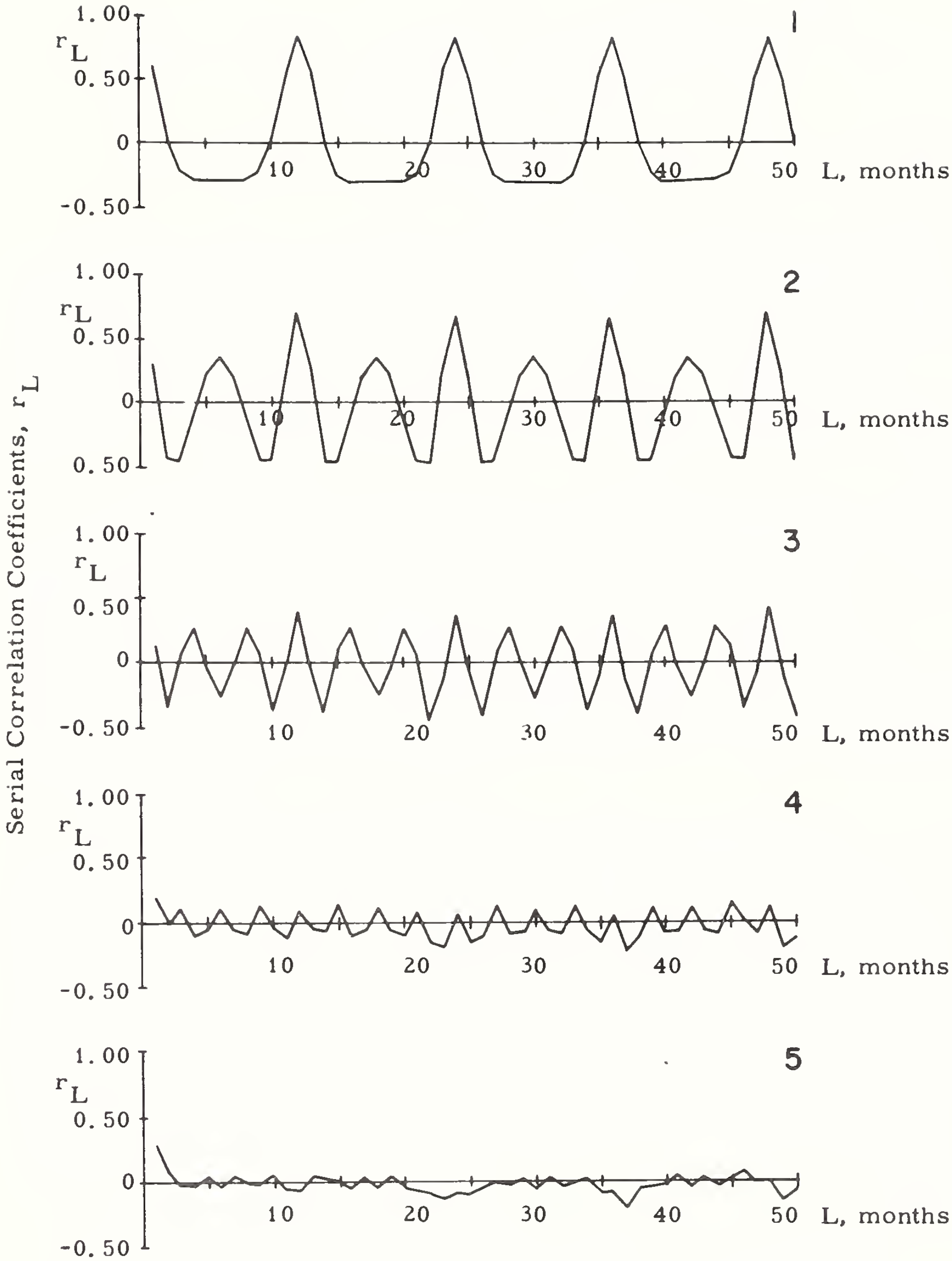

Figure 3. Effects of removing periods from the time series on the correlogram for station 9.378, Elk River at Clark, Colorado: (1) Correlogram with 12-, 6-, 4-, and 3-month periods present; (2) Correlogram with 6-, 4-, and 3-month periods present; (3) Correlogram with 4- and 3-month periods present; (4) Correlogram with 3-month period present; and (5) Correlogram after all periods have been removed.

DETERMINISTIC APPROACH TO WATERSHED MODELING[1/]

by David A. Woolhiser

INTRODUCTION

"No substantial part of the universe is so simple that it can be grasped and controlled without abstraction. Abstraction consists in replacing the part of the universe under consideration by a model of similar but simpler structure. Models, formal or intellectual on the one hand, or material on the other, are thus a central necessity of scientific procedure."[3/]

To this quote I might add that models are also a central necessity of engineering design and of watershed engineering in particular. Engineering models usually contain components derived from the social science of economics as well as elements based upon the physical or biological sciences. The engineering design criterion is economic rather than physical and for this reason "good" engineering may involve "bad" physics but will not involve "bad" economics.

One must realize that in the modeling context, good and bad are not absolute but in fact are subject to reversal. Consider the following example. Models A, B, and C are mathematical models of the infiltration process. All treat the soil as a porous medium subject to Darcy's law for unsaturated flow. Model A includes only the vertical space dimension, Model B has two space dimensions and Model C includes three space dimensions. From a strictly physical viewpoint, we might rank these models C, B, A because Model C more nearly approaches the physical complexity of the real system than do B or A. Depending upon

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1/ Contribution from the Northern Plains Branch, Soil and Water Conservation Research Division, Agricultural Research Service in cooperation with the Colorado Agricultural Experiment Station, Fort Collins, Colorado.

2/ Research Hydraulic Engineer, USDA, Fort Collins, Colorado.

3/ Rosenblueth, Arturo and Norbert Wiener, "The Role of Models in Science," Philosophy of Science XII, No. 4, October, 1945, pp. 316-321.

the problem, the engineer might rank them A, B, C because the added complexity of B and C increases computation costs more than the additional benefits received by more accurate predictions. In fact, the engineer may find a purely empirical expression to be the best. Models must be judged only in the context in which they are to be used.

In this paper, I will consider a limited class of models, e.g., physically-based, deterministic models. Physically-based, in that the models have a theoretical structure based primarily on the laws of conservation of mass, energy or momentum; deterministic in the sense that when initial and boundary conditions and inputs are specified, the output is known with certainty. This type of model attempts to describe the structure of a particular hydrologic process and therefore may be helpful in predicting what will happen when some change occurs in the system. For this reason the results should be readily transferable.

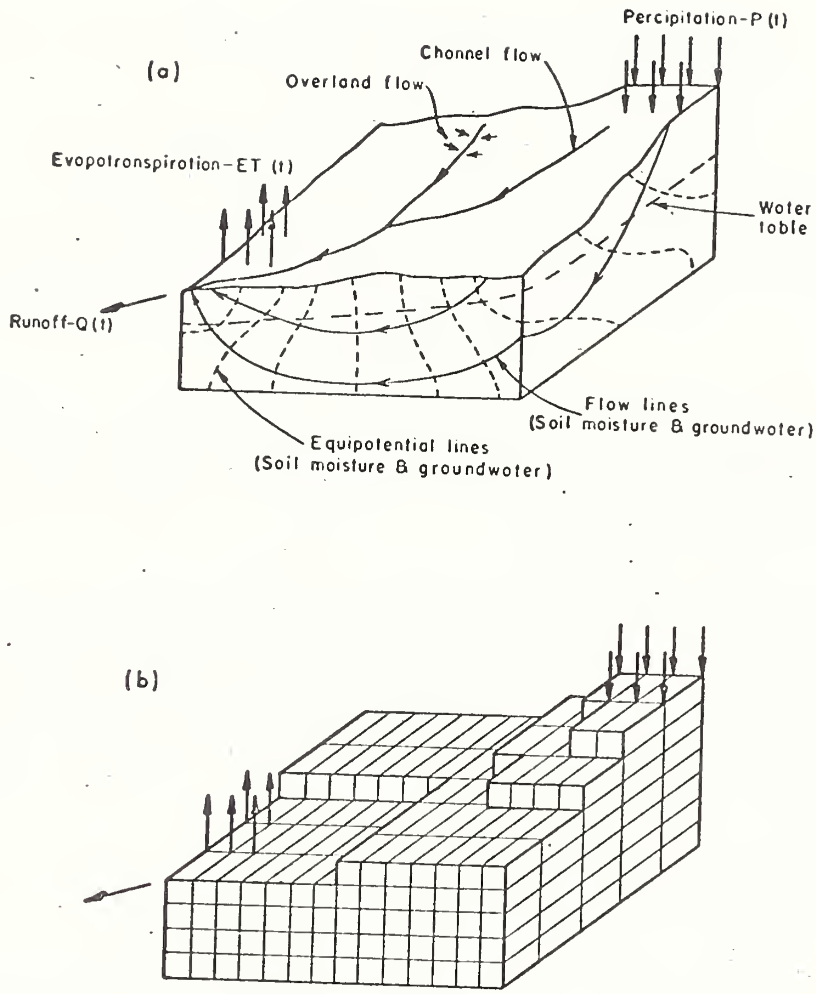
As predictive models in situations where no substantial changes will occur and representative input and output data are available, there is no reason to believe that physically-based models will be superior to the "black box" approach.

GENERAL, PHYSICALLY-BASED WATERSHED MODEL

The general question being asked in research is this: "Is it possible to develop a physically-based, distributed model of watershed behavior?" Such a model would consist of a set of linked partial-differential equations in up to three space variables and time. These equations, with boundary conditions defined by the surface and subsurface geometry of the basin, would operate upon distributed rainfall input data to estimate streamflow and groundwater flow from the basin as well as soil moisture levels. A conceptual, three-dimensional hydrologic model proposed by Freeze and Harlan is shown in Fig. 1.

Although a general watershed model is desirable for engineering design, the model should be widely adaptable so that it can be used for different sized projects and for locations with varying amounts of available data. The general model is composed of a number of components, each of which may be represented in several ways. The general research question is not directly amenable to experimental test because it is very difficult to prove (or disprove) the validity of a general model. For this reason it is good strategy to subdivide the general problem

(a)



(b)

Figure 1. Schematic diagram of (a) hydrologic basin and (b) three dimensional nodal model of hydrologic basin (from Freeze and Harlan).

into first order questions that can be tested. Some examples of these questions are as follows:

(1) Can we develop a deterministic model of infiltration?

(2) Can we develop a deterministic model of overland and open-channel flow?

(3) Can we develop an evapotranspiration model for conditions of less than potential ET?

There are obviously several other important components in a general watershed model but the remainder of this paper will be devoted to a brief discussion of these three.

INFILTRATION

An infiltration model is perhaps the most critical component in a general watershed model because small errors in infiltration predictions may result in large errors in runoff predictions.

The physically-based approach toward an infiltration model has developed around the theory of single-phase flow in a porous medium. The equation for vertical, unsaturated flow in a porous medium can be readily derived from a mass balance in an element of soil along with Darcy's law relating soil moisture flux to the unsaturated hydraulic conductivity and the potential gradient.

If we consider the volume of unit cross-sectional area and thickness dz, shown in Fig. 2, we can write the continuity equation for an incompressible fluid as

$$V = V + \frac{\partial V}{\partial z} dz + \frac{\partial \theta}{\partial t} dz \tag{1}$$

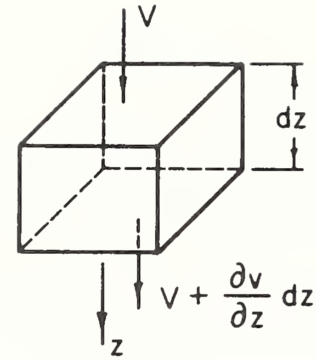


Figure 2. Elemental soil volume.

Where V is the macroscopic volumetric flux of liquid water and θ is the volumetric water content.

From equation (1) we have

$$\frac{\partial\theta}{\partial t} = \frac{-\partial V}{\partial z} \tag{2}$$

Darcy's law for unsaturated flow states

$$V_z = -K_\theta \frac{\partial\psi}{\partial z}$$

where K_θ is the hydraulic conductivity and ψ is the soil water potential,

$$\psi = h + z$$

where h is the soil water (matric) pressure head and z is the height above an arbitrary reference level. By substituting the expression for V into equation (2) we obtain

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left(K_\theta \frac{\partial\psi}{\partial z}\right) = \frac{\partial}{\partial z}\left[K_\theta\left(\frac{\partial h}{\partial z} + 1\right)\right] \tag{3}$$

Equation (3) cannot be solved analytically except for some special cases; it can however be solved by finite-difference techniques. The solution gives θ as a function of z and t as shown in Fig. 3. The mass infiltration during the time interval $t - t_o$ is given by

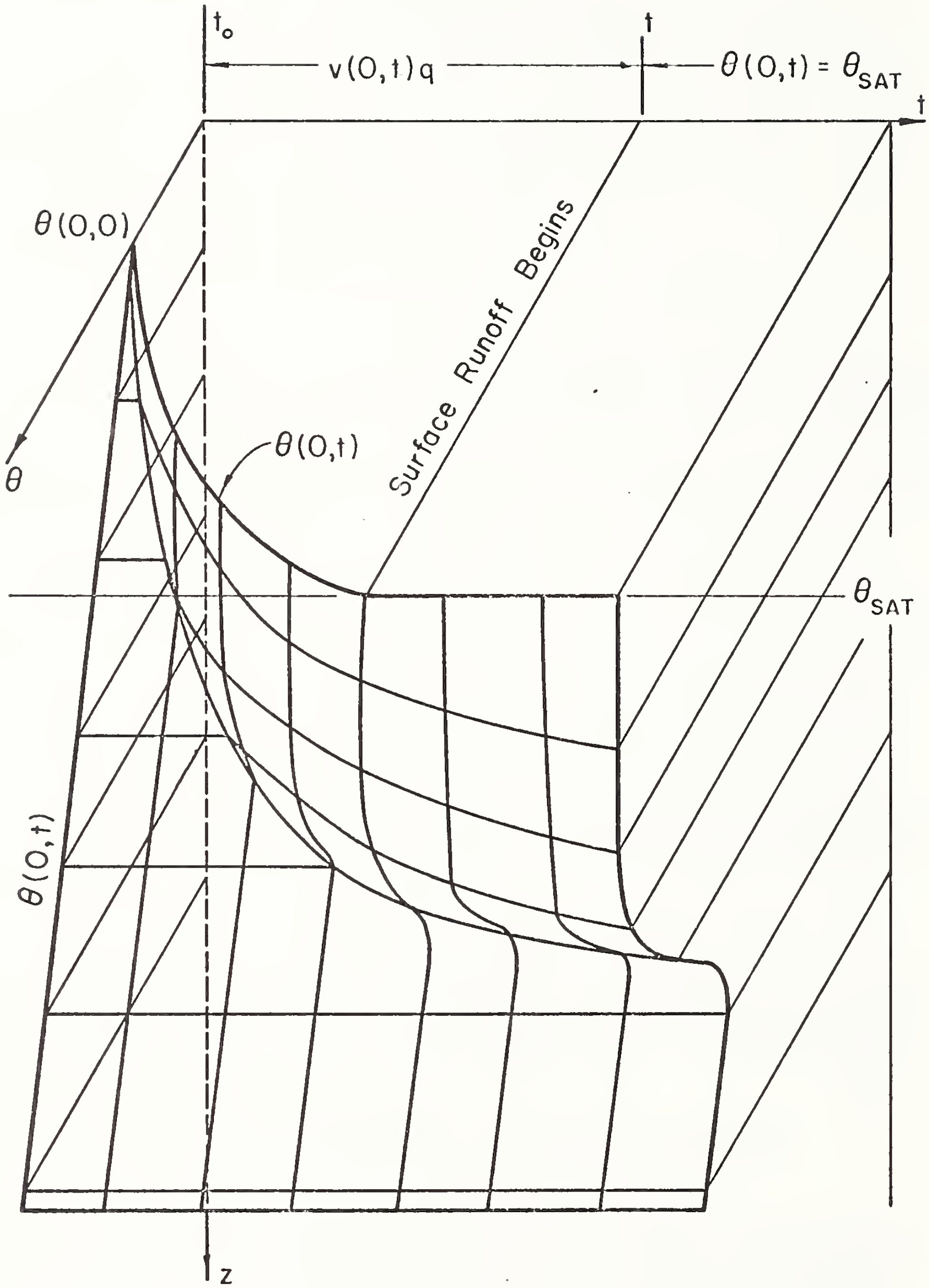


Figure 3. Solution surface of Richards Equation.

$$F = \int_0^\infty \theta(z,t)dz - \int_0^\infty \theta(z,t_o)dz$$

The boundary conditions required are also illustrated in Fig. 3. The initial condition $\theta(z,0)$ is arbitrary, but must be specified. During the time interval $t_s - t_o$ the soil surface is unsaturated and the infiltration rate is controlled by the rainfall rate. Thus we have

$$-K_\theta \left(\frac{\partial h}{\partial z} + 1\right) = q \qquad (4)$$

where q is the rainfall rate.

At time t_s the surface is saturated and the boundary condition becomes

$$\theta(0,t) = \theta_{SAT} \qquad (5)$$

This condition holds as long as free water remains on the surface. When surface runoff ceases, the flux through the surface is zero, or

$$-K_\theta \left(\frac{\partial h}{\partial z} + 1\right) = 0 \qquad (6)$$

The example shown in Fig. 3 is for the semi-infinite case $(0 \leq z < \infty)$ so no lower boundary condition is required. Under a shallow water table condition, the one-dimensional equation may not be appropriate because saturated flow occurs predominantly in a lateral direction and the water table elevation would fluctuate. A water table at a fixed depth may serve as an artificial boundary condition and not have a profound impact on infiltration rates.

To obtain solutions of equation (3) for real soils, we must know the functional relationship between h and θ and between K and θ or K and h. These relationships can be obtained for small disturbed samples in the laboratory. No method has yet been devised to measure these properties in the field. Even the laboratory procedures are difficult and time consuming. Examples of $h - \theta$ and $K - h$ curves are shown in Figs. 4 and 5.

Note that these are not single-valued functions but exhibit the characteristic of hysteresis; e.g., with constant θ, h depends on whether or not the soil is wetting or drying and upon the past history.

The infiltration model specified by equation (3) is quite general in that soil properties can vary in the vertical direction so that various layered situations can be modeled successfully. However, equation (3) includes a number of simplifying assumptions which may invalidate it for some applications.

(1) The one-dimension equation implies that the soil surface is a plane. On a rough cultivated area, initial moisture flow may be strongly three-dimensional.

(2) Air movement is ignored. Strictly speaking, infiltration represents a two-phase flow problem; the simultaneous movement of water into the soil and the flow of air out of the soil. Air effects are probably insignificant when water tables are deep or where water on the surface is not continuous. Under ponding conditions with a shallow water table, infiltration rates computed by equation (3) would be too high.

(3) The soil remains a continuous porous medium irrespective of the moisture content and does not change in volume. Equation (3) is not applicable to shrinking or swelling soils or to soils that crack upon drying. It also could not be applied to a newly plowed or cultivated field.

(4) The properties of the soil as defined by the soil curves are assumed to be time invarient. In fact, soil properties near the surface can change as a result of raindrop impact, erosion or freezing and thawing. These changes can have very important effects on runoff.

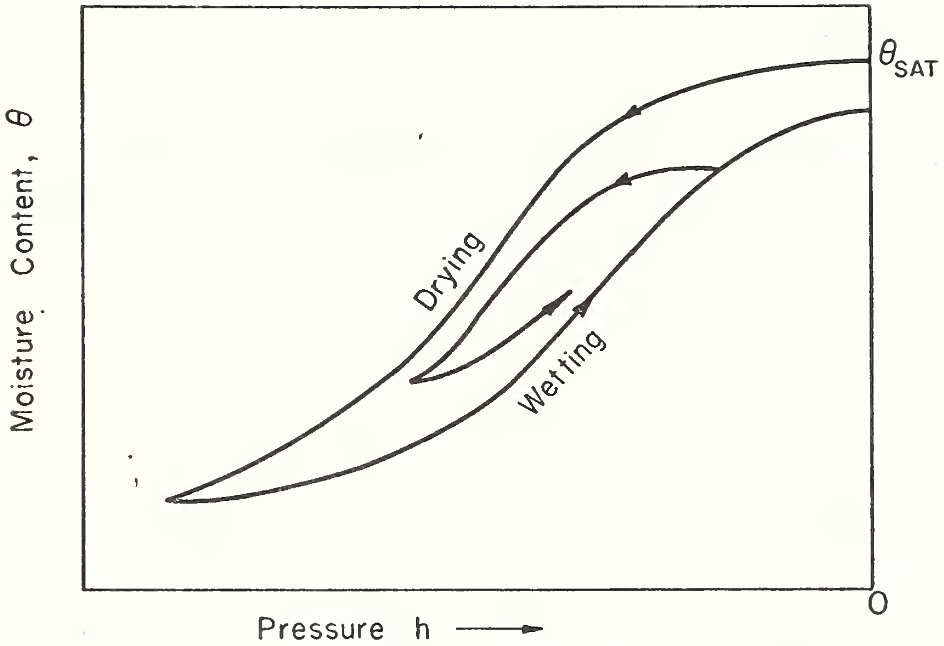


Figure 4. Typical moisture content-pressure relationship.

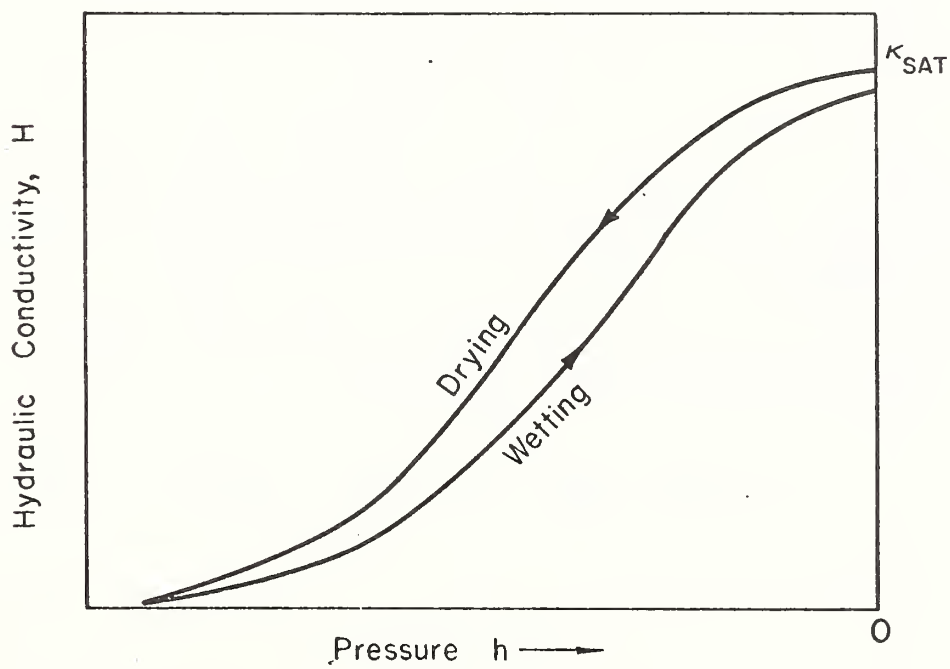


Figure 5. Typical hydraulic conductivity-pressure relationship.

(5) It is also assumed that flow does not occur because of temperature or electrical gradients or in the vapor phase.

Numerical solutions to the one-dimensional equation of unsteady flow in unsaturated porous media have aided in understanding the process of infiltration. In the present state of development, this model may be useful in predicting the behavior of experimental plots and very small watersheds. Application to large watersheds with substantial spatial variations of rainfall or soil properties appears to require an excessive amount of input data and computational time.

OVERLAND AND OPEN CHANNEL FLOW

The most rapid response of a stream to rainfall on the watershed is that due to surface runoff. When rainfall rates are in excess of the infiltration rate, water accumulates on the surface and under the influence of gravity flows overland toward the stream channels. Both overland flow and open channel flow can be classified as unsteady, free-surface flow with lateral inflow (or outflow).

For a complex channel as shown in Fig.6, we can apply the principle of continuity (conservation of mass) and obtain the equation:

$$\frac{\partial A}{\partial t} + \frac{\partial Au}{\partial x} = q\ (x,t) \tag{7}$$

where A is the cross-sectional area, u is the local velocity, $q(x,t)$ is the lateral inflow rate per unit length of channel and x and t are the space and time coordinates.

By applying the principle of conservation of momentum, we obtain the equation of motion

$$\frac{\partial u}{\partial t} + \frac{u\partial u}{\partial x} + \frac{g\partial h}{\partial x} = g(S_o - S_f) - {}^{q}/_{A}\ (u-v) \tag{8}$$

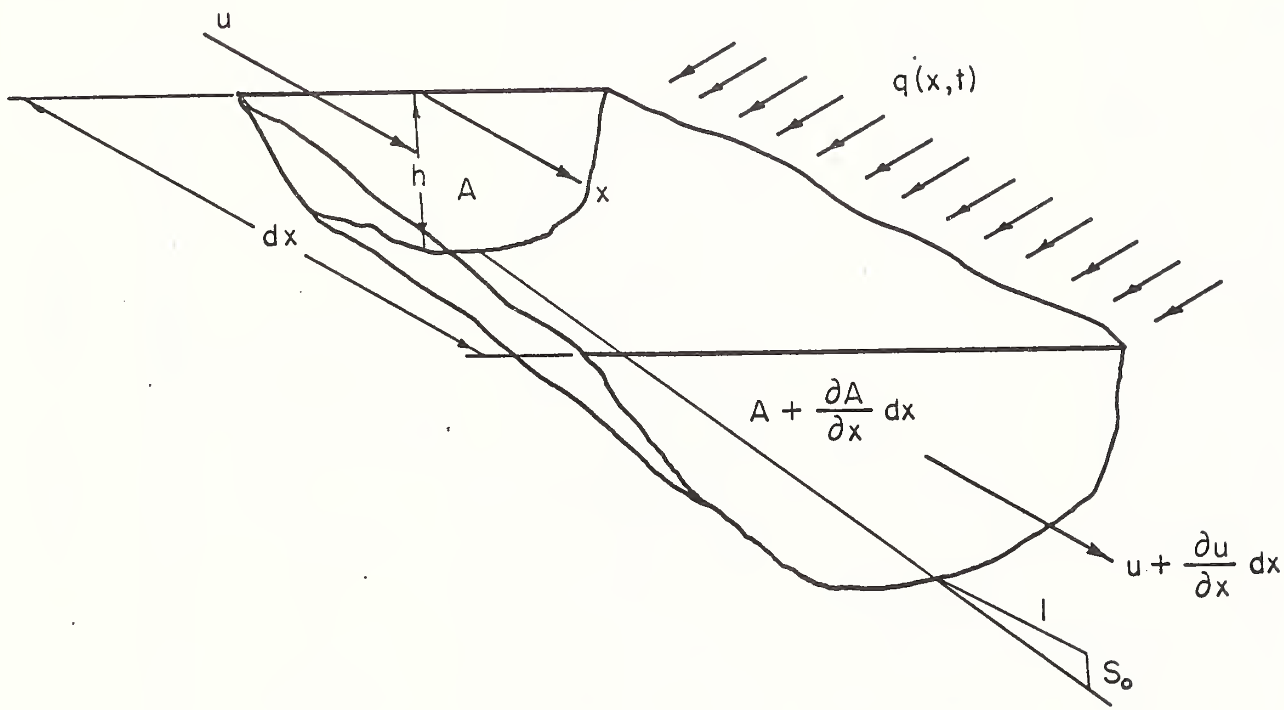


Figure 6. Definition sketch unsteady, nonuniform channel flow.

where h is the local depth, g is the acceleration of gravity, S_o is the bed slope, S_f is the friction slope, and v is the x-component of the velocity of the lateral inflow (usually assumed to be zero). The friction slope S_f is given by either the Manning or Chézy formula. These are the fundamental equations of one-dimensional, open channel flow. By far the largest part of books on open channel hydraulics is concerned with the solution of these equations or of special cases of these equations.

Analytic solutions are not available for these equations so they must be solved numerically for all but some special cases. With the second and third generation digital computers now available and the advancing knowledge of numerical methods, unsteady flow computations can be readily carried out.

One of the advantages of physically-based models is that we can often find simplified approaches that give reliable results and furthermore we can develop physically-based objective criteria for determining when the simplified approach is permissible. As an example, consider the kinematic-wave approximation to equations (7) and (8) for an overland flow problem. A definition sketch for overland flow is shown in Fig. 7. Equations (7) and (8) apply but h can be substituted for A in both equations. The initial conditions are

$$\begin{aligned} h(x,0) &= 0 \\ u(x,0) &= 0 \end{aligned} \tag{9}$$

the upstream boundary condition is

$$u(0,t) = 0 \tag{10}$$

and a possible downstream boundary condition is critical depth

$$u(L_o,t) = \sqrt{gh(L_o,t)} \tag{11}$$

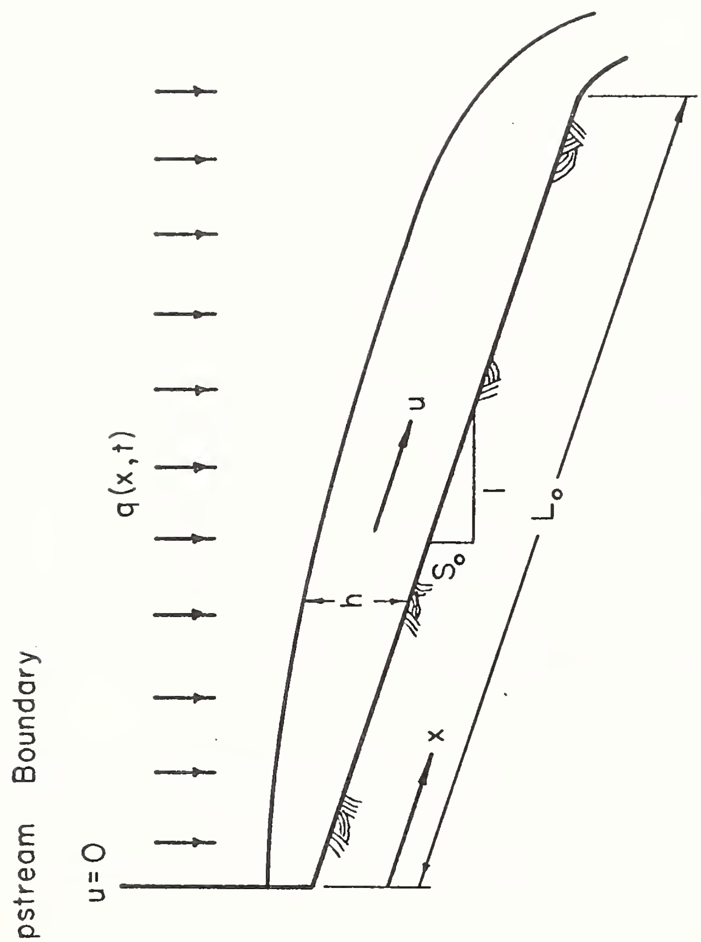


Figure 7. Definition sketch, overland flow.

The kinematic wave equation utilizes equation (7) in its entirety but eliminates all of the derivative terms and the terms accounting for the momentum of the lateral inflow from equation (8). Thus equation (8) becomes

$$S_o = S_f \tag{12}$$

or using the Chézy formulation

$$u = C\sqrt{S_o\ h} \tag{13}$$

which is the relationship between u and h for uniform flow. This expression for u can be substituted into equation (7) and we obtain a single equation in one dependent variable, h ,

$$\frac{\partial h}{\partial t} + \frac{\partial (C\sqrt{S_o}\,h^{3/2})}{\partial x} = q \tag{14}$$

The upstream boundary condition is

$$h(0,t) = 0 \tag{15}$$

No downstream boundary condition is required.

Equation (14) retains all of the physical parameters present in equations (7) and (8) but is much easier to solve although numerical methods are required for general cases.

It has been shown that the kinematic approximation is very good when a dimensionless parameter, k , called the kinematic flow number, is greater than 20. The kinematic flow number is defined as

$$k = \frac{S_o L_o}{H_o F_o^2}$$

where S_o is the bed slope, L_o is the length of the flow plane, H_o is the normal depth at the downstream boundary at the maximum steady-state flow rate and F_o is the Froude number at the downstream boundary at the same flow rate. For most hydrologically significant cases, k is in the order of 100 to several thousand. Therefore, it would be very difficult to distinguish between the solution to equation (7) and (8) and that of equation (14).

Of the physically-based components of a watershed model, that describing unsteady flow is probably the most advanced and the most used in engineering practice. This is true for several reasons. The shallow-water equations were known as early as the 19th century and engineers have been solving special cases of these equations since then. With the advent of the digital computer, complete solutions became possible. Furthermore, the parameters for this model are quite straightforward with the possible exception of the friction coefficient. Even so, engineers have been selecting Manning's n for a long time and while it is by no means completely satisfactory, it can be estimated quite accurately for some channels. This is not true for overland flow where the roughness elements penetrate the free surface and the flow may resemble flow through a porous medium. However, experimental evidence indicates that solutions to either the shallow-water equations or the kinematic-wave equations are adequate models of the phenomenon. More research needs to be done on the problem of estimating roughness coefficients for overland flow.

EVAPOTRANSPIRATION

In the time interval between precipitation events, storage of water in any watershed element is reduced by unsaturated and saturated lateral flows of water and by the process of evapotranspiration. Given some initial configuration of volumetric moisture content as a function of depth, the subsequent evapotranspiration will,to a large extent, govern the amount of ground water recharge and will also control the soil moisture distribution at the beginning of the next precipitation event. Through this influence on base flow and on surface runoff, evapotranspiration exerts a controlling effect on water yield.

Evapotranspiration rates from soil and vegetation depend upon the energy available to vaporize water, the unsaturated flow of water through soil to the surface and to plant roots, and on the type, density and state of growth of plants.

It is possible to analyze the evapotranspiration process and to formulate a mathematical model that helps in understanding the process. Unfortunately these models are extremely complex and require so much input information that they are not useful as predictive tools. This complexity can be illustrated by considering a volume containing soil and the plant canopy as shown in Fig. 8. The evapotranspiration rate at any time is the net flux of water vapor through the sides and top of the above ground portion of the volume element. Water movement is primarily in the liquid phase in the soil. In this form it moves through the soil to the surface and to the plant roots in response to tension gradients and is vaporized at the soil surface and at the surface of the plant leaves. The energy required to vaporize the water must come from radiation, from the advection of sensible heat or from heat stored in the soil. Water vapor is transported out of the canopy zone primarily by turbulent diffusion.

Many of the component processes can be described mathematically, but a general model linking all of the components for realistic plant geometries would be extremely complex. It would include a soil moisture model, a model of plant behavior and a model describing the turbulent transport of water vapor and heat. Input for such a model would include continuous radiation data, wind profiles and directions, air properties and rainfall.

Because of the complexity of a general model, various simplifications have been made to develop physically-based models that apply under restricted conditions. One assumption that results in great simplification is that water supply to the plant and soil surface is not the limiting factor. Evapotranspiration then occurs at the potential rate and is governed primarily by the climatic conditions although the albedo and the surface roughness characteristics also have an effect. Potential evapotranspiration can be calculated from (1) the energy budget, (2) the aerodynamic approach and (3) the combined aerodynamic and energy budget approach. These models are discussed in several of the references cited at the end of this paper and will not be described herein.

Actual evapotranspiration is influenced by soil moisture content and the stage of plant growth. This relationship is a complex one and so far has resisted theoretical treatment. Empirical correlations of the ratio of actual evapotranspiration to potential ET have been used with some success for prediction.

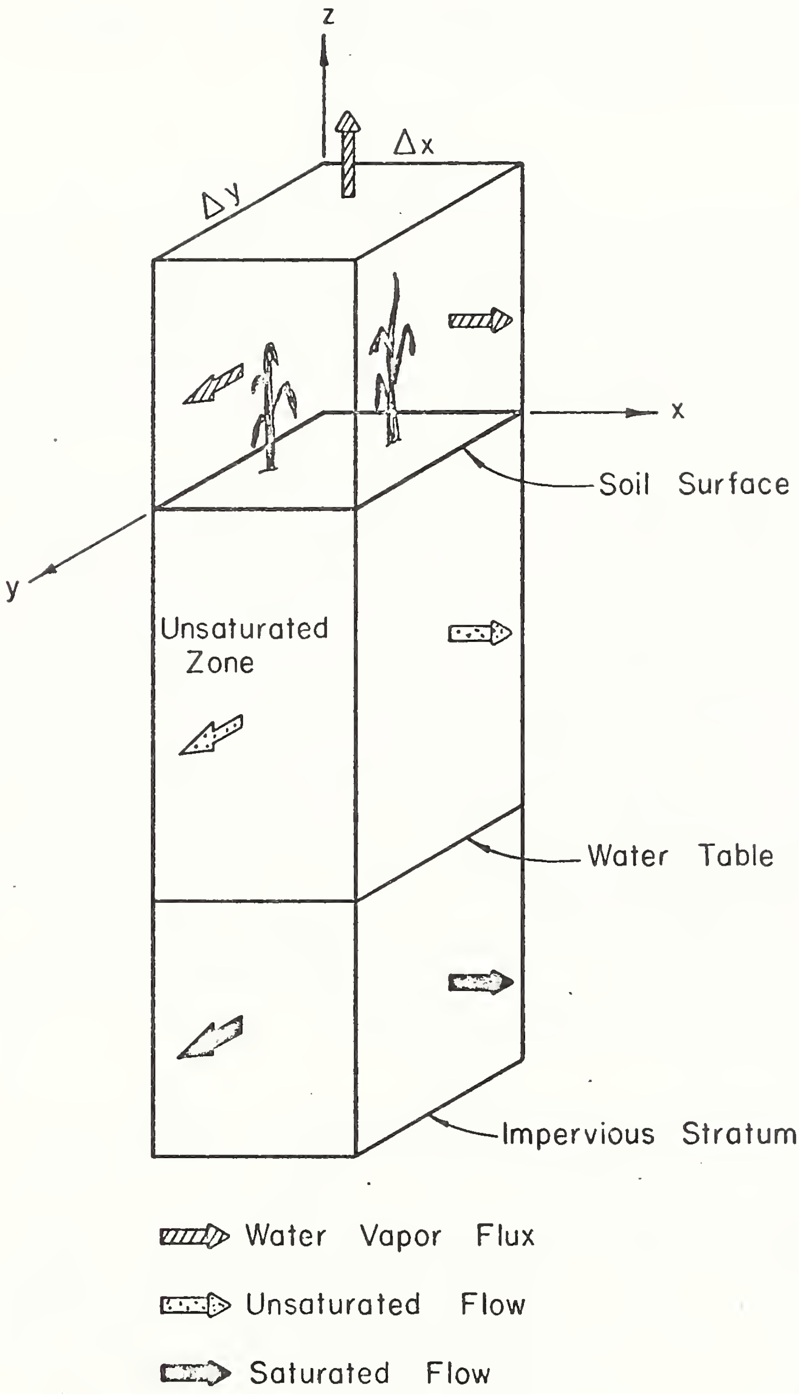


Figure 8. Soil-plant canopy volume.

At present, evapotranspiration models that can be used as components in watershed models consist of both physically-based and black box components. There are no indications that this situation will change appreciably in the near future.

NUMERICAL METHODS

Physically-based models of watershed components invariably involve ordinary or partial differential equations. Solution of these equations frequently requires the use of numerical methods. Such methods are required because the equations cannot be solved analytically or because the initial and boundary conditions cannot be described conveniently as a continuous function.

Fig. 9 illustrates the problems involved in numerical solutions of any differential equation used to describe some physical phenomenon. The box on top represents the "real world" system in all its natural complexity. We might visualize this system as a river reach and the system behavior is defined by observed variables such as depth and velocity in space and time. The second box represents the partial differential equations that have been developed by physical reasoning as a representation of the real system. There are many assumptions and approximations involved in these equations so the true, continuous solutions to these equations would not coincide perfectly with reality. Finally, the third box represents the finite difference or discrete approximation to the differential equations. This discrete approximation to continuous functions also introduces errors so in general the solution of the finite-difference equations will not coincide with the "true" solution to the differential equations.

If we had a perfect instrument for measuring discharge, we could obtain the hydrograph shown as a solid line in Fig. 10. If we had an analytic solution to the differential equation, we could obtain the continuous solution to the mathematical model shown as a dotted line. The solution of the finite-difference equations is shown as a series of discrete points. Now having only the discrete solution, we must know something about the difference between this solution and the continuous "true" solution before we can come to any conclusion as to how well the mathematical equations represent the physical system. This is the problem of convergence. Clearly for a desirable finite-

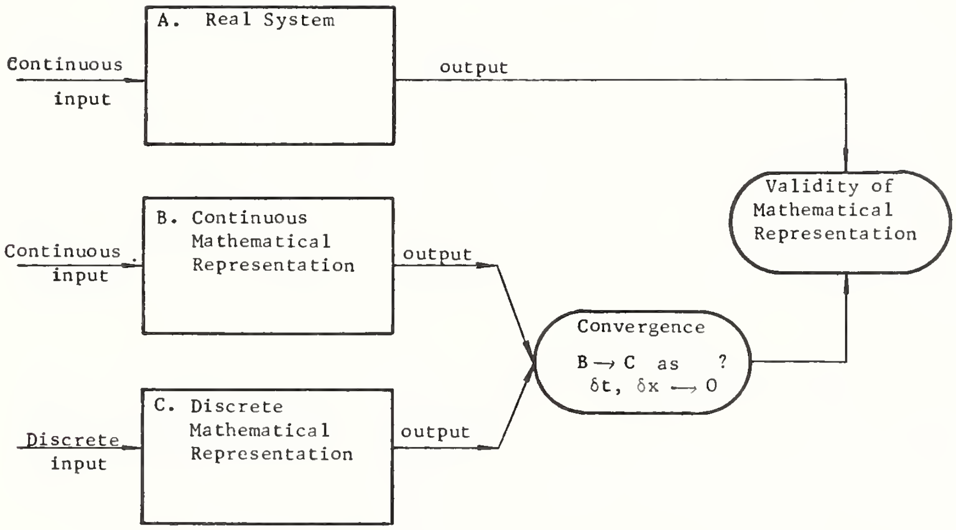


FIG. 9. LOGIC OF FINITE APPROXIMATIONS

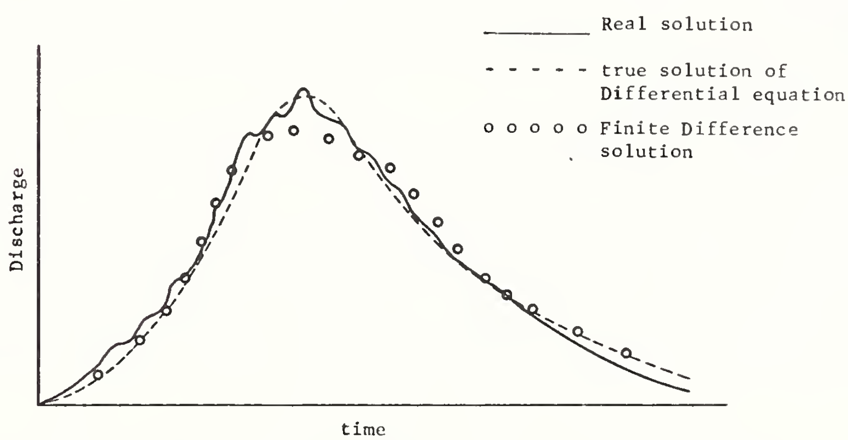


FIG. 10. COMPARISON OF SOLUTIONS

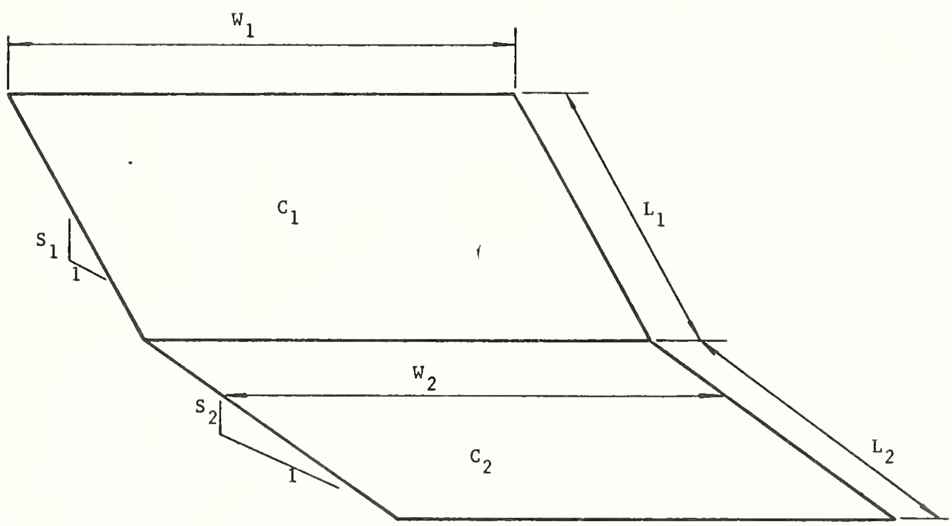
difference scheme, the discrete solution should approach the true solution of the differential equation in the limit as the step size approaches zero.

The general problem of convergence involves two separable aspects: approximation and stability. It is beyond the scope of this paper to discuss these in detail but at a risk of oversimplification we can say that the question of approximation deals with how well the finite-difference equations represent the continuous differential equations in the limit as the increments Δx and Δt approach zero. In an unstable scheme, small rounding errors are amplified by the finite-difference scheme until they completely dominate the solution. It is possible to have stable difference schemes that are poor approximations as well as unstable schemes that appear to be good approximations. In either case, the finite-difference solutions do not converge to the true solutions.

As an example of problems with numerical methods, consider the problem shown in Fig. 11. In this example a two-plane cascade receives a uniform lateral inflow. The upper plane is steeper than the lower plane; all other parameters are the same. The kinematic equations were used to describe the unsteady flow. The hydrograph at the lower boundary was computed by four methods: (1) the single-step Lax-Wendroff; (2) the upstream differencing method; (3) Brakensiek's four-point implicit method, Brakensiek (1967), and (4) by numerical integration along the characteristics. The finite-difference formulations of the rectangular grid schemes are shown in Table 1, and the results are compared in Fig. 12.

This is a very severe test case in that the solution contains a discontinuity or kinematic shock as well as a very sharp peak. The characteristic solution is the most accurate. As one should expect, the second-order scheme gave the best results of the rectangular grid methods. The first-order schemes have errors of approximately 20% at the peak. All rectangular schemes anticipate the shock and delay the peak. This is typical of such methods because the disturbance must follow the grid points.

The importance of this demonstration lies in the physical significance to be attached to parameters obtained by fitting computed hydrographs to observed data. Let us suppose that we had a real world situation where the kinematic approximation was very good and we had observed the hydrograph shown as the solid line. Now using some objective criterion, such as least squares, we could adjust the Chézy



$L_1 = L_2$; $W_1 = W_2$; $C_1 = C_2$

$\alpha_1 = 10$ $\alpha_2 = 2.5$

FIG. 11 TWO PLANE CASCADE

Table 1. Rectangular Grid Finite Difference Schemes

Method	Finite Difference Equation	Order of Approximation	Linear Stability Criterion
Single-Step Lax-Wendroff	$h_j^{i+1} = h_j^i - \Delta t\,\frac{k}{n}\left[\frac{\left(h_{j+1}^{i^N} - h_{j-1}^{i^N}\right)}{2\Delta x} - \frac{1}{2}\left(q_{j+1}^i + q_{j-1}^i\right)\right] + \frac{\Delta t^2\,Nk}{4n\Delta x}\left\{\left(h_{j+1}^{i^{N-1}} + h_j^{i^{N-1}}\right)\left[\frac{k}{n}\frac{\left(h_{j+1}^{i^N} - h_j^{i^N}\right)}{\Delta x} - \frac{1}{2}\left(q_{j+1}^i + q_j^i\right)\right] - \left(h_j^{i^{N-1}} + h_{j-1}^{i^{N-1}}\right)\left[\frac{k}{n}\frac{\left(h_j^{i^N} - h_{j-1}^{i^N}\right)}{\Delta x} - \frac{1}{2}\left(q_j^i + q_{j-1}^i\right)\right] + \frac{2n\Delta x}{Nk\Delta t}\left(q_j^{i+1} - q_j^i\right)\right\}$	$0(\Delta x)^2$	$\frac{\Delta t}{\Delta x} \leq \frac{n}{\sqrt{Nkh}^{N-1}}$
Upstream Differencing	$h_j^{i+1} = h_j^i - \frac{Nk}{n}\,\frac{\Delta t}{\Delta x}\left(h_j^{i^N} - h_{j-1}^{i^N}\right) + q_j^i\,\Delta t$	$0(\Delta x)$	$\frac{\Delta t}{\Delta x} \leq \frac{n}{2.75kNh^{N-1}}$
Brakensiek's Four Point Implicit	$\frac{h_j^{i+1} - h_j^i + h_{j-1}^{i+1} - h_j^i}{2\Delta t} + \frac{k}{n\Delta x}\left(h_j^{i+1^N} - h_{j-1}^{i+1^N}\right) - \frac{1}{4}\left(q_{j-1}^{i+1} + q_j^{i+1} + q_{j-1}^i + q_j^i\right) = 0$	$0(\Delta x)$	Unconditionally stable

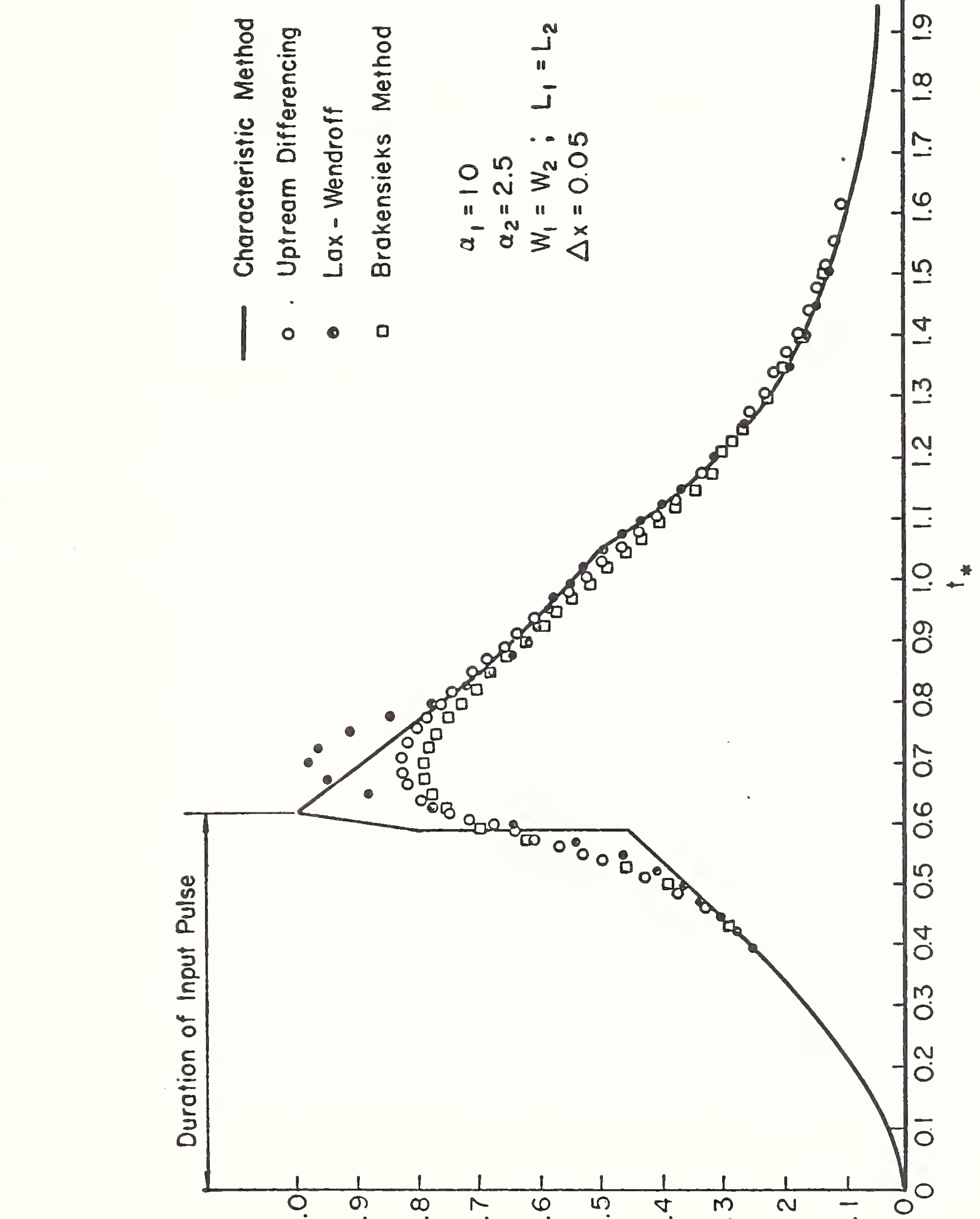


Figure 12. Comparison of Finite- Difference Methods

roughness parameter until we reached an optimum, $\hat{C}$. This optimized parameter, $\hat{C}$, would then be related to the "true" parameter, C, in the following manner:

$$\hat{C} = C + \varepsilon$$

where ε is an error term that depends on the difference scheme used as well as the Δx and Δt interval. I think it quite likely that the error term will not have a mean of zero so that our estimates will be biased and will differ from those obtained by other techniques. If one obtained optimized infiltration parameters as well as roughness parameters, it appears quite likely that the infiltration parameters will be strongly affected by the accuracy of the difference scheme used.

SUMMARY

In this paper I have briefly considered three physically-based, deterministic components of a general watershed model: infiltration, overland and open-channel flow, and evapotranspiration. The basic theory underlying each of these models is well developed. Progress in applying these models is quite different. Of the three, the overland and open-channel flow model is being used in many practical applications, and will replace the black box approach in many cases. Because of stringent data requirements or limitations imposed by some assumptions, the infiltration model cannot be used in some cases. It may be applied for computations of infiltration for plots and very small watersheds. Physically-based models for actual (as compared with potential evapotranspiration) are extremely complicated and do not offer an attractive alternative to combined theoretical-empirical methods.

Solution of the ordinary or partial differential equations describing physical processes on a watershed generally involves numerical methods. The problem of convergence of numerical methods and the possible effect of finite-difference solutions on optimized parameters must be considered.

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THE PARAMETRIC APPROACH TO WATERSHED MODELING[1]

Willard M. Snyder[2]

INTRODUCTION

Parametric hydrology has been defined previously as follows:

> "Parametric Hydrology is defined as the development and analysis of relationships among the physical parameters involved in hydrologic events and the use of these relationships to generate, or synthesize hydrologic events. Historical hydrologic data and known physical data generally are utilized to develop the relationships" (1).

It is desirable to modify this definition slightly to describe the parametric approach to watershed modeling. For purposes of this paper it is proposed that the parametric approach to watershed modeling be the development and analysis or relationships among the hydrologic and physical characteristics of the drainage area contributing streamflow. Utilization of historical hydrologic data and known physical data remain part of the definition.

The change of wording from "physical parameters" to "physical characteristics" is deliberate. If the word "parameter" is limited to a mathematical connotation, the development of a fuller meaning of "a parametric approach" is made easier.

Parameters are defined to be mathematical terms in a functional relationship between variables. These terms operate to scale, shift, or shape the relationship. Consider the very elementary general function shown in equation 1:

$$y = c(x - a)^b \qquad \text{Eq. 1.}$$

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In this equation c is a scaling parameter, a is a shifting parameter, and b is a shaping parameter. In equation 1 the variable x could represent some physical property of a drainage area and y could represent some hydrologic characteristic. For example, equations of this general form are often used to relate flood peak, y, to drainage size, x.

Equation 1 can be called an explicit algebraic relationship. Not all hydrologic or physical characteristics are so explicitly expressed. A unit hydrograph can be represented as a set of tabular values, instead of an equation. The area-elevation characteristics of a watershed may be represented as a graph--the hypsometric curve--instead of an equation. Whether the hydrologic and physical characteristics are given in explicit algebraic form, or whether they are expressed as tables or curves, the parametric approach still means the development of relationships between characteristics. The parametric approach must be made up of two major phases; first, a concept of the desired relationship of characteristics must be developed; and second, the parameters of the relationship must be quantified.

The parametric approach to watershed modeling must be placed in proper relationship to the other approaches, stochastic and deterministic. One comparison results from consideration of the different approaches as giving the position of a researcher on some scale of information. If the researcher is in a position near zero information of cause-and-effect, then he must regard the hydrologic characteristics as stochastic events. Without any information on flood-causing mechanisms, floods are regarded as chance results of a toss of some hydrologic dice. If, on the other hand, the researcher is in a position near perfect information of cause-and-effect, then he regards the hydrologic characteristics as unique consequences of the situation. The flood peak now results from the hydrodynamic solution of flow into and through a reach of channel.

Following the idea of an information scale, parametric hydrology lies between stochastic hydrology and deterministic hydrology. Quite frequently we have some notion of cause-and-effect so we are not in a purely stochastic position. However, we frequently do not have the required perfection of information for the rigorous mathematics of determinism. The compromise position is the domain of parametric hydrology. The goal of parametric hydrology is to retreat from stochasticism

and advance toward determinism.

Since the goal of parametric hydrology is advance toward determinism, one could ask the question, "Why not proceed directly to construction of deterministic models of a watershed?" For situations where it is presently possible, a deterministic approach should be taken. But, a close scrutiny of our system reveals roadblocks to this direct approach. These roadblocks are revealed by comparing man-made and natural systems. Man-made systems follow man-made designs. Such designs are, whenever feasible, the deterministic direction of the system to its objective. The system components are normally rigorously expressible. Such rigor does not normally carry over into natural systems. Particularly in watershed studies, the lack of spatial homogeneity and continuity in characteristics such as soil type, land use, geology and topography results in a system which defines precise definition.

The complexity of the detail of natural systems requires that analytical approaches alternative to the rigor of classical mathematics be developed. At the risk of oversimplification, it might be said that a macro-scale of cause-and-effect is necessary to substitute for the micro-scale of the differential calculus. For example, it may be necessary to impute the gross action in streamflow generation of a significantly sized drainage area instead of deducing such action by areal integration starting from an infinitesimal control area. As another example, we may need to average the flood-attenuating effect of a reach of natural channel when the channel is so complex in configuration as to preclude solution of the simultaneous partial differential equations of continuity of mass and momentum.

The macro-scale concept of physical logic is, of course, an attempt to make respectable the so-called "black-box" procedures in input-output analysis. Black boxes have their purpose when they allow satisfactorily approximate solutions to problems not amenable to classic deductive procedures. After all, we can regard Manning's "n", or a diffusion coefficient, or any other coefficient requiring empirical determination, as a black box. But far more important than this utilitarian aspect is the idea that black boxes are stepping stones to a higher level of understanding. With properly constructed experimental design, the consistency and adequacy of black box structure can be verified. Such experi-

ments, with successful replication, serve to define our black boxes as macro-scale components and linkages of a hydrologic system. Successful experimentation allows us to move toward determinism on our information scale.

PARAMETRIC APPROACH TO ANALYTICAL MODELS

Before proceeding further in a parametric approach to modeling, it is necessary to define our level of understanding of the various processes we wish to model. If we have a concept of the model, and have it formalized and quantified, then we may use our model to predict, or synthesize new data. If, on the other hand, we have not quantified our concept, we have only a hypothesis. It is necessary to verify the hypothesis against real-world data to move it from abstraction to reality. Verification of the hypothesis is a resultant rational quantification. An irrational quantification would obviously mean non-acceptance of the concept.

When we proceed to verify a hypothetical concept, we are following an analytical approach. When we have arrived at a position of sufficient understanding to quantify our concept and then apply it, we are following a simulative approach.

The steps in the parametric approach presented and illustrated below have to do with analysis of data rather than simulation of data. The steps are listed sequentially, but obviously many of the steps overlap and are worked on simultaneously.

Data Processing

Since analytical models presuppose data for verification, the organization of data into usable form is given as the first step. Following instrumental and manual recording, it is necessary to check the data for accuracy against reference information, and to fill in missing data by estimation when feasible. In modern usage the above steps imply computer procedures and the end product is a magnetic tape library of edited data.

Model Formulation

It is hardly conceivable that a research watershed would be established without a stated research objective. Therefore,

it is necessary to regard the watershed as a means to provide data in the approach to the objective. The objective, until it is attained, must be in the nature of a hypothesis. The information forms into which the data will be cast to test the various hypotheses under the objective establish the conceptual analytical models. The information forms range from simple means and variances to mathematical components of a system to generate streamflow or other outflow.

Fitting Routines

The verification and quantification of a hypothetical conceptual model require numerical procedures to optimize the correspondence between the concept and the data. We must fit the model to the data by getting "best" estimates of the numerical values of the parameters. The optimizing processes range over tremendous differences in complexity just as concepts range in structural complexity. The basic techniques of conventional statistics and operations research, as well as the more exotic techniques from such areas as multivariate statistics and systems theory, should all be considered potentially useful in optimization.

Examination of Results

Following the quantification of the concept by fitting to data, the numerical results are checked to establish whether the hypotheses have been verified. Three general modes cover most methods of examination. First, tests of significance from probability theory can be employed where the numerical elements have known statistical properties. Second, the physical rationality of the resultant model structure should conform to the investigator's prior knowledge based on experience and judgment. Thirdly, the methods of simulation can be used to test the sensitivity of the various parametric controls of the model.

Association of Characteristics

The steps of parametric approach to modeling to this point have dealt almost exclusively with treatment of data from one research area. However, in the previous section, Examination of Results, there is a strong implication that the investigator has access to additional information. Most research is not an isolated activity and most modern researchers are not isolated personages. The results of

research form a total pool of information. From this pool of information it is necessary to develop cause-and-effect relationships which support the results of individual contributions to the pool.

The function of the pooled information can be stated in another manner. In any one research watershed it is possible to describe the physical situation--the watershed characteristics--in which the experiment is conducted. However, to apply the results of the research to some other watershed --implying a different set of physical characteristics--the variation of results with variation of characteristics must be established. The parameters of the various relationships among hydrologic and physical characteristics will vary because the models are not perfect. The satisfactory explanation of the parametric variation is the heart of the parametric approach.

Conversion to Prediction Procedures

The final goal of a parametric approach to analysis is prediction. It is at this step that our level of understanding has moved from hypothetical concept to quantified concept. Prediction has a remaining requirement that sampling and observation provide the data necessary to use in prediction. These data are input data to the model, and describe the physical factors of any new--possibly ungaged--watershed for which we wish to establish levels of hydrologic characteristics needed for design purposes.

EXAMPLES OF WORK IN PROGRESS

Some examples from the various steps in a parametric approach are given below. The steps as listed are somewhat idealistic in their sequence. Practically, work in the Southeast Watershed Research Center is proceeding at several levels simultaneously. All of the examples given show the concentration upon computer-oriented methods. This concentration is based on three major considerations:

1. To capture the efficiency and time-saving of automated procedures on large masses of data.
2. To establish objectivity of the various analyses and reduce the bias of personal subjectivity.
3. To provide processing continuity even with changeover of personnel.

Data Processing

Data as observed or recorded are rarely in the form necessary for analysis. Instrumental failure and instrumental error must be anticipated. Algorithms for correcting data and estimating missing data are a necessary part of any approach involving experimental verification.

Gage Height Estimation and Correction

Continuous recording of stream stage with digital-punch gages is hampered by two main types of trouble--an error in gage height due to debris blocking the weir or control, or else clock stoppage. We are presently developing an error and estimation procedure based on simple polynomials (2). Such polynomials can be structured to be common and tangent to good record before and after the record break. Additionally, polynomials of higher degree may be required to pass through intermediate points, if a field check is made by the observer.

Figure 1 shows a 70-hour period during stream recession where the constrained polynomials were used to estimate a record. Since this is a test case, the true record is shown as a series of data under the estimation. Correspondence is excellent.

Figure 2 shows the correction of a record using constrained polynomial. The result is a smooth transition to eliminate an abrupt change such as might be caused by resetting a gage or clearing a weir-notch.

Selection of Storms

In selecting storm events for use in model verification, attention should be paid to the statistical requirements of the data sampling technique. Special attention is particularly necessary when a model contains a component expressing the volumetric reduction of storm rainfall to storm streamflow. The selection of storms should be based on the rainfall, not on the runoff, since a significant rain can conceivably have little runoff, and consequently, significant volumetric reduction.

An algorithm for selection of rainfall events above some threshold has been proposed previously (3) and will be employed or modified for future studies. The algorithm is shown schematically in Figure 3. Starting from some arbi-

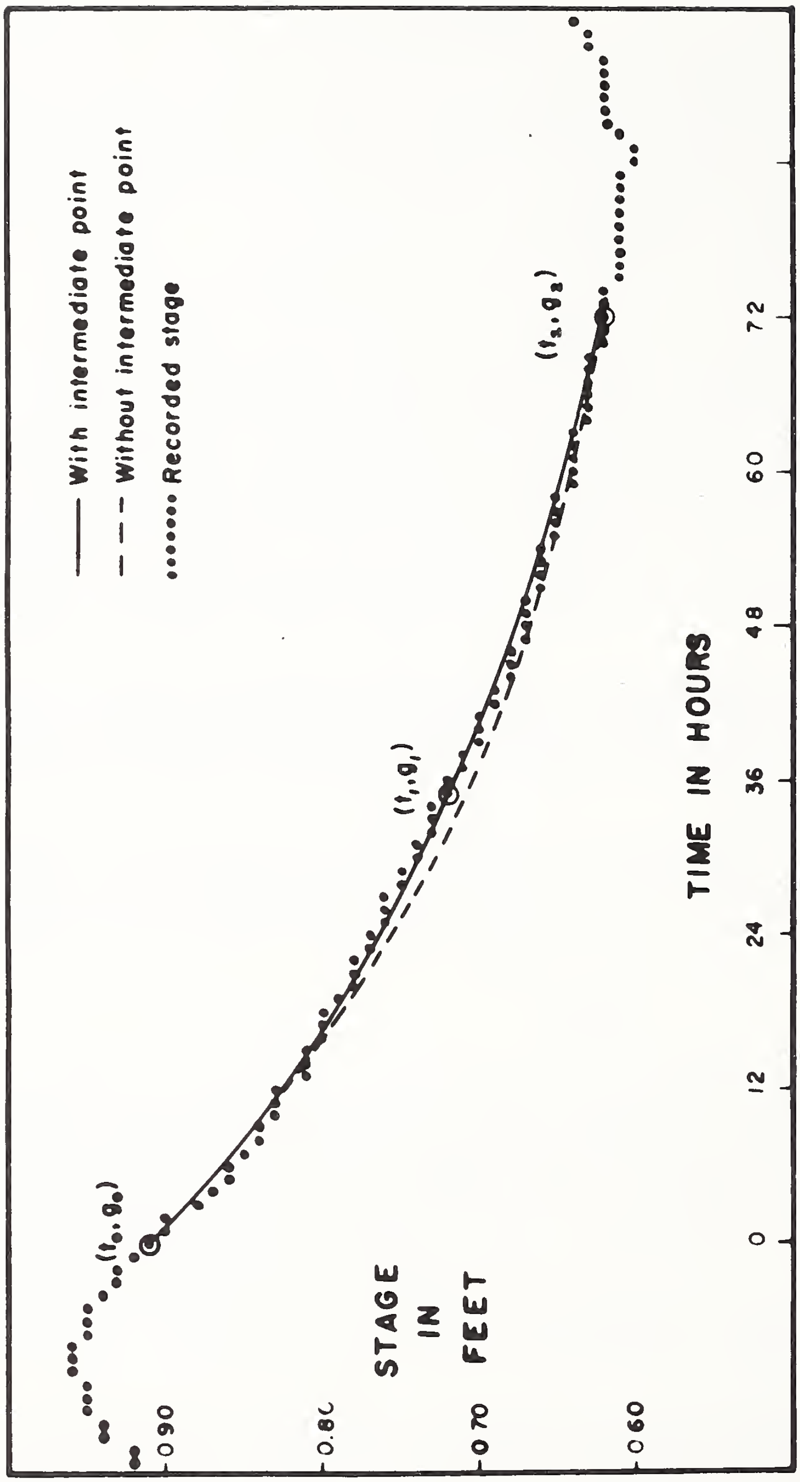


Figure 1. Stage estimates for hydrograph recession.

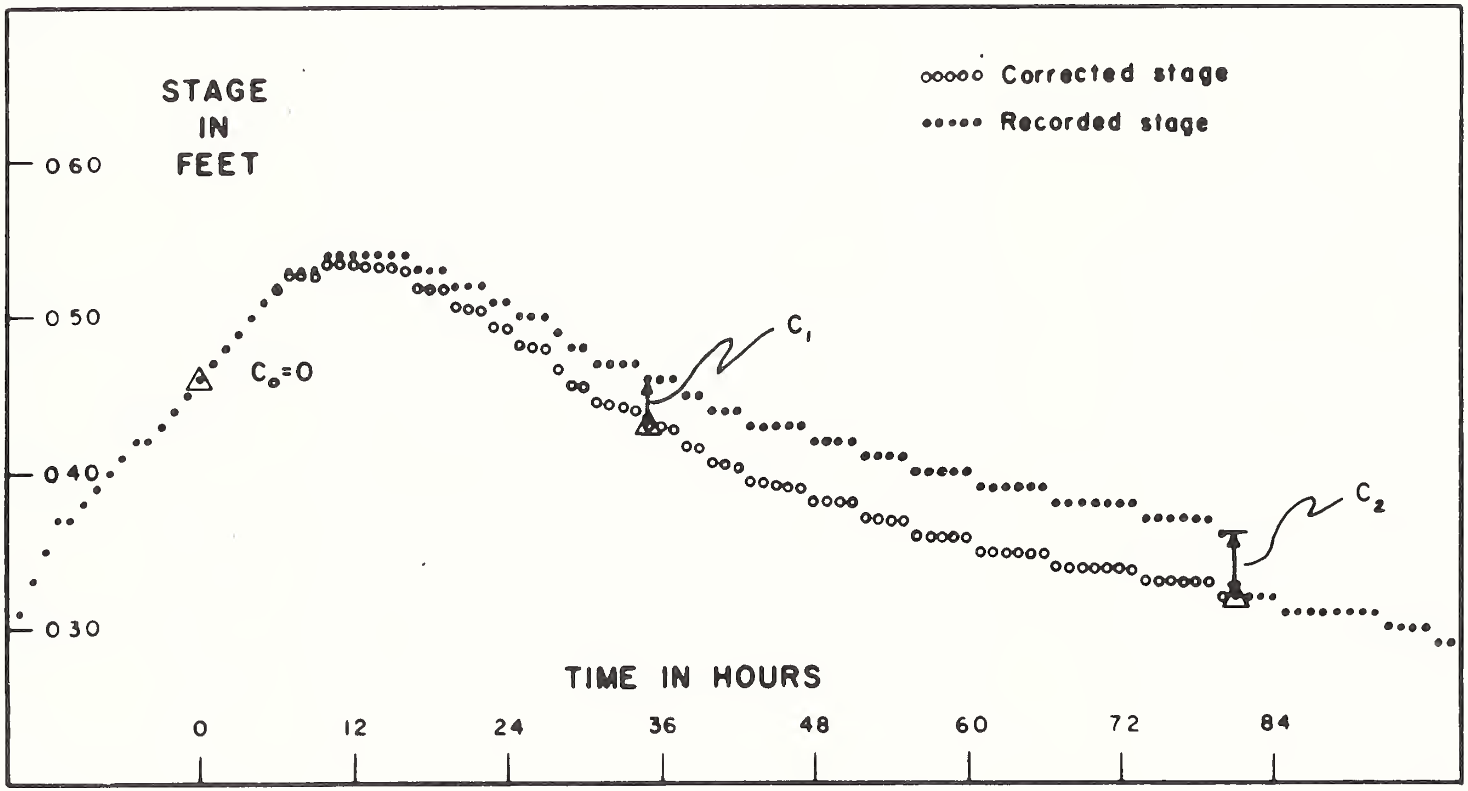


Figure 2. Stage correction with known intermediate point.

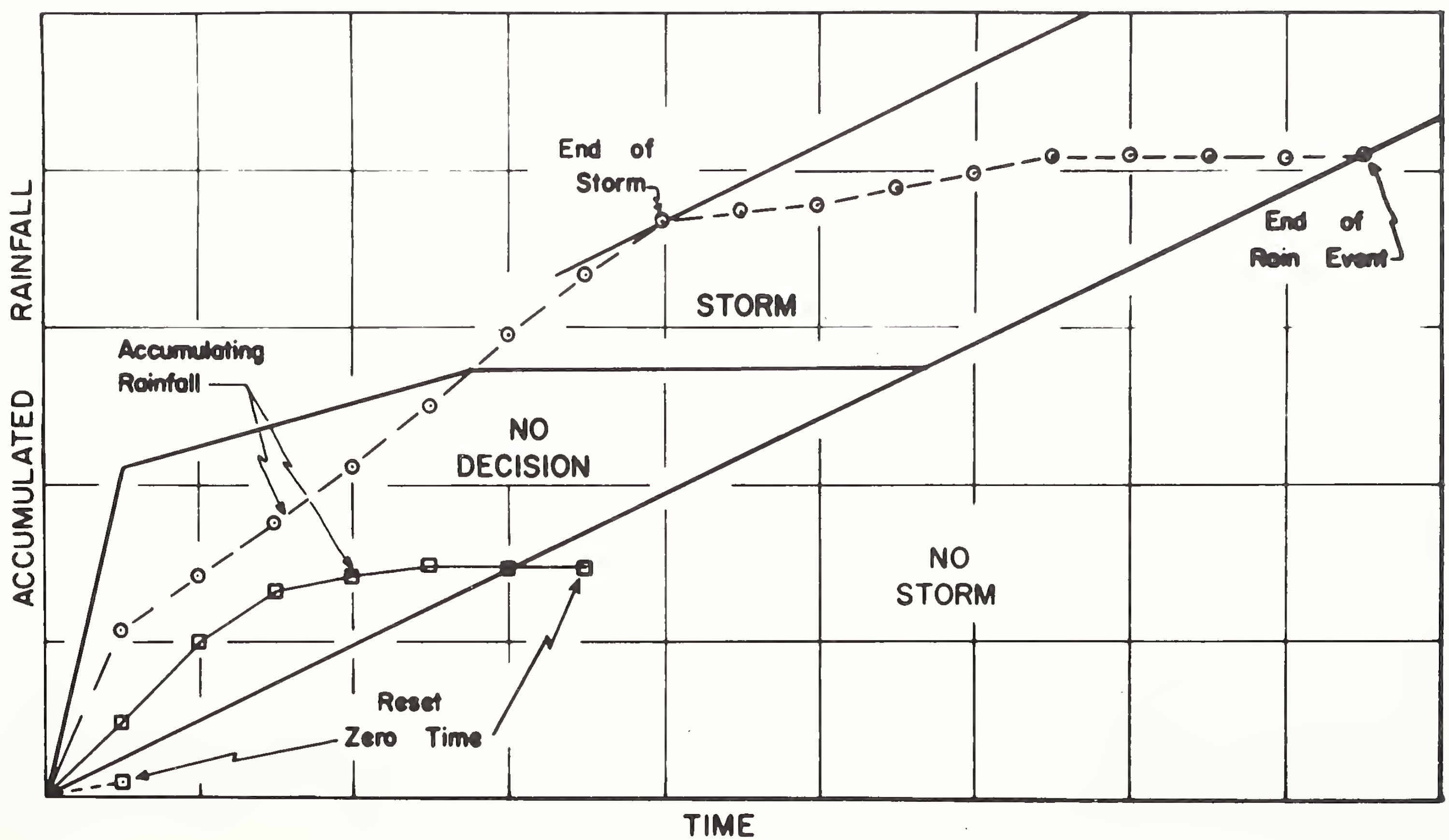


Figure 3. Decision algorithm for selection of storms.

trary point of zero time, the accumulating amount of rainfall--say hourly amounts--are checked against previously set decision boundaries. For insignificant rains the event is rejected and the time zero point advanced through the record. For significant rains a storm event is selected and the amount and duration of the storm rainfall are given automatically. The advantage of such an algorithm is that all storms above the decision thresholds are selected from the record. There can be no subjective tendency to reject a storm because of analytical complexity of the consequent hydrograph.

Model Formulation

Modeling of several hydrologic elements significant in watershed analysis is being attempted. Each of these models should be regarded as representative of subsystems. A model representative of the total watershed system hopefully will evolve.

Water Retention

A mathematical form is presently under development for quick and easy calculation of rainfall proportion that is effective in the generation of a storm hydrograph (4). This form is defined as watershed retention for two reasons. First, it is hoped that the model will be usable in urban as well as non-urban areas, and the infiltration approach is obviously a poor starting point for urban runoff estimation. Secondly, storm hydrographs are presently being analyzed without first separating so-called surface runoff from groundwater runoff. In other words, the total response of the stream to an input of effective rainfall is analyzed. This eliminates the subjective separation of flow prior to analysis. The definition that surface runoff is rainfall in excess of infiltration no longer applies. The rate of retention can be thought of as the rate at which dead storage on and in the soil profile is satisfied.

The mathematical form of the retention model is given in equation 2:

$$r_{t+\Delta t} = r_t - (a + br_t)AB(r_t - r_c)\Delta t \qquad \text{Eq. 2}$$

$$\text{where } A = \frac{R_{\Delta t} + r_u - r_t}{R_{\Delta t} + r_u - r_c}$$

$$B = \frac{R_{\Delta t} - r_c}{R_{\Delta t} + r_c}$$

$r_{t+\Delta t}$ is rate of retention at time $t + \Delta t$

r_t is rate of retention at time t

a and b are shape parameters

$R_{\Delta t}$ is rainfall rate during time increment Δt

r_u is maximum rate of retention

and r_c is minimum rate of retention.

Equation 2 is a finite difference form of an initial value problem. Given the retention rate at time zero, the rate a short time later can be computed from rainfall and the equation parameters and site characteristics. More work needs to be done in calibration of this model against selected small watersheds and subwatersheds. Figure 4 shows the retention function evaluated for a set of parameters and two different starting values. Effective rain is any rain greater than the retention rate.

Time-Separated Hydrographs

It was mentioned above that an attempt is being made to analize total storm streamflow and eliminate arbitrary separation of flow into categories. When information is needed about relative timing of varying proportions of the streamfow, the total hydrograph can be apportioned following analysis. Lines separating the hydrograph into various flow delay times are constructed. It is necessary to derive a unit hydrograph--or more appropriately here, a unit total hydrograph --from the storm, or else utilize one already available. The method is probably best illustrated by using a discrete convolution example.

Consider a unit response function defined by its ordinates

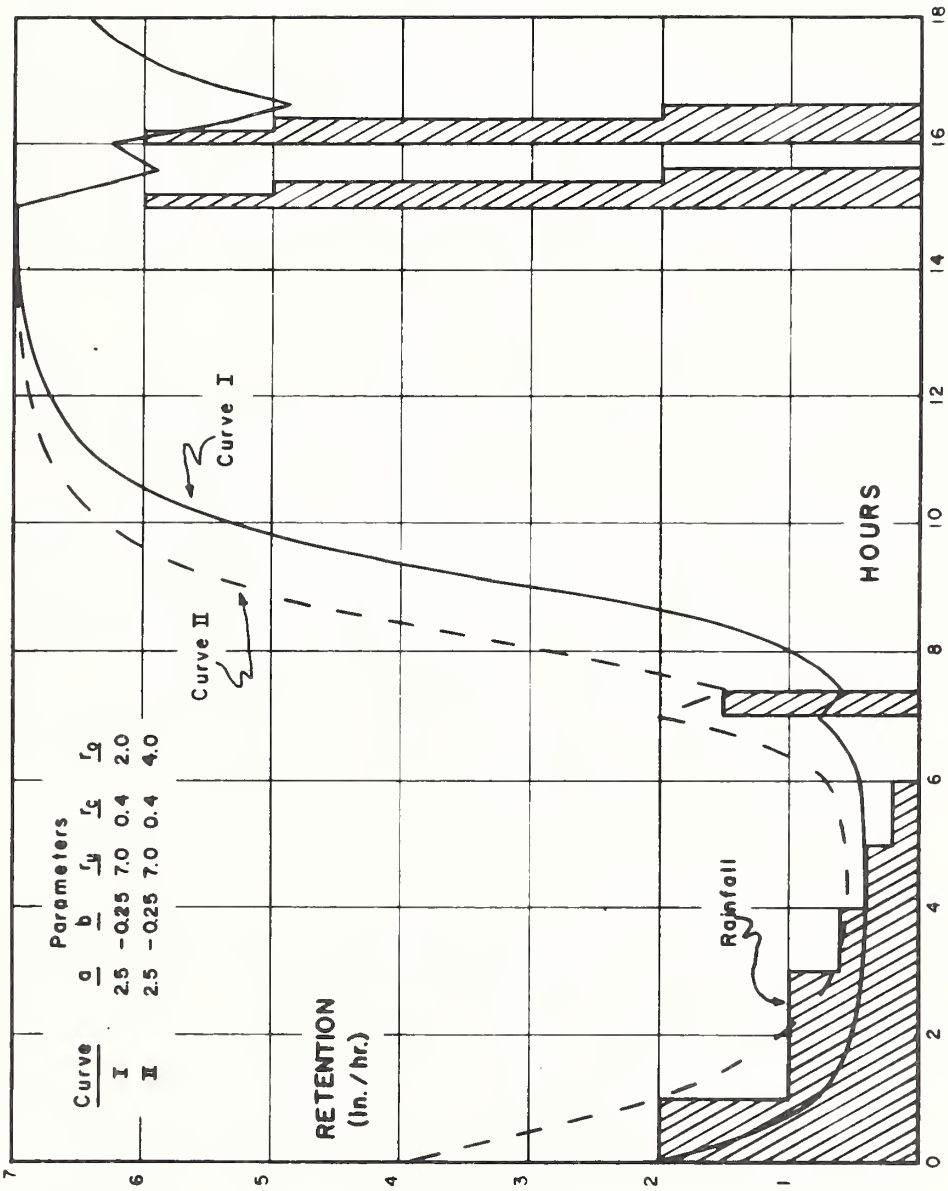


Figure 4. Example computations with watershed retention function.

at 1-hour intervals. The area under the response function from time zero to the 1-hour ordinate represents the proportion of flow having between zero and one hour travel time from runoff source to the gage. The area between 1-hour and 2-hour ordinates represents the proportion of flow having from 1 to 2 hours travel time, and so on. If now, one reconstructs the storm hydrograph using the 1-hour increments of effective rain and the unit response with the 1-hour ordinate deleted, the water with zero to 1-hour travel time is eliminated, and a partial storm hydrograph results. Deleting the 2-hour ordinate from the response function additionally eliminates from the total storm hydrograph the water with one to two hours delay time.

Figure 5, which is Figure 9 from reference (3), shows a constructed example of time-separation of hydrographs. The rising dashed lines are partial hydrographs which define the patterns of travel time given the effective rainfall indicated at the top of the figure. The areas in the subdivisions of the hydrograph represent volumes of flow in the different timing categories. More important, however, is the condition that at any elapsed hydrograph time the total discharge can be fractionated into proportions by delay time. This is an important feature if one were, for example, sampling the flow for suspended sediment or some chemical or biological constituent. If such constituent were related to some proportion of rapid flow rather than total flow at sampling time, such proportions are readily computed.

Two-Stage Convolution

Conventional procedures for outflow hydrograph construction, or for hydrologic flood routing, use a single stage of convolution. Under this procedure an input effective rain or an inflow hydrograph is transformed to an outflow function by operation of a unit response function--usually called in hydrology a unit hydrograph. Such procedures are relatively simple if the response function is assumed to be invariant during the progress of the storm.

It has been amply demonstrated by many researchers that the response function does not remain constant. However, the resulting non-linear systems are difficult of mathematical expression and solution. To overcome some of these difficulties, a discrete computational system involv-

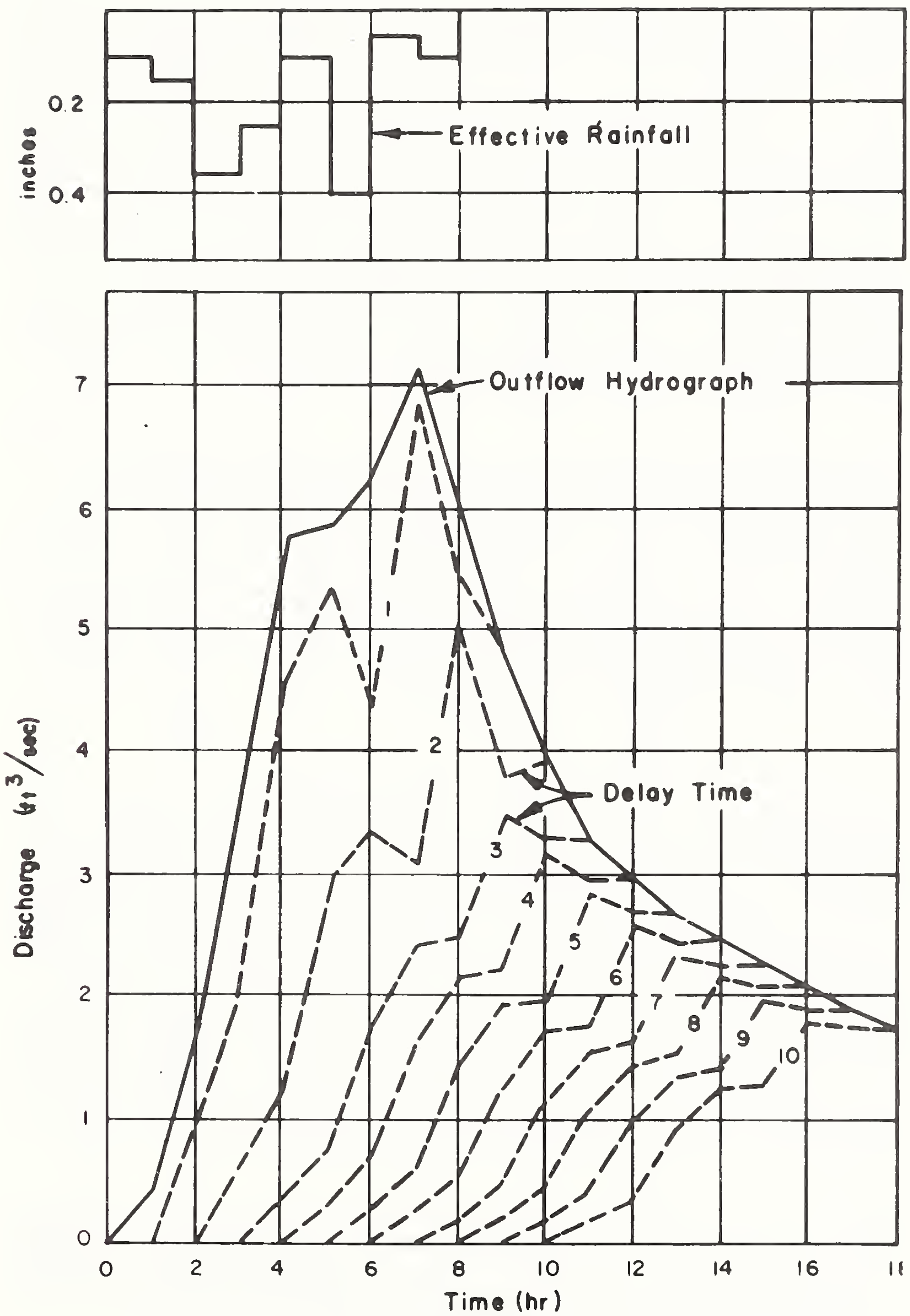


Figure 5. Example of time-separated hydrograph.

ing two stages of convolution has been developed. A schematic comparison of one-stage and two-stage convolution is shown in Figure 6. In the first of two stages a static characteristic function is convolved with a variable state function to produce variable unit response functions. The characteristic functions may be regarded as a one-dimensional representation of the potential of the drainage area to produce runoff. The state functions represent the capability of the area to translate this runoff potential to the stream gage under varying states of "wetness" of the area. Each finite time period during a storm thus has a separate and distinct response function. The second stage of the procedure convolves these response functions, each with its respective rainfall increment.

This concept of two-stage convolution with the first-stage variable has been used in storm hydrograph analysis. A non-linear yield model utilizing the same concept is under development. It is expected that the potential of the concept in flood routing in natural channels will also be explored. The concept may also find utility in predicting "waves" of sediment suspended in the stream during stormflow. Some results of quantification of the concept will be presented in the next section.

Fitting Routines

In early hydrologic usage, curve fitting often meant no more than matching a straight line to a set of points on a graph. If the points did not line up along a straight line, one looked to see if their logarithms did. Usually little thought was given as to whether a straight line was proper because any other form was considered too complicated to fit. In modern usage, curve fitting means optimization. For analytical models optimization means achieving a high level correspondence betwen the concept and the experimental data. The important modern characteristic is that the concept comes first, from the concept comes the form, usually mathematical, that is quantified.

A discussion of optimization is not possible here. The methods are many but the fundamental principle is the same. The principle is expressed in the following direct quotation: "Whatever route a mountain climber takes, he recognizes the peak when he gets there" (5).

The "path to the peak" that has been used quite successfully

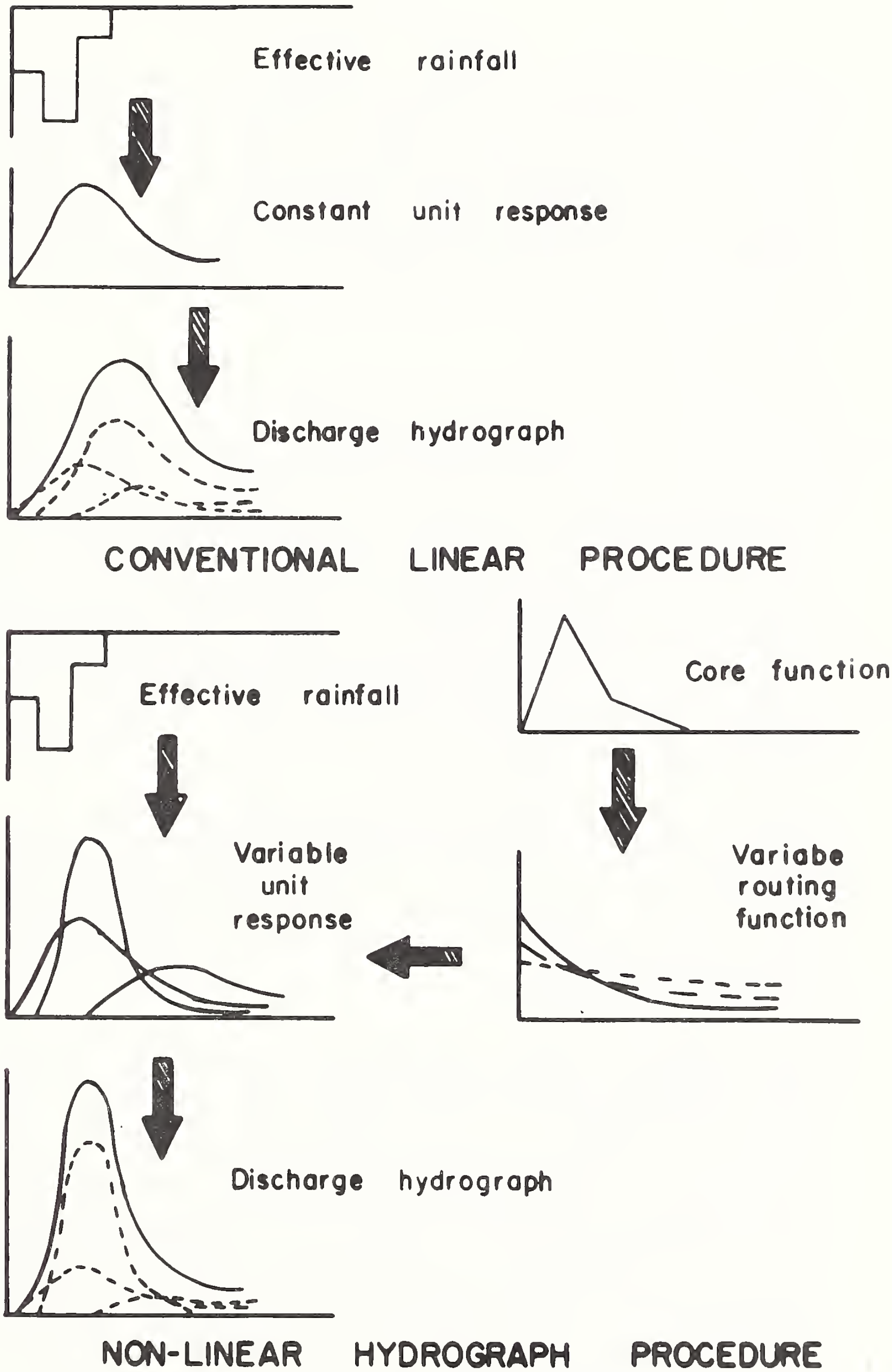


Figure 6. Schematic comparison of hydrograph procedures.

in our modeling work is the principle of least squares. This is expressible as multiple regression when optimization of only linear coefficients is required. Components regression is useful for linear coefficients involving multiple variables instead of the fixed variates of ordinary regression. And lastly, least-squares differential correction coupled with components regression has been found to be an extremely powerful and versatile numerical method (6).

Watershed Retention

It has not been possible to quantify the retention function, Equation 2, as desired, since retention data of the type needed are not presently available. In order to test the flexibility of the equation, it was fitted to published plot infiltration data (7). Table 1 shows results of fitting to certain of the Edwardsville, Illinois plot data. Four elements, the two shape parameters, the maximum value, and the initial value, were optimized. The results are remarkably consistent except for one run on Plot 8, which arouses suspicion as to interaction between shape parameter "b" and the maximum value, r_u. The values of the correlation coefficients indicate almost exact correspondence between infiltration data and model.

Since the retention model to this time appears to be a satisfactory form, it is expected that attempts at quantification will be continued. It will be necessary either to incorporate the form in an input-output model or else first develop retention-rate data by input-output analysis.

Two-Stage Convolution

The numerical method of non-linear least squares coupled with components regression has been used to evaluate two different mathematical models embodying the concept of two-stage convolution. A storm analysis model has been programmed to derive two components from data (8). The first component is the characteristic function expressed in mathematically free form as a series of connected linear segments. The second component is the variable routing function, expressed as the simple descending exponential. The shape parameter of the exponential curve is found by fitting, using the streamflow antecedent to each period of effective rainfall as a feedback variable. Both components are evaluated simultaneously by the fitting process.

Table 1. Optimized parameters based on plot infiltration

Plot	Run Date	Optimized Values: a	b	r_u	r_o	r_c	Δt	Cor. Coef.
				in./hr.	in./hr.	in./hr.	hr.	
1	6/17/40	1.576	-0.099	5.744	1.489	0.19	0.25	99.6
1	6/10/41	1.404	-0.085	5.202	2.064	0.01	0.25	99.7
6	9/13/40	1.610	-0.034	7.328	1.849	0.25	0.25	99.1
6	7/23/40	1.465	-0.115	5.204	2.041	0.40	0.25	99.3
8	6/26/40	1.870	0.128	21.437	1.871	0.49	0.25	99.5
8	6/27/41	1.556	-0.052	6.650	1.874	0.53	0.25	99.8
16	3/26/41	1.513	-0.025	8.984	1.677	0.17	0.25	99.9
16	7/12/40	1.737	-0.059	14.107	2.093	0.30	0.25	99.7

Figure 7 shows the degree of correspondence between the model and the observed outflow hydrograph for a 16-square-mile watershed in the Taylor Creek basin just north of Lake Okeechobee in Florida. Figure 8 shows the extreme variability in unit response functions within two different storms on the Florida watershed. The lowest peak response and the highest peak response within each storm are shown. These unit response functions for 4-hour duration of effective rain are the result of the first stage of convolution.

A non-linear water yield model based on the concept of two-stage convolution is under development. This model differs only in mechanical detail from the storm hydrograph non-linear model. The unit of time of input and output is 5 days instead of a few hours. The input-output process is continuous through a long record, 4 years in our analysis, instead of starting with effective rain and ending during recession as in storm analysis. The water yield model differs in one other detail from the storm hydrograph model. It contains a characteristic function and a state function as in storm analysis, but additionally it contains a third component which allows simultaneous solution for non-effective rain. More work needs to be done on the mathematical detail of the characteristic and state functions of the yield model, but in preliminary trials it has been found that the seasonal rain reduction or loss function is not strongly dependent on the form of the other two model components.

Figure 9 shows the seasonal loss curves for two different drainage areas which were derived from rainfall and runoff data for two different watersheds. It might be thought at first inspection that the two loss curves shown in Figure 9 are so different as to cast doubt on the results of fitting. The differences have a rational explanation, however. The main difference can be explained by the difference in the seasonal distribution of rainfall. Total rainfall for a 5-day period was doubly averaged across 6 successive periods and across the four years of record used in the analysis. These averages are shown as bar diagrams in Figure 9. At Oxford the rainfall is almost uniformly distributed throughout the year. The soil profile begins to dry out in spring and early summer. Therefore, the capacity of the soil to store water--and hence the loss curve--increases rapidly at this time. A double peak of loss occurs, the first in June near the time of maximum solar energy, the second in October

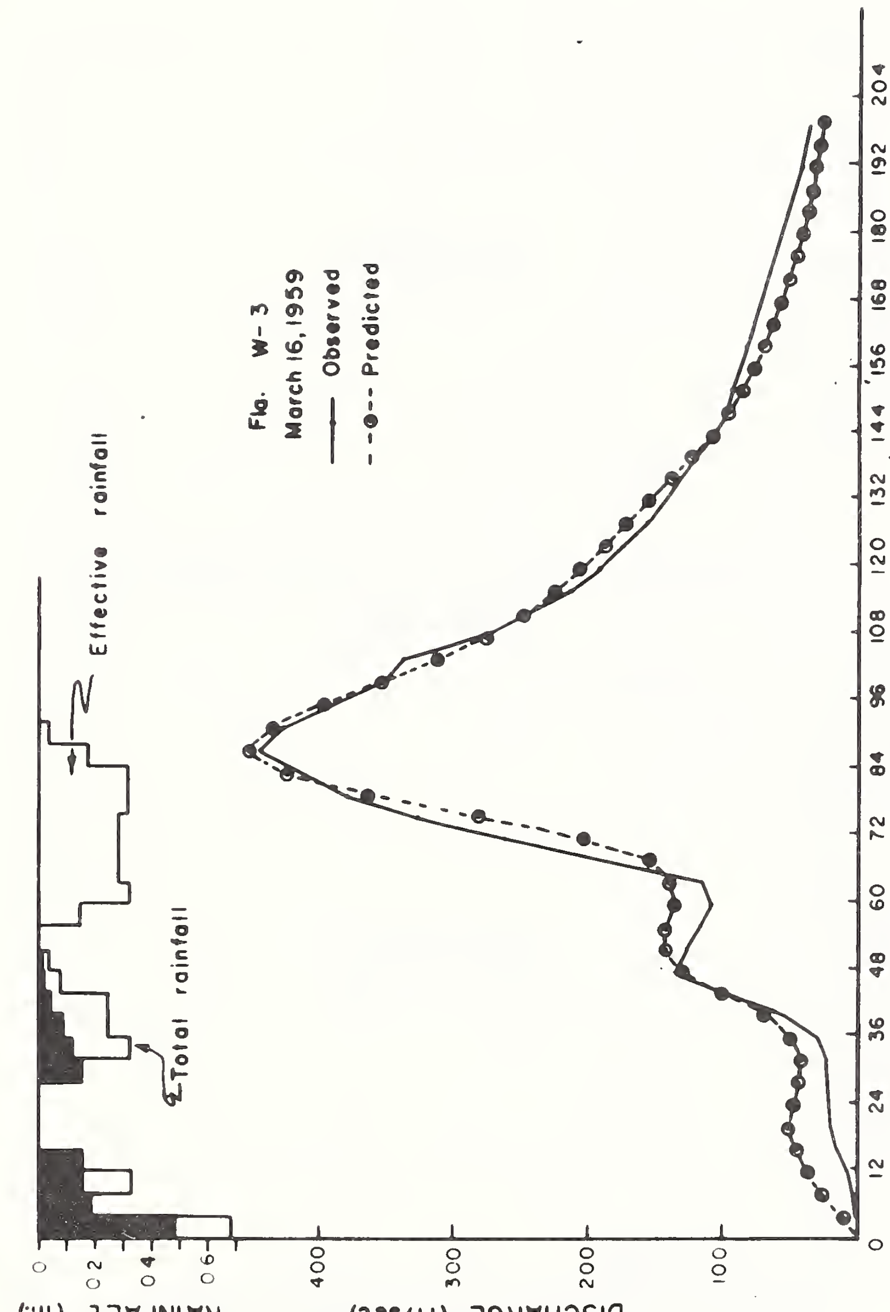
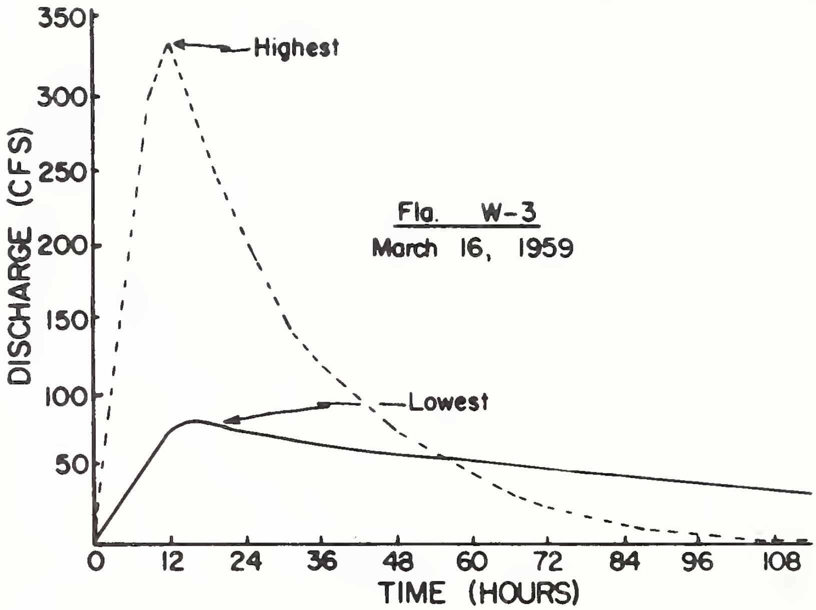


Figure 7. Comparison of predicted and observed hydrographs.



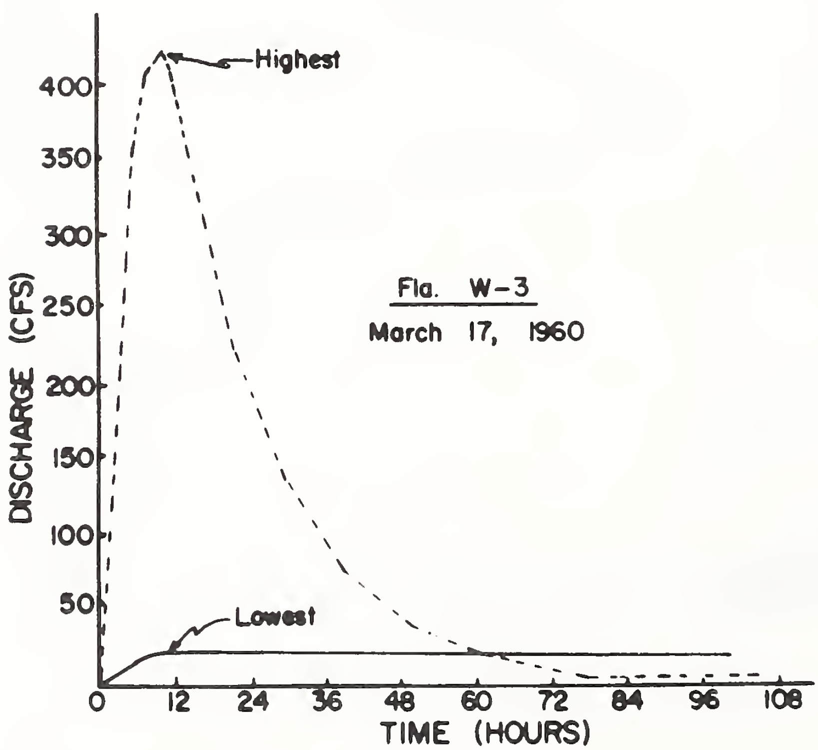


Figure 8. Derived variable unit response functions.

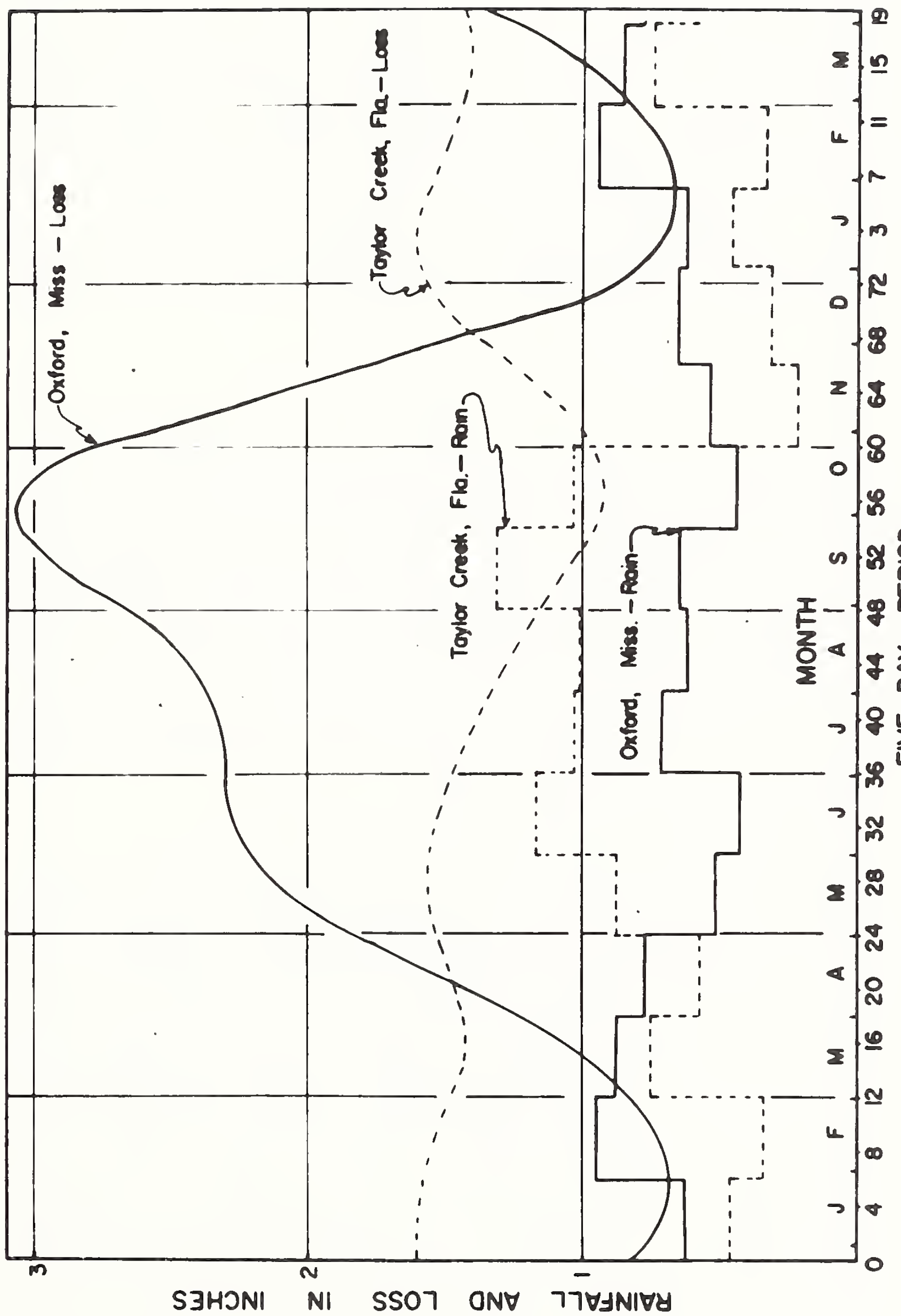


Figure 9. Derived seasonal loss curves.

when the soil reaches its lowest moisture following a summer of drying. The period of little loss is comparatively short and is in the winter season.

For Taylor Creek in Florida, rainfall is highest during the summer months. The loss curve shows the same tendency toward a maximum in June as shown at Oxford. But then loss decreases rapidly and reaches a minimum at about the time Oxford is at a maximum. This is due to increasing water stored in the sandy Florida soils under the abundant summer rain. Under this condition the storage capacity of the soil is satisfied, little additional water can be infiltrated, and hence a minimum of loss occurs.

SUMMARY

The parametric approach to watershed modeling has been presented as "black-box" input-output analysis. Such input-output analysis cannot be done blindly. Each black box must have a rational, identifiable function in representation of watershed hydrologic processes. The mathematical form of the black box must be a formulation of the researcher's concept of the process. The blind choice of a straight line or a simple parabola without regard to its suitability as rational black-box structure will usually produce only fitted curves. Such blind choice will not produce rational model structure leading to improved methods of hydrologic prediction for design purposes.

The parametric approach was outlined in sequential steps of procedure culminating in improved forecasting. Some of these steps were illustrated by examples taken from work in progress in the Southeast Watershed Research Center. There is no intent to imply that these examples represent the only attempts at the parametric approach to watershed modeling. But it is felt that examples drawn from one location do have cohesion and relationship as parts of a total research objective. The long standing interest of many researchers in finding relationships between the hydrologic characteristics and physical characteristics is shown by the listing in the bibliography. The relatively recent emergence of parametric hydrology is a consequence of growing recognition that limited empirical association of crudely expressed characteristics will produce limited improvement in prediction techniques. The rational development of the form of the relationship between the sets of characteristics, and the quan-

tification of the mathematical parameters of these relationships using modern optimization techniques offers much more promise.

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OBJECTIVES AND TECHNIQUES OF WATERSHED MODELING [1/]

M. H. Diskin [2/]

INTRODUCTION

Watershed modeling is a procedure by which watersheds and processes taking place in them are represented by simplified systems which can be conveniently handled by digital or analog computers or, in some cases, by analog or physical small-scale models. The processes represented by the models are usually the parts of the hydrologic cycle that involve rainfall on the watershed and some transformations of the rainfall that take place within the boundaries of the watershed.

Watershed models are a fairly recent development in hydrology. While a number of works may be cited as marking the beginning of this development, there is no doubt that the 1962 report by Crawford and Linsley on the Stanford model was the most important of the early developments of models representing the relationship between total rainfall and total runoff. One of the notable models developed before the Stanford model is the linear reservoir model proposed by Nash (1957) for the relationship between rainfall excess and direct surface runoff.

Models are constructed for a variety of reasons. Two of the most important objectives of model construction are: (a) to gain a better understanding of the functioning of the watershed and the effects of changes in the watershed; and (b) to provide a design tool for generation of synthetic hydrologic data, either as an extension of existing

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data or as a substitution for nonexistent data. While the first of these objectives calls usually for detailed and complicated models that follow as closely as possible the physical phenomena, the second objective allows more freedom in simplification of the model and elimination or lumping of many intermediate steps in the transformation of input to output.

The development of a watershed model calls for two major steps. One involves a decision about the structure of the model and the second is the choice of the numerical values of the various parameters that are needed for the description or the use of the model. These two steps are usually interrelated; the structure chosen for a model influences the values of the parameters, and vice versa, the evaluation of model parameters may point out deficiencies or redundancies in a proposed structure and the need for changes in the structure of the model.

The purpose of the present paper is to discuss some of the models available or proposed for rainfall-runoff relationships and some of the techniques for the evaluation of the parameters of these models. The models considered will be mostly those in which the main purpose is to provide a tool for generation of synthetic data, and to a smaller extent, models designed to follow as closely as possible the physical processes that take place in the watershed. Stochastic models which generate synthetic data using only statistical parameters of the prototype data as input will not be discussed.

OBJECTIVES OF WATERSHED MODELING

As stated above, the two main objectives of modeling in general and watershed modeling in particular are: (a) to achieve a better understanding of the prototype system represented by the model;and (b) to provide means of extending available data and of generation of data for watersheds where measurements do not exist.

The idea behind the first type of objective is to test our concepts about the physical processes that take place in the watershed, to reaffirm those that are valid, and to reject the concepts that are contrary to reality. The natural processes involved in converting rainfall to runoff as well as in any other input-output conversion that

takes place in the watershed are highly complex and interdependent. Any set of differential equations and boundary conditions used as a mathematical statement of the behavior of the watershed or any breakdown of the complex processes into a set of simpler processes constitutes a statement of our concepts of the behavior of the watershed or of our concepts as to how the watershed behavior may be approximated.

When these concepts are formalized in the structure of a model and the rules of its operation, it is possible to test the validity of the concepts by subjecting the model to some sets of specified input data, which may be historical data observed on the watershed or special data prepared for testing the model. The behavior of the model is judged in terms of the output obtained from the model as a whole as well as the outputs at a number of intermediate points within the model.

If the output from the model, as well as the outputs at the intermediate points, agree with measured outputs at the prototype watersheds or with output expected by independent considerations, the model is assumed to be a true representation of the prototype, and the concepts used in constructing the model are considered to be validated. By varying the structure of the model, the values of its parameters or the values of the input data, it is possible to get an estimate of the sensitivity of the model to the various factors and an indication of the relative importance of the elements of model. It is also possible to study the results obtained with extreme or unusual combinations of input data such as would be obtained in nature only on rare occasions. It is these types of studies on watershed models that lead to a better understanding, or at least to more confidence in the formalized concepts, of the behavior of prototype watersheds.

The second type of objective (extrapolation and extension of existing data) requires less details in the internal structure of the model as it does not normally involve comparisons or verifications of output at intermediate points. It is, of course, true that a model constructed for the first type of study could be used also for prediction purposes, but in most cases it is not economical to do so. A model developed specifically to produce synthetic data of a specific nature, such as values of monthly runoff or values of maximum yearly discharge, will in general be more efficient in terms of computer time and in most cases will also produce more accurate results of the specific data for which it was designed.

Ideally, the structure of a predictive model and the values of its parameters should be the most efficient for the purpose for which the model is constructed. The term most efficient may be interpreted here in terms of a simplest structure of the model and the optimal set of parameters that will minimize some objective function related to the type of output for which the model is designed. The objective function may be defined in terms of the sum of squared deviations between model output and observations, the sum of absolute deviations, the magnitude of the maximum deviation, or some other function related to the deviations between observed and computed results.

If the model will be used to produce more than one output, for example a model designed to predict values of monthly runoff and monthly contributions to regional groundwater, the objective function must be formulated so that the deviations of the two outputs from their respective measurements are considered. The two types of deviations may be taken with equal weight, or one of the outputs may be given a larger weight if this is considered to be appropriate.

Predictive models that are expected to yield more than two or three types of output may become so complex that their structure will resemble that of models constructed for the first type of objective. It may well be more efficient in such cases to start with a model of the first type and simplify or modify it so that it will produce the types of outputs needed.

CHARACTERISTICS OF RAINFALL-RUNOFF WATERSHED MODELS

Examination of a number of models for the rainfall-runoff relationships in watersheds indicates that they have some common characteristics which are useful in the discussion of their properties. One basic characteristic of these models is that their inputs and outputs represent quantities of water -- the input being the rainfall over the watershed and the output the runoff at the outlet of the watershed. Internal inflows and outflows of the models also represent flows of water between the various elements of which the models are composed.

Another important characteristic of watershed models is the fact that their operation is based on some accounting procedure that keeps track of the quantities of water entering and leaving the various parts of the model. The equation that governs this accounting procedure for each element of the models, as well as for the whole model, is the equation of continuity. This is expressed either in terms of instantaneous values of flow

$$I - Q = ds/dt \tag{1}$$

or in a finite difference form in terms of volumes entering and leaving the element concerned in a finite time interval Δt

$$P - R = \Delta S \tag{2}$$

In the above equations, I represents the instantaneous rainfall intensity and P the precipitation during the time interval Δt

$$P = \int_0^{\Delta t} I\, dt \tag{3}$$

Similarly, Q represents the rate of discharge and R the volume of runoff in time Δt

$$R = \int_0^{\Delta t} Q\, dt \tag{4}$$

The units of the above quantities are assumed to be compatible so that no conversion factors are needed in the equations.

An additional characteristic of a great number of models is that they operate in terms of discrete finite time intervals which may be a day, an hour, a month, or any other convenient time unit. The unit chosen in each case is the one most significant for the problem being studied. The results produced by the model are expressed in terms of volumes involved as input or output of the various elements during each time increment. Alternatively, the results are expressed in terms of average rates of flow of the quantities considered during these time intervals, but practically the two forms are the same.

The various elements of the rainfall-runoff models perform, for each time interval, one of the following operations:

(a) Addition of a number of inputs during the time interval and production of an output equal to the sum of inputs during the same time interval.

(b) Proportioning of the input received during a time interval between a number of outputs during the same time interval

(c) Storage of the input received at a given time interval and its redistribution in time so that output will be produced in the given time interval and also in a finite number of subsequent time intervals.

Some elements may be complex in the sense that they perform addition and proportioning as in Items (a) and (b) above, combined with a storage and time distribution with respect to one or more of the outputs.

Each of the above operations is carried out according to fixed operating rules for the element which may include reference to the state of the element as expressed by the storage accumulated in it. The state of an element may also be specified with reference to some external information such as,for example, the mean temperature for the time increment which, in turn, affects the evapotranspiration for the period concerned.

A typical example of the structure and functioning of a watershed model for rainfall-runoff relationships is shown in Figure 1. The model shown is that proposed by Dawdy and O'Donnel (1965). It is a simplified version of the Stanford Model developed by Crawford and Linsley (1962). The overall model is represented by the operator ϕ and shown schematically by the dashed line box in Figure 1. It receives the daily precipitation P as input and produces as its main output the daily runoff R; a byproduct output is the daily evapotranspiration E.

The model may be considered as made up of six elements,each performing one of the types of operations listed above. The operators $\phi_1 \ldots \phi_6$ of the elements are described briefly below, and they require nine parameters to fully specify their properties and rules of operation.

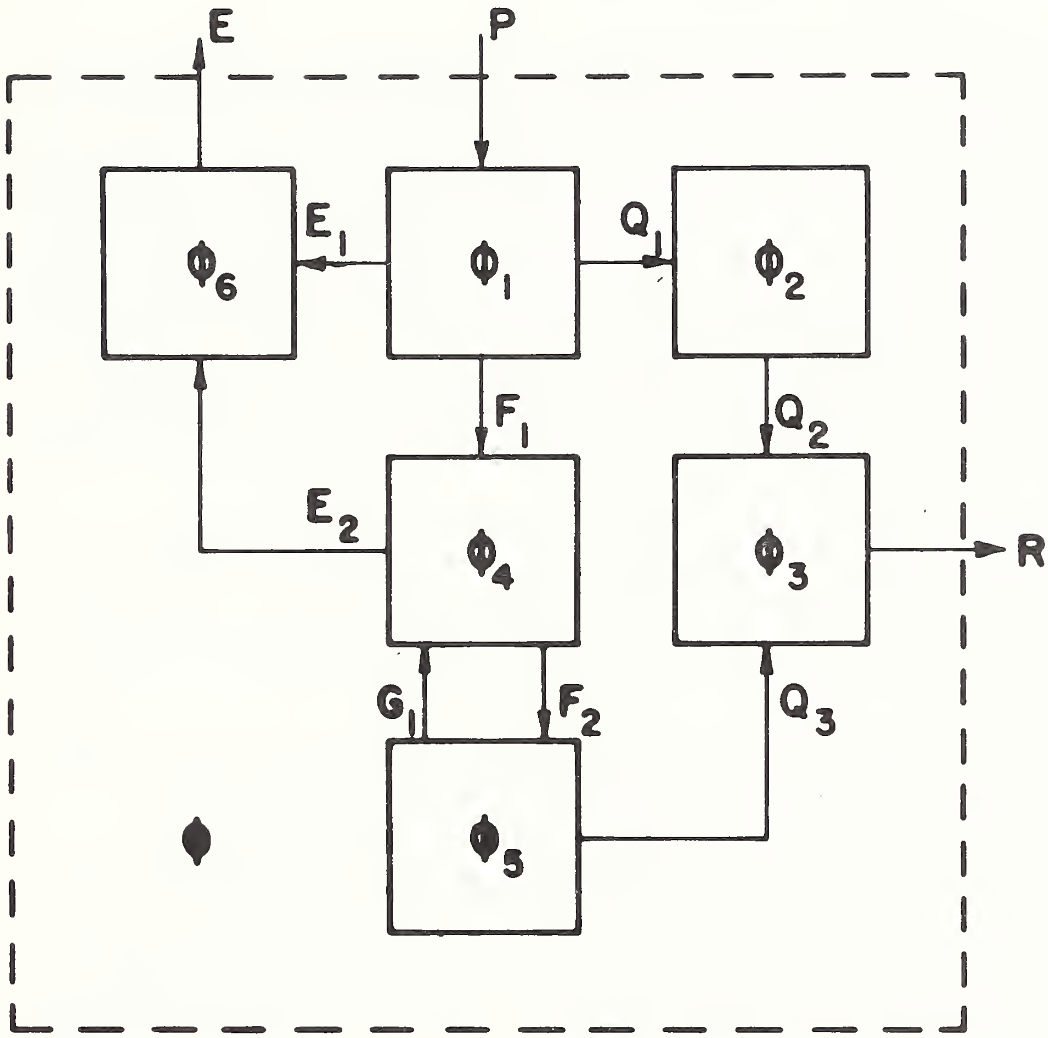

FIGURE 1. RAINFALL-RUNOFF WATERSHED MODEL (DAWDY AND O'DONNEL).

The operator ϕ_1 is a proportioning element receiving input P and producing 3 outputs: E, evaporation from surface storage; F_1, infiltration to an intermediate zone; and Q_1, surface runoff. Operator ϕ_2 is a distributing element receiving Q_1 as input and releasing it as surface runoff Q_2 distributed over a number of time intervals. Operator ϕ_3 is an additive element which receives two inputs, the modified surface runoff Q2 and the groundwater runoff Q_3, adds the two, and produces the model output R, which is the total daily runoff. Operator ϕ_4 receives as inputs the infiltration F_1 and possible capillary rise of groundwater G_1. It produces as outputs evapotranspiration E_2 and deep percolation F_2. The operator ϕ_5 receives the deep percolation F_2 as input and produces two outputs, capillary rise G_1 and groundwater runoff Q_3.

The number of parameters involved in the description of the various elements is as follows: ϕ_1 - 4 parameters, ϕ_2 - 1, ϕ_3 - 0, ϕ_4 - 1, ϕ_5 - 3, ϕ_6 - 0, or a total of nine parameters.

EVALUATION OF MODEL PARAMETERS

The first step in the construction of any model is to determine the structure of the model and the operating rules for each of the elements of the model. These rules include also the sequencing of mathematical or logical manipulations that are involved in the operation of the model. The sequencing adopted affects the results obtained with any model because computations are carried out for discrete intervals of time, and it makes a difference, for example, if input to a storage element is admitted before or after distributing its contents between a number of possible outputs.

Once a decision has been reached on the structure and operating rules of a model, the next step is to evaluate the various parameters needed for the operation of the model. There are basically two different approaches to the problem of evaluation of model parameters. One approach, related mostly to "physical" models, assigns the values of the parameters according to measurements of some physical entities in the watershed without regard to the goodness of fit between the observed data and the output of the model. The second approach is based on making the output of the model agree as closely as possible with recorded data. This is done with reference to a specified objective function which depends on the values of the parameters and which will attain either a minimum value or a maximum value as a result of choosing the optimal set of parameter values.

The selection of the numerical values of the various parameters is done in the second approach so as to obtain the "best" agreement,in terms of the objective function, with a set of data that is considered to be representative for the watershed investigated and for the purposes for which the model will be used. In many cases the parameters are not allowed to assume any value that will produce good agreement. The values are restricted,in such cases, within limits that are considered to be consistent with the physical concepts involved in the construction of the model. Thus, the threshold storage of an element cannot assume a negative value or exceed some higher limit even if such values may improve the prediction. Similarly, ordinates of a unit hydrograph are restricted to have only positive values.

The optimal set of values for the parameters of a model of given structure and operating rules depends on two important factors. One is the data selected for the comparison of the performance of the model, and the second is the form of the objective function that was adopted for definition of best agreement between observed and synthetic data.

The data used in the comparison are, of course, only a sample of the population of possible data that the prototype watershed is capable of producing given sufficient time. It is, of course, assumed that the prototype is stationary and that its structure does not change with time. Being a sample of a limited number of observations, the characteristics of the sample differ from those of the population to an extent depending on the number of observations. Obviously, the larger the number of observations, the nearer will the sample represent the original population and the better will the model parameters be for generating synthetic data that are presumed to be equivalent to data from the original population. The effect of choice of data is evident in those cases where a set of data is split so that one half can be used for determination of model parameters and the second for testing the adequacy of the model. If the roles of the two halves are interchanged, the optimal parameters derived from the two halves will not be equal and will be different from values obtained with the complete set of data. The effect of the data is also noticeable in cases where the parameters of a model are revised after a few years during which additional data are gathered.

The second factor influencing the values of the optimal parameters of a model is the choice of the objective function that defines what is meant by a good agreement between the synthetic data generated by the model and the historic data available for the watershed concerned. Comparison between observed and synthetic data is done with reference to the deviations between the two types of data at a finite number of discrete points. Even if the output is continuous, the comparisons are usually based on a number of discrete points along the continuous record. The objective function is defined in terms of these deviations, but any definition adopted is only one of the many definitions that can be formulated.

The most common definition for an objective function $\mathcal{S}$ is the sum of the squared deviation between the historic data y_h and the synthetic data y_s produced by the model.

$$\mathcal{S} = \Sigma\,(y_h - y_s)^2 \tag{5}$$

Another common definition is in terms of the sum (or the mean) of the absolute magnitude of these deviations

$$\mathcal{S} = \Sigma\,|y_h - y_s| \tag{6}$$

Other definitions used or proposed include the maximum absolute deviation, the sum of the absolute values of the ten largest deviations, etc.

Another element of variation of the objective functions is the introduction of weighting factors. Thus, it may be felt that extra weight should be given to deviations in certain parts of the record than to other deviations. The above definition based on a sum of the square deviation would become

$$\mathcal{S} = \Sigma\,W(y_h - y_s)^2 \tag{7}$$

where W signifies the weight assigned to the particular values. Examples for such weighting can be found in cases where a better fit is desired for ordinates of the runoff hydrograph near the peak of the curve than in other parts.

A more complicated problem of objective function definition presents itself if the model produces more than one output. In such cases, a decision must be made on the relative weights assigned to the deviations of the various outputs and a method for combining deviations that may be of different dimensions.

None of the definitions of the objective function is, of course, better than the other definitions, at least not from a theoretical point of view. Each definition will produce a distinct set of values which are optimal values of the parameters of the model in terms of the objective function employed. The only guide for a choice of an objective function in any particular case is possibly some considerations of the future use of the model. It is thus quite conceivable that a model of a given structure will have several sets of optimal parameters depending on the type of information it is producing.

PARAMETER EVALUATION TECHNIQUES

The problem of evaluation of model parameters has been formulated above in terms of minimizing (or maximizing) an objective function while the values of the parameters are subjected to a set of constraints determined independently from the physical considerations related to the structure of the model. With the above formulation, many of the techniques available in linear or dynamic programing can be applied to obtain a set of optimal values of the watershed parameters.

One of the simplest techniques available for parameter evaluation is the simultaneous solution of a set of equations obtained when the objective function is differentiated with respect to each of the parameters defined by the model, and the resulting expressions are equated to zero. Thus, if the objective function S is a function of n independent parameters $A_1, A_2, \ldots A_n$

$$S = f(A_1, A_2, A_3 \ldots A_n) \tag{8}$$

it is possible to obtain n equations of the form

$$\frac{dS}{dA_i} = 0 \qquad (i = 1, 2, 3 \ldots n) \tag{9}$$

and to solve for the n values of the parameters that will simultaneously satisfy the set of equations.

The method is applicable mostly to those cases where the functional relationship between the objective function and the n parameters is continuous and can be differentiated, and where the resulting equations are linear with respect to the parameters.

Where these conditions are not met and if the number of parameters is not large, a trial and error or a systematic mapping technique may be found useful. In this technique, the parameters are assigned arbitrary values, and the value of the objective function is calculated. The values of the parameters are then varied either in some systematic manner or randomly, and the value of the objective function is recalculated for each new set of parameter values. The resulting "map" of the values of the objective function is then inspected, and the set of parameters giving the minimal value of the objective function is chosen as the optimal parameters.

The random choice of parameters is used in cases where the range of possible values of the parameters is large or in cases where local minima of the objective function may be present. After finding the minimum by the random procedure, an additional systematic search for the set of optimal parameters is usually advisable in the vicinity of the optimal set found by the random procedure. In this search, the value of each of the parameters is varied systematically above and below the solution found by the random process.

A natural development of the above mapping technique is that of gradient climbing techniques. In these techniques the values of the parameters are changed from one computation to the next in such a way that the value of the objective function is continuously decreasing (or increasing) along the direction of its steepest rate of change. The first step is to compute, for a given set of parameter values, the partial derivatives of the objective function with respect to each of the parameters. The partial derivatives G_i are computed by incrementing each of the parameters A_i in turn by a small amount ΔA_i, noting the resulting change ΔS_i in the objective function while the other parameters are held constant at their original values and computing the partial derivative

$$G_i = \frac{\Delta S_i}{\Delta A_i} \tag{10}$$

If the changes in the parameters are all made of equal relative magnitude, for example by making the increment of each parameter equal to 10% or 5% of the value of the parameter, the resulting values of the change in the value of the objective function will be a measure of the sensitivity of the model to changes in the values of the parameter. Comparing the sensitivity of the model with respect to the various parameters will indicate those parameters that influence the value of the objective function to the largest extent.

After computing the partial derivatives there are a few possibilities that are available for proceeding to find the set of optimal parameters. One is to hold all parameters constant except for the one having the largest partial derivative. This parameter is varied by an arbitrary amount in such a way that the value of the objective function is decreasing. At each stage the value of the objective function and the values of the partial derivatives are recomputed, and the process is repeated until the changes in the value of the objective function or in the values of the parameters are less than some prescribed limits.

An alternative procedure is to change at each stage of the computations the values of a few or all the parameters. The relative change in the value of each parameter is made proportional to the sensitivity of the model with respect to this parameter. After each change, the value of the objective function is evaluated and if it has not reached a minimum value, further changes in the values of the parameters are made, keeping the relative changes as before. After reaching a minimum value for the objective function in the direction adopted for the changes, a new set of partial derivatives or sensitivities are computed, and the above scheme of computations is repeated until the desired optimal set of parameters is obtained.

A number of algorithms for carrying out the computations for optimal parameters are available. They are related to the gradient climbing technique discussed above, but each proceeds to achieve the goal of finding the optimal parameters by its own methods. Methods developed are oriented towards solution by digital computers, taking advantage of the capabilities of such computers. The descriptions of two recent methods are quoted below (with changes in notation to agree with that used herein). The first quote is from a paper by Dawdy and O'Donnel (1965):

> Of several optimizing procedures available, one well suited to the catchment model problem is that developed by Rosenbrock (1960). The particular class of problems for which the method was developed is one in which (1) the parameters, A_i, are restricted by physical considerations and must fall within specific limits, and (2) the function, S, dependent on those parameters, and whose value is to be

maximized or minimized, is such that partial derivatives of S with respect to the various A_i cannot be stated analytically in usable forms.

If there are n parameters on which the function S depends optimization consists of a search in an n-dimensional vector space (formed by n orthogonal parameter axes and bounded by limits set on the n parameters) until the optimum value of S is found. Rosenbrock's method is recursive in that it makes this search in a series of repetitive stages. Each stage is terminated by evaluating a new set of n orthogonal directions along which the search during the next stage is conducted. The evaluation of the new directions is based on the movements made along the n directions of the current stage. Only in the first stage are the orthogonal directions coincident with the n parameter axes. In subsequent stages, the first component of the new directions lies along the direction of fastest advance.

During each stage, movement is made along each orthogonal direction in a series of steps. A step of arbitrary length, e, is attempted first. This is treated as successful if the resulting new value of S represents an improvement of, or is equal to, the previous value. If a success, the step is allowed, and e is multiplied by $\alpha > 1$; if a failure, the step is not allowed, and e is multiplied by $-\beta$, in which $0 < \beta < 1$. A new attempt is then made. These attempts are terminated as soon as at least one successful attempt, followed by one failed attempt, has been achieved in each of the n directions. Then the new orthogonal directions used in the next stage are evaluated. An attempt in the end must succeed for each direction, because e becomes so small after repeated failures that it causes no change in S.

The second quote is from a paper by DeCoursey (1968), reporting on work by DeCoursey and Snyder (1969).

After the form of the model is established, and the data to which it is to be fitted are collected, initial estimates of the parameters in the model are made. The data are then processed through the model and the

difference, residual error, between the predicted and observed value of the dependent variable is recorded for each set of observations. Optimum values of the parameters are found by an iterative reduction of this residual error. The method of optimizing the parameters is based on the "Method of Differential Correction" (Nielsen, 1957) with the technique of "Principal Component Analysis" (Kendall, 1957) used to relate the residual error to the parameters (see DeCoursey and Snyder, 1969).

The residual error in any prediction is the cumulative result of errors in each of the parameters. A measure of the size of the error associated with any one parameter is given by the change in the dependent variable caused by an incremental change in the parameter. If the ratios of the changes for each of the parameters are used as weighting factors, then the following equation can be used to relate the total error to corrections for each of the parameters.

$$E_i = h_1 \frac{\Delta \hat{y}_i}{\Delta A_1} + h_2 \frac{\Delta \hat{y}_i}{\Delta A_2} + \ldots + h_m \frac{\Delta \hat{y}_i}{\Delta A_m} \tag{11}$$

where

E_i the prediction error for each observation, i;

h_j a correction to the initial estimate of the numerical parameter, A_j;

A_j a numerical parameter in the functional relationship between the dependent variable y and n independent variables, X_1;

$\Delta\hat{y}_i = \hat{y}_i - \hat{y}_{iAj}$ the difference between the predicted value of y_i and the predicted value of y_i with A_j incremented a small amount, and

$\Delta A_j = A_j - (A_j + \Delta A_j)$ the increment by which A_j has been changed.

Equation 11 is linear; however, it is highly probable that the "independent" variables will not be independent, thus it would not be advisable to use multiple regression techniques to solve the equation for h_j by minimizing the

sums of squares of E_i. An alternative to the problem is to use components regression, in which the orthogonal, truly independent, components of the system are used to solve for h_j.

Components regression solves equation 11 for a minimum sum of squares of E_i by assigning values to h_j that are orthogonal with respect to each other. This is accomplished by developing the correlation matrix of the $\Delta \hat{y}_i / \Delta A_j$ data. The orthogonal components h_j used to correct the initial estimate of the parameters are transformations of the eigenvectors of the characteristic equation of the correlation matrix. (See DeCoursey and Snyder, 1969).

The correction terms h_j are added to initial estimates of the parameters and the new values are used as estimates for a second pass. This iterative reduction of the residual error is repeated until changes in the parameter values reach a minimum value.

PARAMETER EVALUATION FOR SURFACE RUNOFF MODELS

The parameter evaluation techniques discussed above are applicable to most watershed models. There are, however, some special techniques that were developed for the purpose of evaluation of the parameters of particular types of models. The most notable example of such special methods is found in models representing the surface runoff portion of the watershed response.

Surface runoff models receive as input a portion of the total rainfall designated as rainfall excess (I) and produce as output a portion of the total runoff hydrograph which is considered to be direct surface runoff (Q). The relationship between input and output is assumed to be either linear or quasi-linear.

In the first case, the model representing the relationship between rainfall and runoff has constant valued parameters, but in the quasi-linear case the parameters are assumed to be constant only during the occurrence of each event. Between events, the values of the parameters may change depending on antecedent conditions in the watershed.

The relationship between the rainfall excess input and the direct surface runoff output for the linear or quasi-linear model is given by the convolution integral

$$Q_{(t)} = \int_{0}^{t} I_{(\tau)} H_{(t-\tau)} d\tau \qquad (12)$$

where H is the instantaneous unit hydrograph or the impulse response function of the system.

The purpose of parameter evaluation techniques for surface runoff models is to find appropriate values of the parameters of the impulse response function H, while the form of the function is predetermined by the structure of the model adopted to represent the relationship. The basis of the various techniques is to estimate from the rainfall and runoff data a number of numerical characteristics that are related to the parameters sought. A set of simultaneous equations is thus obtained with the parameters as unknowns. Usually the number of equations is made equal to the number of unknown parameters so that a unique solution is obtained. If the number of equations is greater than the number of unknowns, a compromise or best fit solution is obtained.

The best example for this type of technique is that based on matching of moments. It can be shown that the moments of the impulse response function of linear (or quasi-linear) systems can be evaluated from the corresponding moments of the input and output functions of the system. On the other hand, once the structure of the model is fixed, it is possible to derive expressions relating the parameters used in the definition of the model to the moments of the impulse response function.

Considering, for example, a model composed of N equal linear reservoirs in series, the first two moments about the origin of the impulse response function is given by

$$M_1 = NK \qquad (13)$$

$$M_2 = N(N+1)K^2 \qquad (14)$$

where K is the time constant of the reservoirs. The two moments can be evaluated in terms of the moments of the input and output from

$$M_1 = M_{Q1} - M_{I1} \qquad (15)$$

$$M_2 = M_{Q2} - M_{I2} - 2M_{I1}M_1 \qquad (16)$$

where M_{Q1}, M_{Q2}, M_{I1}, M_{I2} are the first and second moments about the origin of the output and input functions, respectively. Solving for the two parameters of the model, the following expressions are obtained:

$$N = M_1^2 / (M_2 - M_1^2) \qquad (17)$$

$$K = (M_2 - M_1^2) / M_1 \qquad (18)$$

Higher moments are employed if the model contains more than two parameters.

Instead of moments, other characteristics such as times to peak or values of Laplace transforms may be used for the selection of the values of the parameters.

THE LINEAR CORRELATION MODEL

As an example of some of the ideas discussed above, the operation and evaluation of a simple rainfall-runoff model is given below. The model used for this illustration may be called the linear correlation model. The model is one of the simplest rainfall-runoff models, and it is suitable for describing the relationship between annual runoff volume and annual depth of precipitation. The structure of the model may be described in terms of only three elements or subsystems shown in Figure 2. The operators that define the three subsystems perform the following operations:

Operator ϕ_1 receives as input the annual precipitation P and divides it into two outputs R_1 and L_1 according to the following scheme:

$$\text{if } P \leq C \qquad L_1 = P \quad \text{and} \quad R_1 = 0 \qquad (19)$$

$$\text{if } P > C \qquad L_1 = C \quad \text{and} \quad R_1 = P - C \qquad (20)$$

where C is a constant parameter.

Operator ϕ_2 receives as input the output R_1 of the first subsystem and divides it into two outputs R and L_2 according to the following scheme

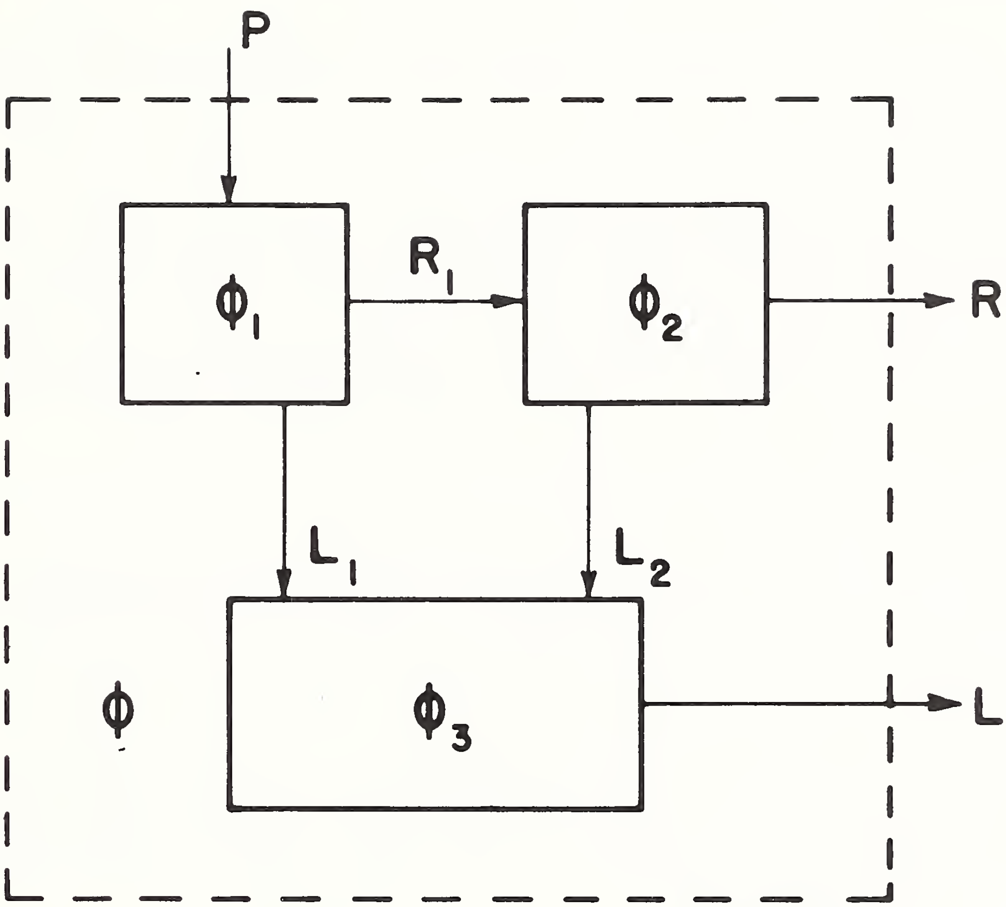

FIGURE 2. LINEAR CORRELATION MODEL

$$R = A \cdot R_1 \qquad \text{and } L_2 = (1-A)\, R_1 \tag{21}$$

where A is a constant parameter.

The third operator ϕ_3 is a summation operator receiving as inputs L_1 and L_2 and producing an output L which is the sum of the two inputs

$$L = L_1 + L_2 \tag{22}$$

The complete system represented by the dashed line box in Figure 1 and by the operator ϕ is thus seen to receive an input R equal to the annual depth of precipitation and to produce two outputs, the annual volume of runoff R and the annual volume of losses L which are mostly losses by evapotranspiration. Combining the operations of the first two subsystems (1 and 2) leads to the following direct relationships between annual volume of runoff and annual depth of precipitation

$$\text{if } P \leq C \qquad R = 0 \tag{23}$$

$$\text{if } P > C \qquad R = A\,(P-C) = AP - B \tag{24}$$

where B = AC is a constant parameter. Similarly, the relationship between annual losses and annual precipitation is given by the following expressions:

$$\text{if } P \leq C \qquad L = P \tag{25}$$

$$\text{if } P > C \qquad L = (1-A)\,P+B \tag{26}$$

If all or nearly all values of the depth of precipitation are larger than the parameter C, the problem of evaluating the parameters A and B (or A and C) resolves itself into a least squares fitting procedure for the straight line described by Equation 24. The values of the parameters are given in this case by

$$A = \frac{\Sigma\,(RP) - N\bar{R}\bar{P}}{\Sigma\,(P^2) - N\bar{P}^2} \tag{27}$$

and

$$B = \bar{R} - A\,\bar{P} \tag{28}$$

where N is the number of observations, $\bar{P}$ is the mean of the precipitation observations, and $\bar{R}$ is the mean of the runoff observations.

The values of the parameters thus obtained are those minimizing the sum of the squared deviations between measured and predicted runoff volume.

The problem is, however, more complicated if the rainfall records are such that the annual precipitation values for an appreciable number of years of record are below the limiting value for the production of runoff. Such may be the case in arid or semiarid watersheds where the annual rainfall has a high variability and its mean value is not far from the limiting, no-runoff, value. An example of such a set of data is given in Table 1 and is shown graphically in Figure 3. The data have been prepared for the example discussed and do not represent any particular watershed.

If the above equations (Equations 23 and 24) for evaluation of the parameters are applied to all observations as given, the values obtained (line A in Figure 3) will indeed minimize the sum of the squared deviations of the whole set of data, but some of the predicted values of runoff for low values of rainfall will be negative. This is, of course, an impossible result, and it also contradicts the structure of the model as described above (line BC in Figure 3). The defect can be corrected by specifying that the predicted value of runoff is set to zero, whenever its value is negative, but doing so will increase the sum of squared deviation and will not correct the slope of the line and its location. The objective of minimizing the sum of the squared deviations between predicted and measured runoff values will thus not be achieved.

The optimal set of parameters can, of course, be obtained by a graphical procedure of passing a straight line (by eye) as in Fig. 3, but if a subjective method is desired, it is possible to employ a mapping technique. Values of the parameters A, B are assumed and the sum of squared deviations $\mathcal{S}$ is computed for each such set of values. The resulting map for the data given in Table 1 is given in Figure 4, indicating that an optimal value of the parameters may be chosen from the points bounded by the line $\mathcal{S} = 5.0$, giving the following ranges for the parameters:

$$0.63 < A < 0.75 \tag{29}$$

$$-5.2 < B < -3.9 \tag{30}$$

TABLE 1.--Annual precipitation, runoff and losses data.

Ovservation No.	Annual precipitation	Annual runoff	Annual losses
	(in)	(in)	(in)
1	2.79	0.00	2.79
2	3.61	0.01	3.60
3	4.14	0.08	4.06
4	4.63	0.00	4.63
5	5.11	0.20	4.91
6	5.20	0.00	5.20
7	5.58	0.00	5.58
8	6.29	0.02	6.27
9	6.45	0.42	6.03
10	6.87	0.25	6.62
11	7.09	0.16	6.93
12	7.12	0.59	6.53
13	7.54	0.40	7.14
14	7.93	1.03	6.90
15	8.41	0.22	8.19
16	8.60	1.82	6.78
17	9.20	1.80	7.40
18	9.55	2.91	6.64
19	10.19	2.63	7.56
20	10.98	2.83	8.15
21	11.60	3.65	7.95
22	12.60	2.99	9.61
23	12.63	4.36	8.27
24	13.21	4.20	9.01
25	13.78	5.61	8.17
26	14.51	5.30	9.21
27	15.99	6.64	9.35
Mean	8.578	1.782	6.796

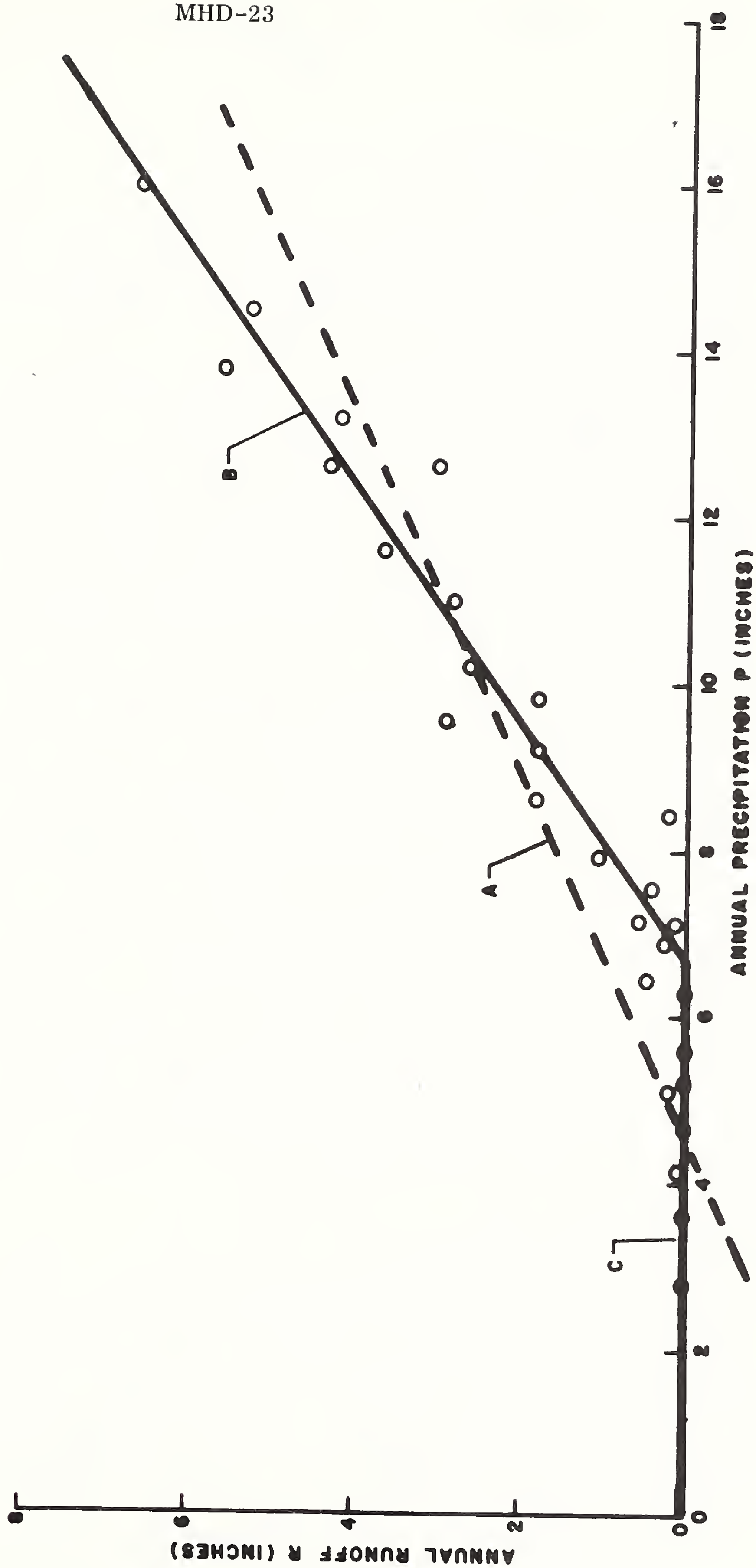

FIGURE 3. ANNUAL RUNOFF—RAINFALL RELATIONSHIP FOR LINEAR CORRELATION MODEL.

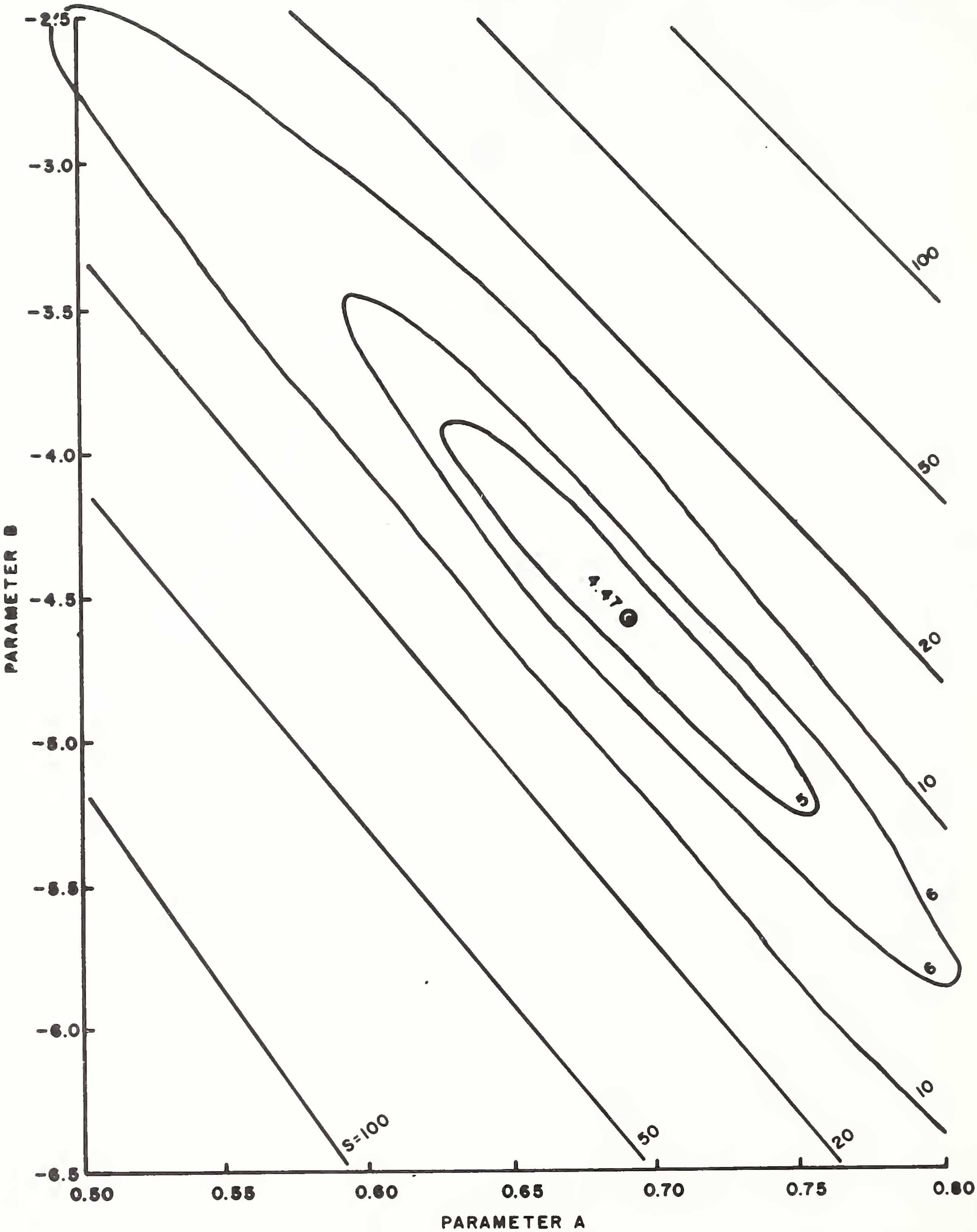

FIGURE 4. SUM OF SQUARED DEVIATIONS FOR LINEAR REGRESSION MODEL S=f(A,B)

As an alternative to the mapping, a gradient climbing technique may be employed. Starting with any assumed values for the parameters, the solution will follow the steepest gradient on the surface shown in Figure 4 until the minimal point, or a point near the minimum, is reached.

A special procedure suitable for this particular model is as follows. The data are arranged in increasing values of the observed precipitation P, as in Table 1, the observed data are then divided into two groups by choosing one of the values of P as a division point P_o and the parameter evaluating equations (Equations 23 and 24) are applied only to observations in the group for which the precipitation is higher than the division point, $(P > P_o)$. The straight line thus obtained will be the best fitting line for the observations within the group so defined. The sum S of the squared deviation of the observed values of runoff and the values predicted by the line

$$R = AP + B \qquad \text{for } P > P_o \tag{31}$$

or by the line

$$R = 0 \qquad \text{for } P \leq P_o \tag{32}$$

will thus be a function of the division point P_o.

$$S = f(P_o) \tag{33}$$

if the point P_o is varied systematically, it is possible to plot the functional relationship between S and P_o and to determine the value of P_o that will minimize the function S. The values of the parameters computed for this division point will be the optimal in the sense that the objective function S has obtained its minimal value consistent with the adopted structure of the model. The curve obtained for the data given in Table 1 is shown in Figure 5. The optimal point obtained by this method is shown also on Figure 4. The solution provided by this method appears to be identical with that provided by the mapping technique.

CONCLUSIONS

A number of hydrologic models for representing the relationship between the total rainfall on a watershed and the total runoff at the outlet of the watershed are available. Some of these models are used to further our understanding of the behavior of the watersheds but some are used only as tools for generation of synthetic data. Methods and techniques are available for the evaluation of the optimal values of the parameters used

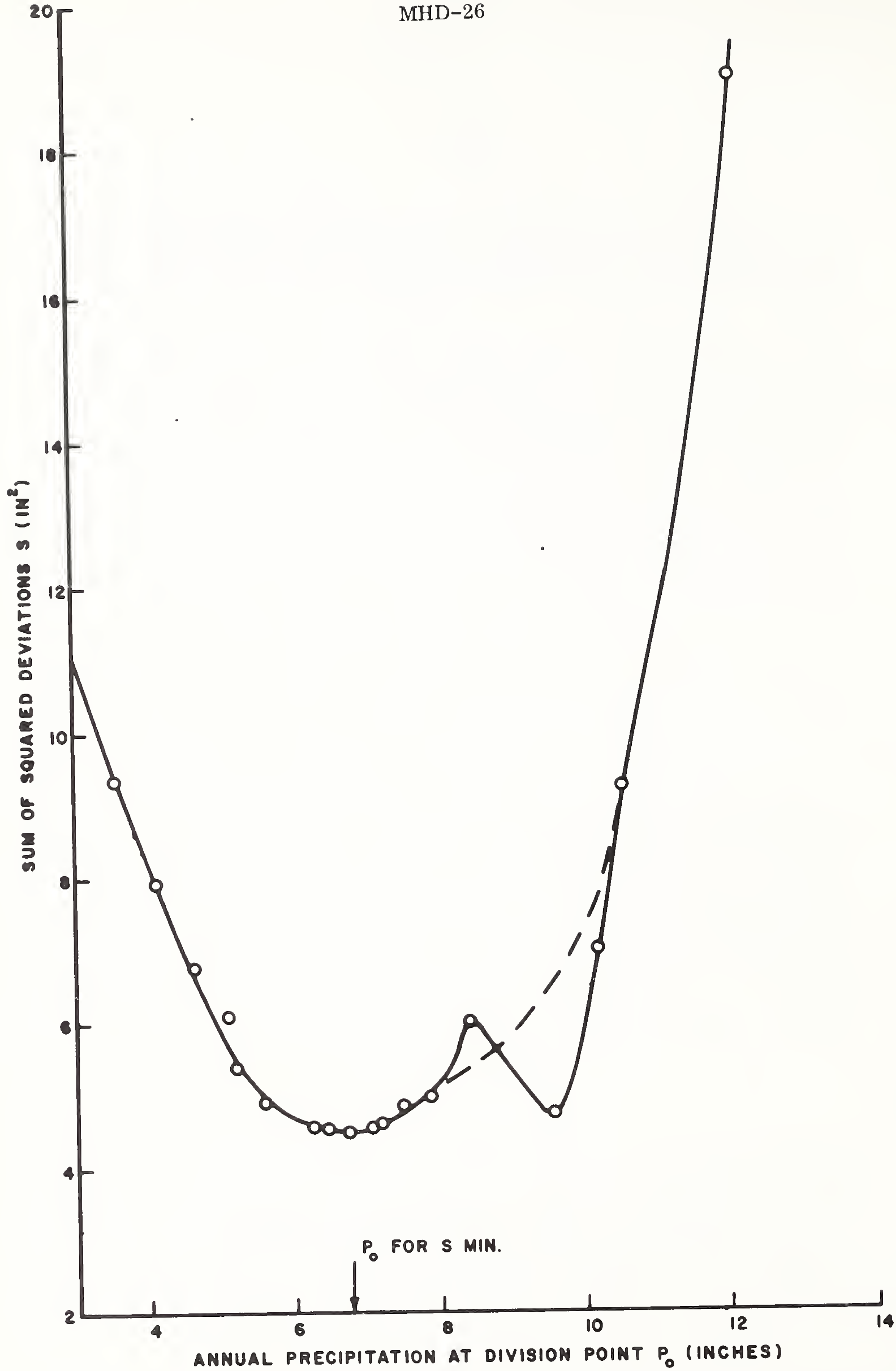

FIGURE 5. OBJECTIVE FUNCTION FOR LINEAR REGRESSION MODEL.

to define the structure and the operation of the model. The optimal set of values depends on the definition of the objective function used to specify the goodness of fit between the synthetic data and the historic data. The values depend also on the quality of the data used and the number of observations available.

Further research is needed to determine the effects of the above factors and the effects of the optimization scheme employed on the optimal values of the parameters of the model. The interrelationship between the structure of the model and the sensitivity and stability of the parameters also deserves some research effort.

A problem which will probably receive attention in the future is that of producing efficient special purpose models. These models may be defined as having the simplest structure consistent with the objectives for which the synthetic data are needed, while retaining some of the physical concepts used to construct the more complicated models designed to reproduce all processes that take place in the watershed.

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PARAMETRIC HYDROLOGY ADJUSTED TO

STREAM GAGE RECORDS

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10

PARAMETRIC HYDROLOGY ADJUSTED TO STREAM GAGE RECORDS

Introduction:

The Soil Conservation Service has been planning flood control measures on small watersheds for more than 20 years. Very little work had been done on the development of procedures for evaluating the effect of these measures at the time planning was started on the 11 authorized watersheds. The first floodwater retarding structure was planned and built in Oklahoma in 1947 on Cavalry Creek, a tributary of the Washita River. Since that time, thousands of structures and hundreds of miles of channel improvement have been planned and built throughout the United States.

These structural measures are evaluated by determining the damages that would occur on the watershed without project and with project usually over a 100-year period. The difference in damages, without project and with project, together with other project benefits, must exceed the cost of the installation and maintenance of the structural measures for the project life.

Purpose:

The purpose of this paper is to point out some of the problems encountered by a hydrologist when making a hydraulic and hydrologic analysis of a watershed for the purpose of evaluating the effects of various structural programs (Floodwater Retarding Structures and Channel Improvement).

The Soil Conservation Service uses a standard approach which is shown in the SCS National Engineering Handbook, Section 4, Hydrology, Part I - Watershed Planning, to develop the hydraulic and hydrologic phase of watershed planning.

Hydraulics:

In solving the hydraulics of a watershed, stage-discharge curves are developed for cross sections located at selected points along the stream by computing water surface profiles for uniformly varied flow.

Hydrology:

Using a dimensionless unit hydrograph, the unit hydrograph theory is used to develop storm hydrographs for incremental areas of the watershed. These storm hydrographs are combined and routed through routing reaches (using the convex method of flood routing) to determine the peak discharge for various sizes of storms under "without" and "with" project condition at each cross section.

The following data are available, or can be developed from other sources or records, for use in developing the hydraulics and hydrology of a small watershed:

1. Base Map - Aerial photographs.

2. Areas of Damages - Delineated by economist on aerial photograph or strip map.

3. Structure Locations - All possible structure sites located by planning engineer based on aerial photographs and topographic maps.

4. Curve Number - Developed by geologist and hydrologist from soils and land use maps.

5. Rainfall - Taken from U.S. Weather Bureau Technical Paper 40.

6. Drainage Areas - Developed from aerial photographs.

7. Peak Discharge Frequency Line - Developed from stream gage record if available; otherwise, computed from U.S. Geological Survey Survey Reports on a regionalized basis.

Using the above data, the hydrologist is faced with making a number of decisions or assumptions that vary from one watershed to another. The following decisions or assumptions based on experience and good judgment play an important part in making a realistic or true evaluation of the structural programs:

1. Cross Sections - Cross sections are located to define the hydraulics of the channel and flood plain. They also serve to represent the area flooded by depth increments for the various floods used to evaluate the structural program.

2. "n" Values - The Manning "n" values assigned to the various segments of the cross section should be based on the best data available.

3. Unit Areas - Areas controlled by structures and uncontrolled areas that make up a routing reach should be as nearly uniform in shape as possible in order to conform to the dimensionless unit hydrograph theory.

4. Routing Reaches - Routing reaches should be selected that will best define the effect of the structural program. An average cross section is selected to represent the reach for use in the convex method of flood routing.

The National Engineering Handbook, Section 4, Hydrology, uses the following data and assumptions which have become basic for all watersheds regardless of size, location, or topography:

1. Unit hydrograph - figure 1(a).

   This dimensionless unit hydrograph has one unit of time to peak, 1.7 units of total time to the point of inflection, and 5 units of total time for the base.

2. Peak discharge

The constant 484 is used in the equation

$$q = \frac{484\ A}{T_p} = \frac{484\ A}{\frac{\Delta D}{2} + 0.6\ T_c}$$

to compute the peak discharge for the above unit hydrograph. The ΔD is 0.20 of T_p and T_p is approximately 0.7 T_c.

3. Rainfall

The rainfall is assumed to be uniform over the entire watershed for all storms used in evaluating the project.

4. Rainfall pattern

A standard 24-hour duration rainfall pattern, figure 4(a), was developed from U.S. Weather Bureau Technical Paper 40, putting the maximum intensity near the center of the rainfall.

5. Runoff

The runoff from all storms is based on a curve, the number of which is computed from the soils and vegetative cover of the watershed. All storms are assumed to have happened with average antecedent moisture condition of the soil.

Using the above data and assumptions, the hydrologist solves the hydraulics and hydrology of a watershed to develop a peak discharge relationship at each cross section for an array of storms ranging from the 1-percent chance down to a storm that will produce very little or no flooding. On most watersheds the smallest storm analyzed produces a smaller flood than the 100-percent chance event.

The peak discharge-frequency relationship developed from flood routing should then be compared with a frequency line developed from a stream gage on the watershed, if available. If no stream gage record is available, the comparison is made with a frequency line developed from regional analyses of stream gages on similar watersheds. The U.S. Geological Survey has

developed magnitude and frequency flood reports on a regionalized basis for each state. If the routed frequency line does not compare closely with the gage record or a frequency line developed from a regional analysis, an adjustment should be made to bring them together.

In using the magnitude and frequency flood reports prepared by USGS, one could also get into trouble on a long narrow watershed. If no gage record had existed on this watershed and the report "Floods in Georgia - Magnitude and Frequency," dated 1962 had been used to develop a frequency at the gage site, figure 3(b) would have been used to compare the routed line to in the original routing shown as figure 3(a). These two lines are not too far apart except near the 50-percent chance flood.

However, a report of "Floods in Mississippi - Magnitude and Frequency," dated 1961 shows a correction for length-width ratio, figure 2. If this correction is applied to figure 3(b), the computed frequency line, the resulting figure 3(d) compares favorably (for floods greater than the 10-percent chance) with the frequency line computed from gage records, figure 3(e).

The following adjustments were made which brought figure 3(a) down to figure 3(c). This new developed frequency line compares favorably with figure 3(e) which was developed from gage records, using the Log Pearson method of developing a frequency from stream gage records, except for the annual flood (660 c.f.s.) which computes to be approximately half of the smallest annual flood (1,300 c.f.s.) recorded in a 25-year period of record.

1. A dimensionless hydrograph, figure 1(b), having approximately 20 percent of the volume ahead of the time to peak was used instead of the one normally used, shown as 1(a), which has 37-1/2 percent of the volume ahead of the time to peak.

2. The peak discharge equation for the revised unit hydrograph is

$$q = \frac{256\ A}{\frac{\Delta D}{2} + 0.6\ T_c}$$

3. The rainfall-runoff curve number for average antecedent moisture conditions was changed from 72 to 68.

4. The rainfall distribution figure 4(b) was used instead of the standard rainfall distribution figure 4(a).

There are no other gages on the watershed for use in comparing the original routing with the adjusted routing. However, figure 5 shows a comparison of the two routings with a frequency line computed from "Floods in Georgia - Magnitude and Frequency," dated 1962. At this point on the watershed where the drainage area is 47 square miles, no length-width ratio correction would be necessary. The 50-percent chance discharge of 1,650 also compares favorably with the 50-percent discharge of 1,500 on another watershed in the same hydrologic area which is located about 40 miles to the west with a drainage area of 50 square miles.

The following tables point out some of the findings as they relate to acres flooded and eventually to the economic phase of the project:

Table 1 shows the acres flooded by various frequency floods for without and with project condition, based on the original and the revised hydrology.

Table 2a shows the average annual flood damage and benefits based on the original and revised hydrology.

Table 2b shows the total average annual benefits and cost of the structural program based on the original and revised hydrology.

Table 2c shows a breakdown of the cost of the two plans. You will note that the cost of channel improvement has been reduced from the original cost of $3,494,432 to $2,079,497, a reduction of $1,414,935.

Economics:

This watershed is typical of much of the Piedmont Plateau area which has been heavily cultivated, much of it in row crops, during most of its occupancy until a relatively few years ago. This has resulted in heavy erosion in upland areas. Much of the sediment so produced has been deposited on the flood plain, reducing its productivity and creating a high water table. As a result, the intensity of flood plain use has declined and floodwater finds only low value crops and pasture to damage. This accounts for the relatively low floodwater damage per acre flooded shown by table 2a.

On the other hand, the proposed project will be effective in holding much of the sediment upstream, and reducing the flooding that deposits sediment. This will permit the forces of nature to heal the damaged land and restore much of its earlier productivity. Reduction of the sediment damage to the land constitutes a much greater share of benefits than does reduction of floodwater damage. However, most of the project benefits are derived from recreation and municipal water supply.

Observations and Conclusions:

The development of upstream hydrology is now, and will continue to be for some time, based on a synthetic approach using sparse data. The only way to determine the quality of the hydrologic analyses of a watershed is to compare the peak discharge frequency distribution (for without project condition) with one developed from a stream gage record. Unfortunately, stream gage records are not available on most of the watersheds planned by the Soil Conservation Service. Thus, the frequency relationships based on regional analyses of similar watersheds must be used as a check on the hydrology for without project condition.

The Soil Conservation Service is fortunate in that the structural measures on a watershed are evaluated on the difference of damages without and with structural measures assumed in place. Therefore, if the hydrologic analyses used for without project condition checks with the actual happenings on the watershed and the same assumptions and data are used to reflect condition with the structural measures assumed in place, figures 3(f) and 3(g), the difference in damages for without and with project condition would be a realistic value of the structural program. It will also serve to point out the actual effects of the program to the sponsoring agencies.

In this paper four parameters were adjusted in order to reproduce the frequency distribution at the gage: (1) Shape of unit hydrograph; (2) Peak of unit hydrograph; (3) Runoff curve number; (4) Rainfall pattern. Adjustment to other parameters or procedures could have been made, or all the adjustments could have been applied to either the unit hydrograph shape and peak discharge relationship or the runoff curve number, and the same goal would have been achieved. However, the author feels that the adjustments made were within the limits for each parameter.

TABLE 1 - ACRES FLOODED BY PERCENT CHANCE

Item	Percent Chance					
	1	4	10	50	100	300
	(Acre)	(Acre)	(Acre)	(Acre)	(Acre)	(Acre)
Original Plan:						
Without Project	7,990	7,519	7,066	5,768	5,205	2,432
With Project	4,883	4,404	3,850	1,460	744	0
Revised Plan:						
Without Project	6,856	6,261	5,816	4,296	3,196	1,416
With Project	4,296	3,685	3,117	1,341	374	0

TABLE 2a - ESTIMATED AVERAGE ANNUAL FLOOD DAMAGE REDUCTION BENEFITS

Item	ORIGINAL PLAN			REVISED PLAN		
	Damages			Damages		
	Without Project	With Project	Benefits	Without Project	With Project	Benefits
	(Dollars)	(Dollars)	(Dollars)	(Dollars)	(Dollars)	(Dollars)
Floodwater	27,615	4,116	23,499	17,080	3,341	13,739
Road and Bridge	2,642	407	2,235	1,696	183	1,513
Fixed Improvements	2,001	545	1,456	1,211	162	1,049
Total	32,258	5,068	27,190	19,987	3,686	16,301

TABLE 2b - TOTAL AVERAGE ANNUAL BENEFITS AND COST

Item	Original Plan	Revised Plan
	(Dollars)	(Dollars)
Benefits	562,487	541,916
Costs 1/	455,850	350,599
Benefit-Cost Ratio	1.23:1	1.55:1

1/ Amortized at 4-7/8 percent and includes Operation and Maintenance.

TABLE 2c - STRUCTURAL MEASURES COSTS

Item	Original Plan	Revised Plan
	(Dollars)	(Dollars)
Floodwater Retarding Structures	1,237,503	1,237,503
Multiple-Purpose Structures	1,974,425	1,974,425
Channel Improvement	3,494,432	2,079,497
Critical Area Treatment	136,950	136,950
Project Administration	829,465	636,196
Total	7,672,775	6,064,571

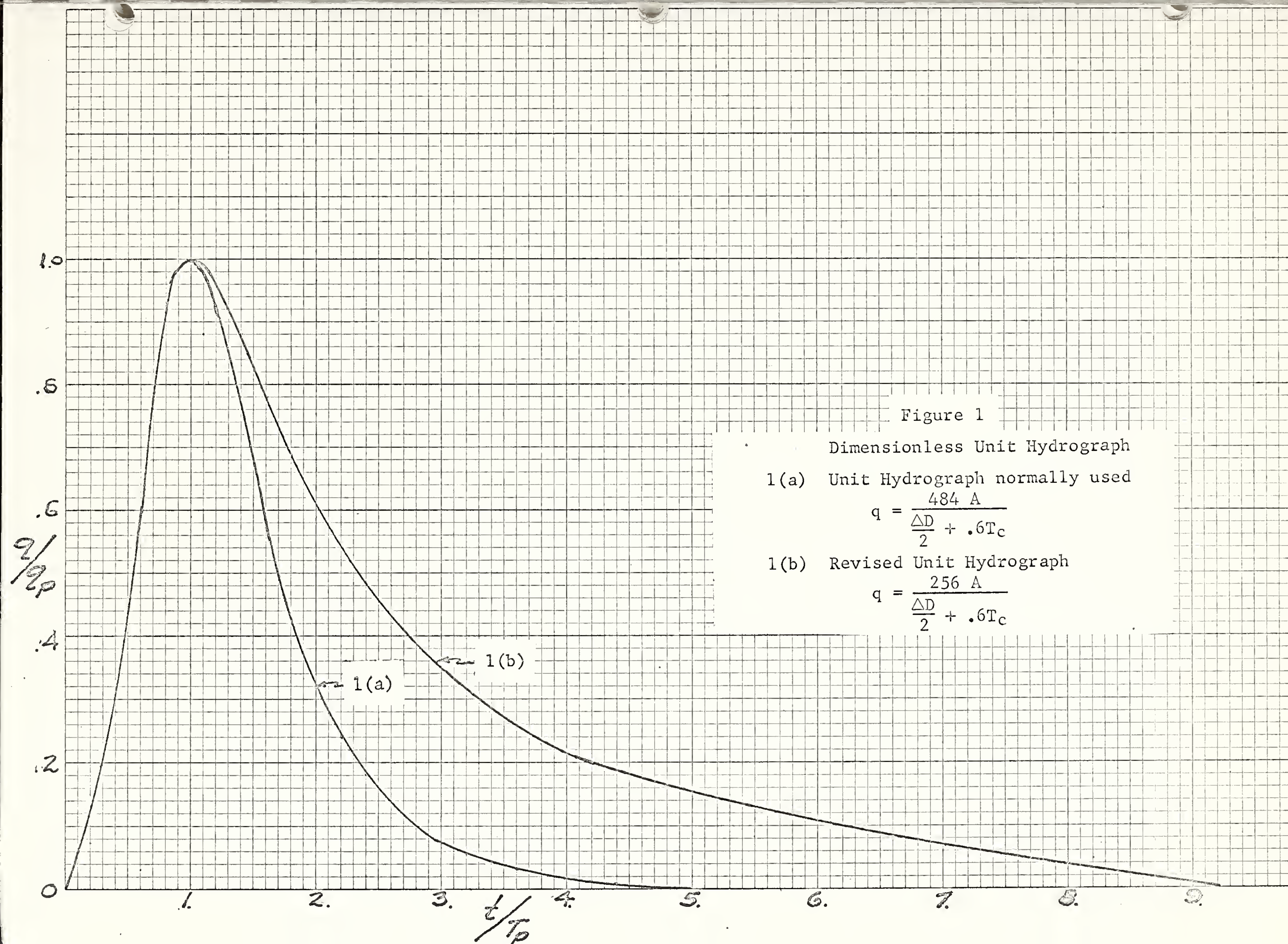


Figure 1

Dimensionless Unit Hydrograph

1(a) Unit Hydrograph normally used

$$q = \frac{484\ A}{\frac{\Delta D}{2} + .6T_c}$$

1(b) Revised Unit Hydrograph

$$q = \frac{256\ A}{\frac{\Delta D}{2} + .6T_c}$$

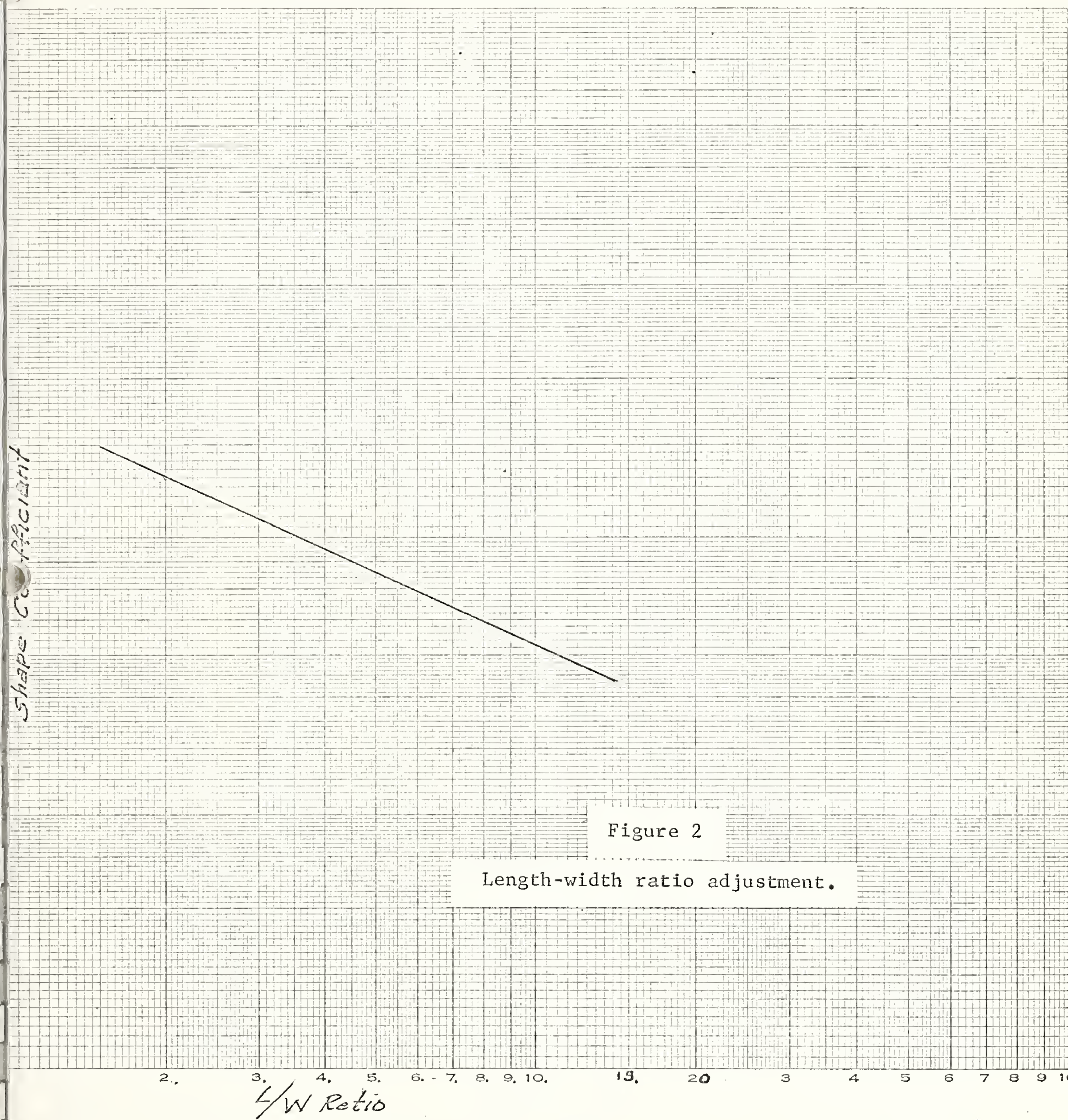


Figure 2

Length-width ratio adjustment.

Note: Taken from Floods in Mississippi Magnitude and Frequency

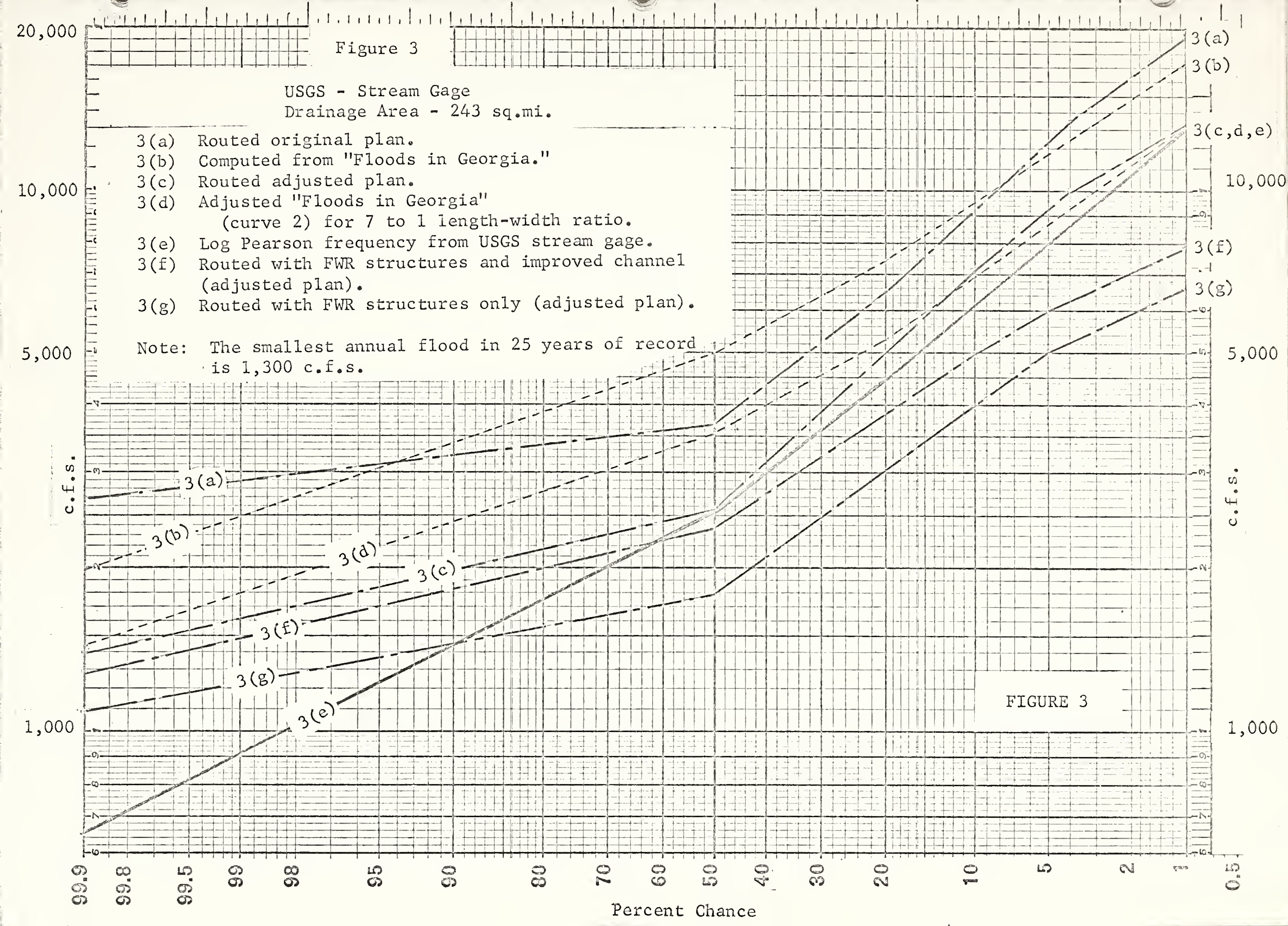


Figure 3
USGS - Stream Gage
Drainage Area - 243 sq.mi.
3(a) Routed original plan.
3(b) Computed from "Floods in Georgia."
3(c) Routed adjusted plan.
3(d) Adjusted "Floods in Georgia" (curve 2) for 7 to 1 length-width ratio.
3(e) Log Pearson frequency from USGS stream gage.
3(f) Routed with FWR structures and improved channel (adjusted plan).
3(g) Routed with FWR structures only (adjusted plan).
Note: The smallest annual flood in 25 years of record is 1,300 c.f.s.
20,000
10,000
5,000
1,000
c.f.s.
3(a)
3(b)
3(c)
3(d)
3(e)
3(f)
3(g)
3(c,d,e)
99.9
99.8
99.5
99
98
95
90
80
70
60
50
40
30
20
10
5
2
1
0.5
Percent Chance
FIGURE 3

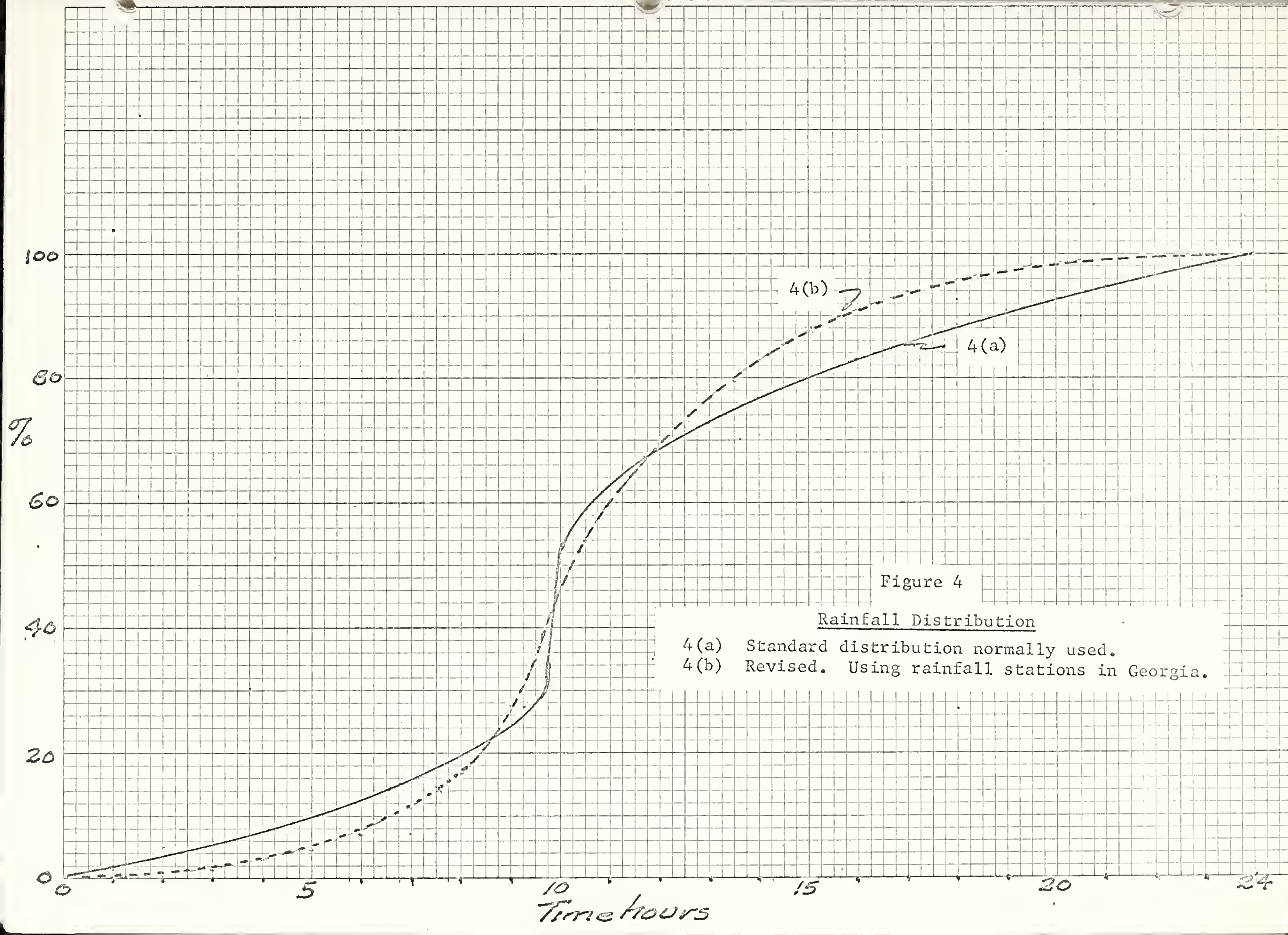


Figure 4

Rainfall Distribution

4(a) Standard distribution normally used.
4(b) Revised. Using rainfall stations in Georgia.

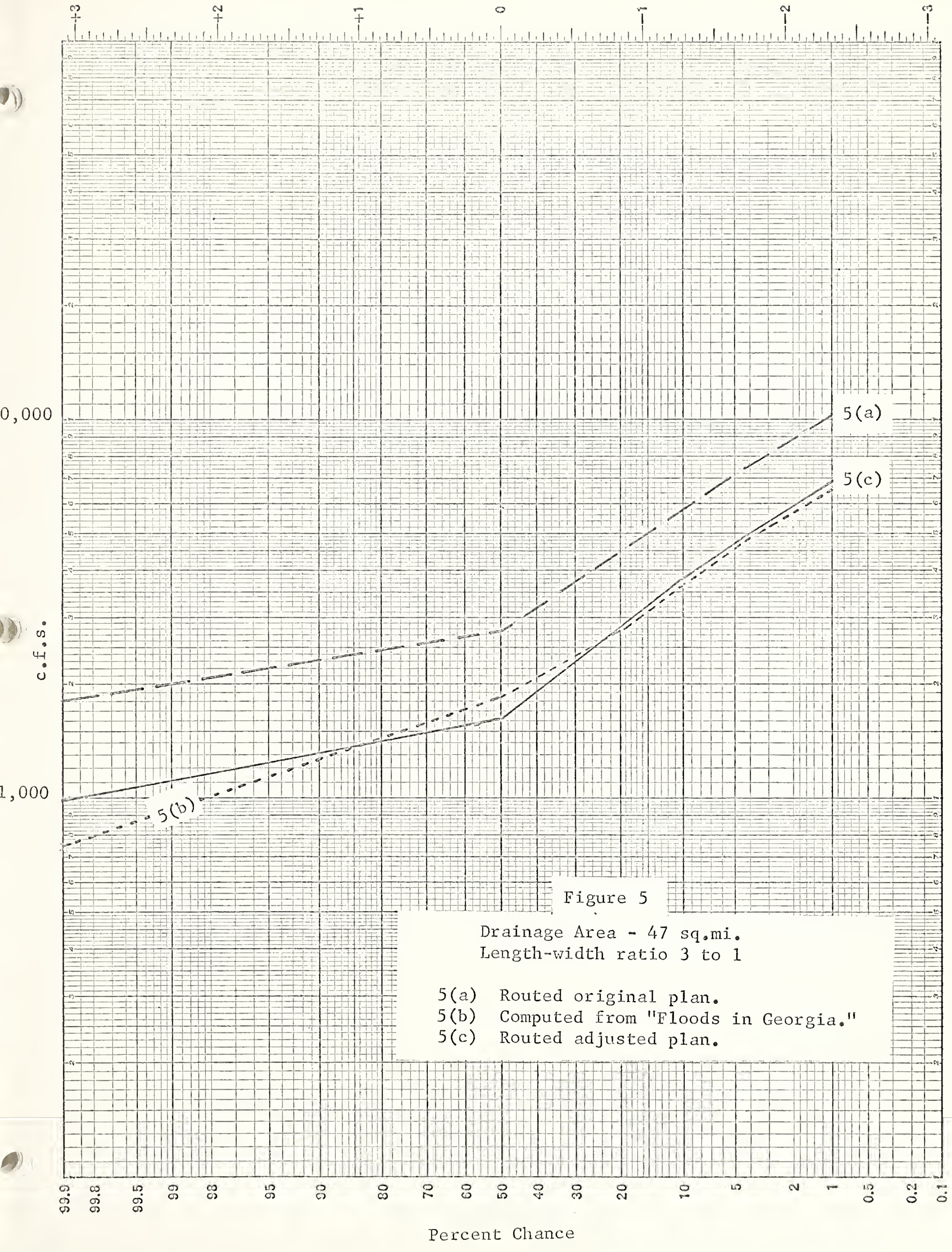


Figure 5

Drainage Area - 47 sq.mi.
Length-width ratio 3 to 1

5(a) Routed original plan.
5(b) Computed from "Floods in Georgia."
5(c) Routed adjusted plan.

SIMULATION OF BASIN HYDROLOGIC PROCESSES USING A MODIFIED STANFORD MODEL

R. E. Rallison

SCS Type IV study USDA coop. w/ State Resource Agency

INTRODUCTION

The increasing demand on existing water supplies in certain western areas is expected to reach crisis proportions within the next few years. Information describing quantity, location and distribution of water is frequently not available in sufficient detail to develop alternatives from which intelligent management decisions can be made. The need for a reliable procedure for not only defining the water resource in quantitative terms but also providing a basis for evaluating effects of land and water management programs, has led to the hydrologic model concept.

Until the last few years most hydrologic model studies concentrated on a segment of the total hydrologic system such as infiltration, snowmelt, consumptive use, overland flow, channel routing, etc. It has become apparent that a fragmented approach is not acceptable where the total water resource of a watershed or basin is under consideration, due to the fact that hydrologic processes are complex and interrelated and are strongly time dependent. To overcome these shortcomings, a hydrologic system approach has been used with varying success. Although a hydrologic system is relatively easy to describe from a qualitative standpoint, to obtain quantitative results is a difficult problem for reasons previously mentioned. Simulation of hydrologic processes by combining the processes into a dynamic system, subject to known or estimated physical constraints, can be an excellent tool for evaluating the interaction between hydrologic processes under different input conditions.

The concept of simulation of a hydrologic system is not a new idea; the simplest manual water accounting system is simulation, but the application of simulation to field problems in which most of the known hydrologic processes are accounted for has become much more practicable due to the development of high speed digital computers.

MODEL DEVELOPMENT

1. Requirements

Certain general objectives associated with basin wide land and water resource development dictated the choice of model

described in this paper. Prior to model development, the followed model characteristics were considered to be necessary.

a. Include the complete hydrologic system.

b. Be flexible enough to apply to a variety of meteorological conditions.

c. Be capable of simulating the hydrologic processes with a degree of accuracy consistent with measured input and output data.

d. Be sufficiently sensitive to reflect changes in output data when input parameters are modified.

e. Be structured so that input data are handled according to the best current understanding of hydrologic processes.

2. Selection of Model

In 1967 SCS entered into a contract with the Desert Research Institute (DRI) at Reno, Nevada to develop a hydrologic model capable of simulating the annual hydrologic cycle for the Central Lahontan River Basin. This area is meteorologically and hydrologically complex, ranging from desert conditions (mean annual precipitation less than 5 inches) in the vicinity of Walker Lake and Carson Sinks, to high mountain alpine conditions along the crest of the Sierra-Nevada range, where mean annual precipitation exceeds 50 inches. A majority of the precipitation at high elevation occurs during the winter months as snow.

Staff at DRI were familiar with the capabilities of the Stanford Watershed Model IV and it was agreed that a modification of this model offered the best potential to handle the types of problems expected to be encountered in the Central Lahontan Basin as well as other areas in the west.

3. Comparison with Stanford Model (1)

The model developed by DRI includes major elements of the hydrologic cycle as illustrated in Figure 1. It differs from the Stanford Model IV in the degree of detail used in the routing process--overland flow and interflow are not considered--and in the method of handling snow accumulation and melt.

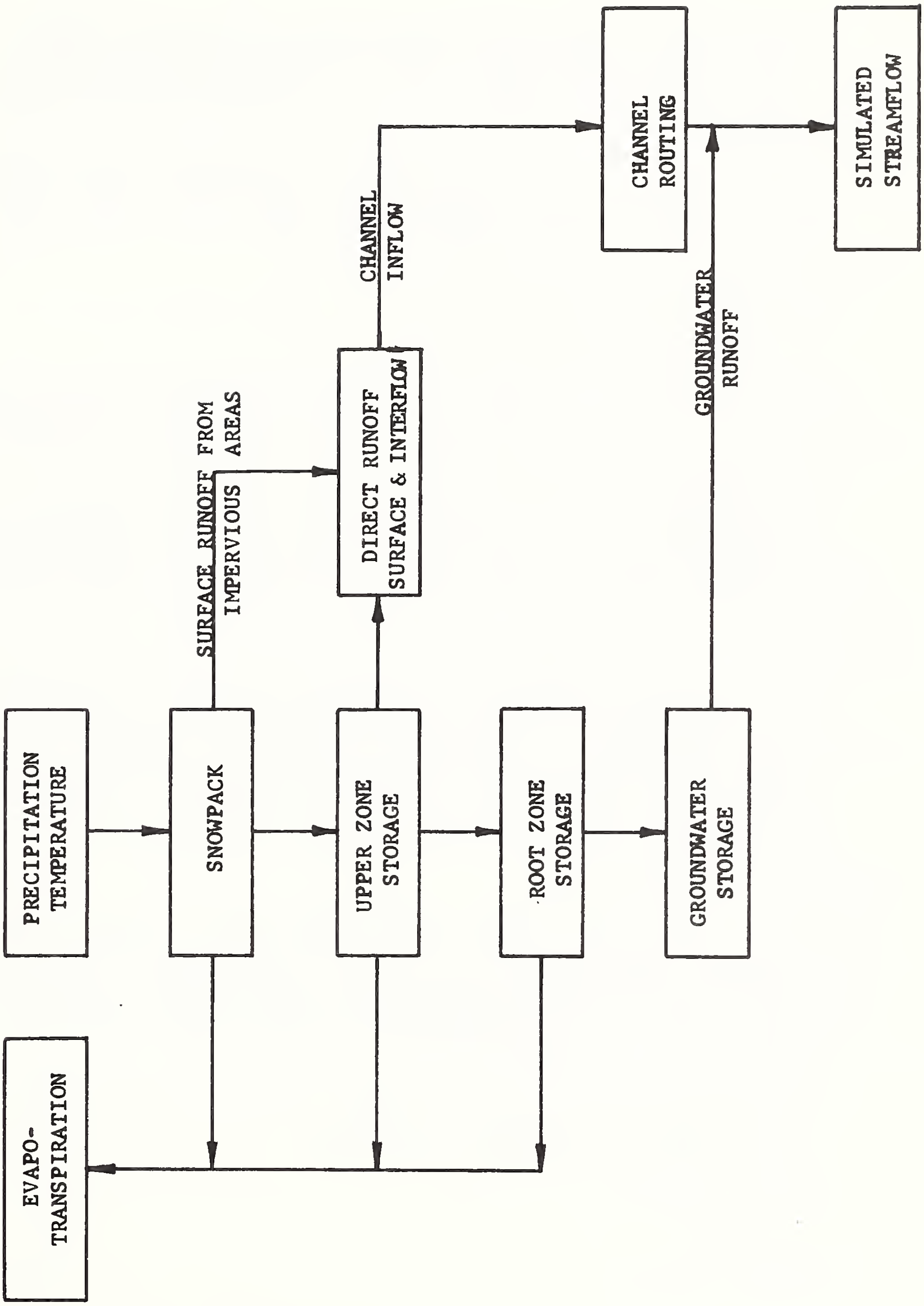


Fig. 1 Basin Simulation Model Flow Chart (2)

Features such as consumptive use by irrigated agricultural crops and phreatophytes and application of irrigation water through either diversion, reservoir or pumping are included in the modified version but are not included in the Stanford model.

The model has been used primarily in mountainous western areas and at present is most adaptable to developing a water budget where snow is the main source of runoff.

4. Description of Input Parameters (2)

a. Basin Characteristics

There is no limit on the size of drainage area that can be handled. Drainage areas are subdivided into elevation zones (usually 1,000 ft. increments) by general soil and cover types. Six general categories of cover type or water users are identified: native vegetation, stream bottoms, irrigated crops, phreatophytes, water surface and impervious area, and municipal and industrial use. Soils are classified by soil groups and subdivided primarily on their soil moisture water holding capacity.

b. Precipitation

In most instances, daily precipitation is not available for mountainous watersheds so precipitation from a valley station must be relied upon. Precipitation amounts are adjusted for any elevation zone by a ratio of average annual precipitation between the base station and the location being studied. Precipitation is assumed to be uniformly distributed throughout the day.

c. Temperature

Both maximum and minimum temperatures are used to develop average temperatures for half day intervals. Any lapse rate can be used to adjust temperatures from a base station to the location under study. The following equations are used to calculate average half day temperatures.

$$T\ day = 0.75\ T\ max + .25\ T\ min$$

$$T\ night = 0.75\ T\ min + .25\ T\ max$$

d. Evaporation

Lake evaporation is used as an index to potential evapotranspiration for native vegetation and is obtained from either adjusted pan measurements or Weather Bureau TP-37, "Evaporation Maps for the U. S." Evaporation from snow pack is estimated from data such as have been collected at the Central Sierra Snow Lab, which is located just west of the Sierra-Nevada crest on the headwaters of the South Fork of the Yuba River.

e. Evapotranspiration (4)

Water use by agricultural crops and phreatophytes is calculated using the modified Blaney-Criddle method utilizing monthly crop coefficients, percent of daylight hours and average monthly temperatures.

f. Infiltration

Soil storage is divided into three zones. The upper zone includes the effects of detention and interception storage with infiltration being a limiting factor in the decision as to whether surplus water goes into the root zone or into direct runoff. Moisture in the root zone percolates into ground water storage up to a maximum percolation rate with the excess moving into direct runoff. For each soil subdivision, upper zone and root zone available water holding capacities are estimated together with maximum and minimum infiltration capacities.

5. Hydrologic Procedures (2)

a. Ripening of the Snowpack

Precipitation occurring during average daily temperatures of 35° or lower is assumed to accumulate as snow. When temperatures cause melting, significant changes within the snowpack can be identified and quantified. The liquid water holding capacity of the pack is significant for small rains, which can be completely stored in the pack. The liquid water holding capacity of the pack is assumed to be a constant percentage of the water content of the pack at any given time.

b. Disposition of Snowmelt

Snowmelt or combinations of snowmelt and rainfall are disposed of through either infiltration or direct runoff. Snowmelt generally enters the root zone in the infiltration route since melt rates are usually less than infiltration capacities. Percolation rates to the ground water zone generally reflect the effects of a least permeable layer in the soil profile which can limit the amount of infiltrated snowmelt.

Melt is estimated in half day amounts in units of inches of melt per degree above 32°F. Although melt is known to vary throughout the year due to the amount of incident short wave radiation and albedo of the snow surface, no precise way is known to calculate these values by months in advance.

c. Handling of Irrigation Water

Irrigated and semi-irrigated cropland can obtain water from several sources. The program permits the diversion of water from either an outside source or a storage reservoir. Pumpage from ground water can also be a source of irrigation water. The supply available to irrigated cropland is defined on a monthly basis. Upper zone storage must be satisfied first to account for detention and interception storage. Infiltrated water is accumulated in the root zone until the irrigation requirement is satisfied. Irrigation requirement is currently calculated as the amount of water required to bring the root zone storage to 75 percent of field capacity. Provision is made to calculate a loss due to seepage from the irrigation distribution system in addition to field losses due to deep percolation and tailwater.

d. Routing of Runoff

Runoff is handled in two components. Direct runoff includes runoff from impervious areas as well as water which is excess to either infiltration capacities, soil storage capability, or percolation rates. Direct runoff is routed through the channel system using the Muskingum method of flood routing. Ground water storage is released at a rate determined from hydrograph analysis of the recession curve of a streamflow record from either the basin under study or from an area with similar meteorologic and geologic characteristics.

WATER BUDGET COMPUTATIONS (2)

For each subarea within the elevation zone, the water budget is computed in the following manner. See flow chart, Figure 2.

1. Water available is divided into runoff, evapotranspiration or soil moisture by the following steps:

a. Runoff from impervious areas is subtracted from the total available moisture.

b. Upper zone moisture holding capacity is satisfied.

c. Evapotranspiration is deducted from the upper zone at a potential rate if there is moisture in the upper zone.

d. Excess available moisture either infiltrates to the root zone or becomes direct runoff.

e. Infiltrated moisture is added to any existing in the root zone. If no moisture exists in the upper zone, evapotranspiration is subtracted from the root zone at a rate proportional to the quantity of available water in the root zone not to exceed the potential evapotranspiration rate for the crop.

f. Water from the root zone is percolated into the ground water zone by a rate which is determined by examination of trial computer runs.

g. Water is released from the ground water zone at a rate which is determined from storage outflow analysis of hydrographs to determine recession characteristic of ground water or base flow.

2. Runoff from impervious areas is added to direct runoff to give total direct runoff which is routed downstream by use of a channel routing procedure.

3. Total runoff from the elevation zone is the sum of direct runoff and ground water runoff.

An excerpt of printout data is shown in Table 1.

Three items considered in the water budget computations and which have their own set of constraints are as follows:

1. Areas containing phreatophytes remove water from root zones and ground water zones at a rate to satisfy potential ET requirements.

BASIN SIMULATION PROGRAM
FLOW DIAGRAM

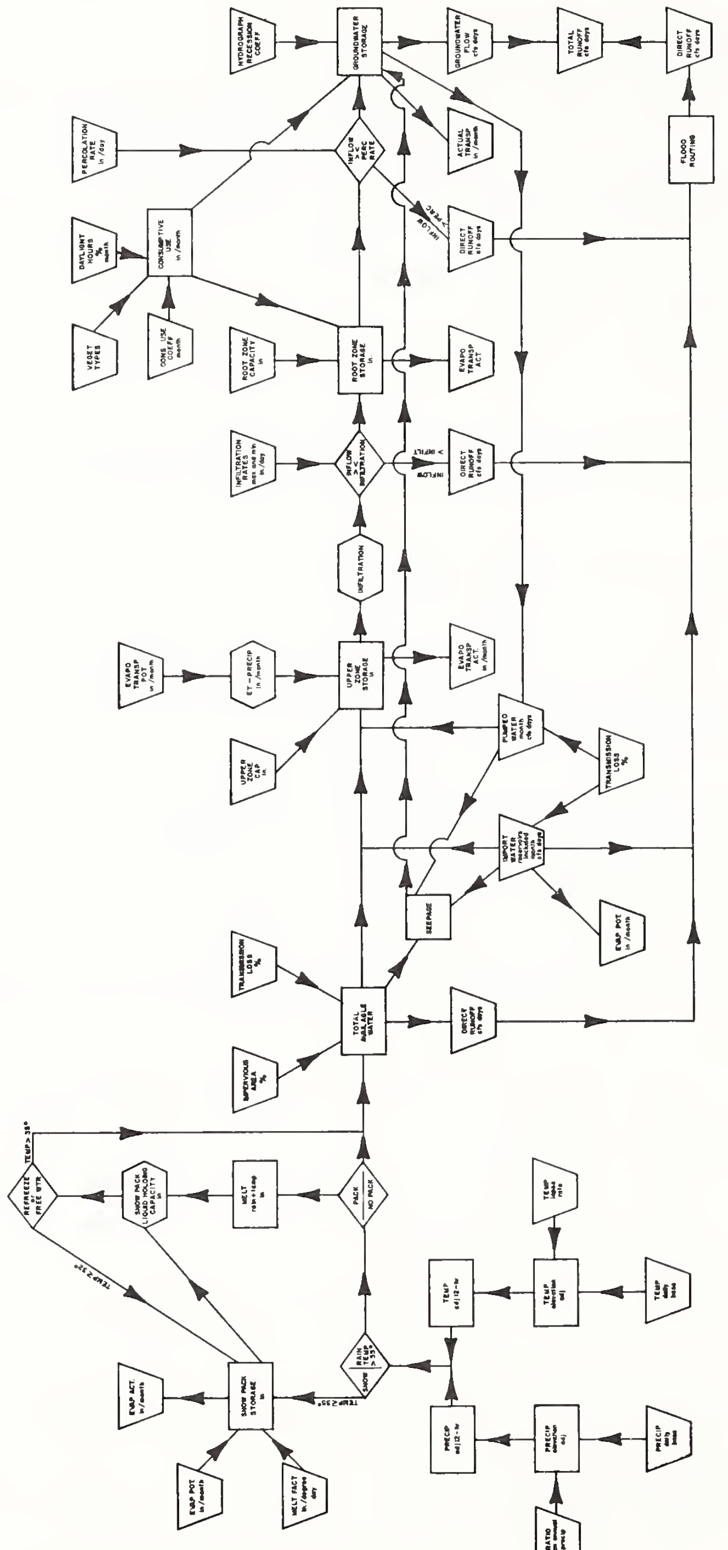


FIG. 2

TABLE 1

SUBAREA SUMMARY

Year 1960 June

Elev 9000 Precip Ratio = 1.20

AREA = 58.10 CIMP = 0.03 UZMAX = 0.50 BZMAX = 6.40 FMAX = 4.00 FMIN = 2.00 ETINDEX = 1

SUBAREA	RAIN	SNOW	POT ET	ACT ET	SURFLOW	GWFLOW	TOT FLOW	IRRIG
1	5.0219	0.9179	4.0515	3.7847	0.5520	0.1723	0.7244	

MONTHLY FLOWS IN INCHES

ZONE	RAIN	SNOW	POT ET	ACT ET	STORAGE	EVAP	MELT	PACK EOM	SUFLOW	GWFLOW	TOT FLOW	IRRIG
2	5.02	0.91	4.05	3.78		0.08	1.75	0.00	0.55	0.17	0.72	0.000000

FLOW TOTAL FOR SUB BASIN IN - AC FT (Includes imported water from large reservoir)

YEAR 1968

	OCT	NOV	DEC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	ANNUAL	
BASINQSU	7092.	10288.	11374.	9930.	6172.	3481.	2546.	2689.	4106.	72893.	57171.	11517.	199266.	AC FT
BASINQGW	1398.	793.	21.	2.	1.	0.	0.	0.	5778.	15329.	3427.	346.	27099.	AC FT
BASINQT	8491.	11081.	11396.	9932.	6174.	3482.	2546.	2689.	9884.	88223.	60599.	11864.	226365.	AC FT
BASIRRIG	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	AC FT

2. A reservoir subroutine operates in the following manner.

a. Total available water to the reservoir is calculated by summing precipitation, import water and runoff from all elevation zones within the basin which contribute to it.

b. Evaporation is subtracted at the potential rate.

c. Reservoir volume is adjusted for releases and seepage loss.

3. Irrigated areas are handled separately through an irrigation subroutine. The irrigation season is set as April 1 to October 31 unless otherwise specified.

a. Irrigation requirement is calculated as the amount of water required to bring the soil moisture profile to 75 percent of field capacity.

b. If no irrigation is required, the analysis returns to the main line program and analyzes the subarea the same as the other areas.

c. If irrigation is required, natural precipitation and known pumped water are checked first. If these satisfy the requirement, the analysis returns to the main line program.

d. If irrigation requirement is not satisfied, diverted irrigation water is used.

e. Diverted irrigation water is obtained first from the watershed itself and then from imported water. Imported water is entered into the program directly as a gross diversion which includes allowances for system efficiency.

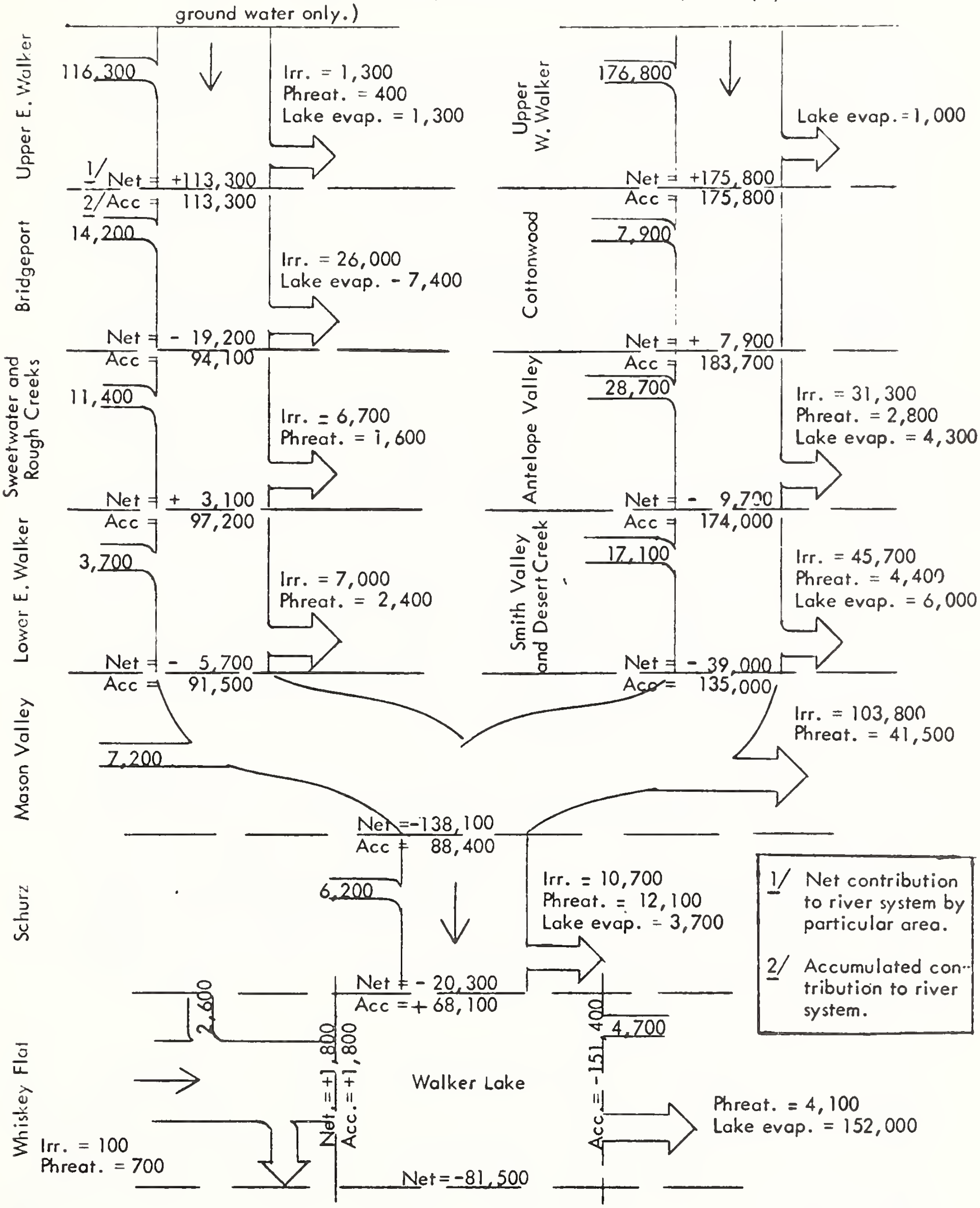
Monthly summaries are computed for all subareas.

A typical presentation of data obtained from the water budget program is illustrated in Figure 3 from the Walker River sub-basin report, Nevada. (3)

EVALUATION OF MODEL

The model as developed can reproduce monthly and annual volumes of runoff within the accuracy dictated by the input data. The accuracy of the model is limited by the assumption that the base

Figure 3 -- Flow diagram of water yield and depletions in acre-feet (50 percent chance), surface and subsurface flows, Walker River Subbasin. (Phreatophyte use from ground water only.)



precipitation station is representative of the entire watershed. The model gives response to storms which may not have covered the entire watershed and no response is shown for storms that were not recorded at the precipitation station.

The temperature index used to determine the form of precipitation over the watershed is generally satisfactory for the snow accumulation and melt season, but for individual storms it may be in error, causing runoff to be calculated at times where actual records show none. The overall simulation is not extremely sensitive to this index, but fall and late spring precipitation could be either rain or snow.

The timing of snowmelt runoff is controlled by the liquid water-holding capacity of the snow and the monthly snowmelt factors which are determined from air temperature. The temperature relationship works well for more periods with exceptions occurring when a variable temperature lapse rate occurs during the melt season.

The use of a greater number of elevation zones (current usage has been 1,000 ft. zones) would permit more precise definition of the area of melt so that responses to temperature change would be reflected in more gradual rises and recessions in direct runoff.

Opportunities for Program Improvements

The original digital computer program developed to handle the basin simulation model was first operated on an IBM 360 Model 65.

The program was restructured in the Portland ADP Unit so that it will operate efficiently on an IBM 1130-8K computer with disk drive capability. The program as currently written for the IBM 1130 computer includes 23 subroutines which permits considerable flexibility for future changes in individual subroutines without restructuring the complete program.

As more complete meteorologic data becomes available for study areas, it should be possible to incorporate these data into improved procedures for estimating rates of snowmelt, ET and other parameters where temperature is currently used as a sole index. For instance, it is known that radiation, wind velocity and temperatures at the specific sites would be desirable but until this type of information is available for areas under study, there is little value in including them at this time.

Because input data on precipitation and temperature are entered on a daily basis, it would be relatively simple to request output in the same detail if there was a need to obtain daily or weekly estimates of runoff. Although this kind of detail has not been needed in the basin areas studied to date, it would have value for items such as flood forecasting of snowmelt streams and to develop detailed reservoir operation schedules under a variety of conditions.

Improvements in other variables of the model such as infiltration capacities and water-holding characteristics of soils can also be expected as more detailed soil survey information becomes available.

FUTURE USES OF MODELING

Study of individual components of the hydrologic cycle within the framework of the total basin simulation model offers some existing challenges and potentially productive studies. The current model is no better than its weakest link so it is obvious that further studies to upgrade the weaker elements of the model are needed.

From an optimum land and water resource development standpoint, future use of a model for simulation of hydrologic processes and particularly as a tool for evaluating effects of alternative land and water resource development proposals is a necessity rather than a luxuary. As competing interests and conflicting purposes further narrow the gap between total available water supply and demand the challenge of providing a truly optimum solution to water development, problems will become extremely challenging. Remembering that a kingdom was once lost because of lack of a horseshoe nail, lets be sure that optimum development is not stymied because of a weak link in our knowledge of hydrologic processes.

1. Crawford, N. H., and Linsley, R. K. - Digital Simulation in Hydrology: Stanford Watershed Model IV, Stanford Univ., Dept. of Civil Engineering, Technical Report No. 39, Stanford, California; July 1966.

2. Fordham, John - Basin Simulation Program - Unpublished report prepared by Desert Research Institute for USDA, Soil Conservation Service; 1968.

3. U. S. Department of Agriculture, Soil Conservation Service - Water and Related Land Resources, Central Lahontan Basin, Walker River Sub-basin, Nevada-California; 1969.

4. U. S. Department of Agriculture, Soil Conservation Service - Technical Release No. 21 - Irrigation Water Requirements; Engineering Division; April 1967.

REGIONALIZATION OF STREAM GAGE DATA FOR MODIFICATION OF A FLOOD MODEL

W. J. Owen[1/]

Watershed planning involves the determination of flood damages under present conditions and for future conditions considering several alternate plans that would reduce flooding. Major items considered in planning are land use, conservation treatment, detention reservoirs, and channel improvement.

Flood damage studies are made to compare the benefits and costs of the plans under consideration. In many states the standard procedure to determine flood damages on an average annual basis involves the use of a partial duration flood frequency series. Runoff estimates for a range of frequencies are based on 24 hour rainfall amounts and the soil cover complex values for each subwatershed. Hydrographs are developed for each subwatershed. They are routed downstream and combined where appropriate to establish the peak discharge and duration of various rates of flow at selected points in the watershed. Damages are determined based on area flooded, depth and duration of flooding, and the type of crops or property involved. Another important factor to consider with crop damage is the season or month of the year floods occur. A standard practice is to use a monthly distribution of flood events based on the monthly distribution of intense rainfall events for a given area.

This procedure was used in the early years of the watershed program because adequate stream gage data were not available for most of the watersheds we were planning. At the present time several factors have changed this situation. Additional gaging stations have been installed on smaller watersheds. With the passing of time the length of record for many stations has increased sufficiently to provide adequate data for frequency analysis. The drainage area of the typical watershed we are planning has increased to a point where the principal damage areas may have drainage areas over one or two hundred square miles.

Several years ago we were making a study of a watershed having a drainage area of about 200 square miles. Standard

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procedures were used. Hydrographs were developed and routed using the SCS TR-20 computer program. Volumes of runoff for selected frequencies were based on 24 hour rainfall amounts and the soil cover complex values for average antecedent soil moisture conditions. One stream gaging station is located in the watershed at a point where the drainage area is 130 square miles. Another station is located on the same stream but outside our study watershed at a downstream point having a drainage area of 460 square miles. The length of record is 27 and 35 years respectively.

After the routings for present conditions were completed, the peak discharge values for the location near the upstream gage were compared with a frequency analysis of the measured peak flows. The measured peak flows were about 75 percent higher than the computed values. Studies were continued to determine the cause of an error of this magnitude. Storm distribution, time of concentration, reach routing, and volume of runoff could be a factor.

From the measured data a detailed study of runoff volumes was made by use of manual procedures. Hourly flow information was available for several major flood events. Hydrographs of one flood event for both stream gages are shown on Figure 1. A study of the hydrographs for several events indicated the upstream hydrograph had a typical base time of about 40 hours and the downstream hydrograph had a base time of about 3 days.

Past studies of a large number of hydrographs from watersheds in the Midwest have shown that the base time of hydrographs at a given location is not highly variable. This is particularly true for drainage areas greater than 200 square miles. In the larger watersheds the constant values of drainage area, slope, and watershed shape have considerably more influence on the shape and base time of the hydrograph than the typical variation in the duration of excess rainfall.

In order to use the complete record where the only readily available information is the average flow for each calendar day, it is necessary to select a base time in whole days to determine the volume of runoff. In the upstream watershed with a base time of 40 hours, 2 calendar days would be adequate if the runoff starts near midnight. This was the case of the flood hydrograph shown on Figure 1. However if the flood starts at noon or 6 p.m., the 40 hour base period will involve 3 calendar days. Flood volumes were determined based on both the 2 and 3 day period. The resulting frequency curve based on the 2 day total equaled 90 percent

of the values based on the 3 day total. Both curves were nearly double the volume obtained using rainfall and soil cover complex values. This indicated that the error involving the peak flows was due to errors in estimating the volume of runoff. Volumes were computed for the downstream gage using a base time of 4 calendar days. Results were similar to those of the upstream gage.

The runoff records indicated a large number of flood events occurring during the winter and spring months. Our standard soil cover complex values are based on an average condition during the growing season. In areas where a substantial share of the floods occur during the dormant season, we should increase our estimates of the soil cover complex values to account for the changing conditions during a year. Another item used in our standard procedure that resulted in the low estimate of runoff is the use of an average antecedent soil moisture condition. Studies of runoff and rainfall data during the dormant season indicated that a high percent of the major rainfall events occurred with antecedent soil moisture conditions above normal or near saturation. Considering the change in curve numbers during the year and the probability of having high antecedent soil moisture conditions during the dormant season result in the conclusion that a curve number in the range of 90-92 should have been used instead of the number 80 that was based on standard procedures. If curve number 92 had been used in the initial routings, we would have had close agreement with the measured peaks for the range of frequencies being considered.

Studies of the gaged data also indicated a considerable difference in monthly distribution of flood events compared to our estimates based on the monthly distribution of excessive rainfall events. The gaged data indicated a higher percent of the floods occurring during the winter months and a lower percent during the summer and fall. This undoubtedly caused by the seasonal variation in curve numbers and the high antecedent soil moisture conditions existing during the winter and spring months. Monthly distribution is an important factor in determining agricultural damages because the factors used for various crops are considerably higher for certain months during the growing season.

The results of this study indicated that substantial errors were involved in this watershed when we used our standard procedures. It was decided that it would be desirable to

expand the areal coverage of the study and include about 200 watersheds located in four adjacent states.

Because the manual procedure is laborious and time consuming, the SCS Central Technical Unit was requested to write a computer program to perform this type of analysis. Records of the average daily flow are on magnetic tape and are available from the U. S. Geological Survey.

A key item for each station is the selection of a base time in whole days. Previous studies by others had determined unit hydrographs for a large number of watersheds in one state. The base time used for these watersheds was the base time of the unit hydrograph rounded off to the next highest day. For other watersheds the selection of a base time involved the study of several individual flood events. The minimum base time used was 2 days considering the probability that even for small watersheds, under 50 square miles, that some of the runoff would occur around midnight.

In the area of interest the flood runoff volumes varied considerably among some of the watersheds. It was decided to use a base volume for the partial duration series that was equal to the lowest annual volume during the period of record. The other alternatives would be to select a different base volume for each watershed or use a constant value for all watersheds.

An additional requirement was that only floods separated by a 15 day interval should be used. This was wanted by our economists; their reasoning being that flood events occurring within 15 days did not permit sufficient time for crop recovery, reseeding or repair to roads, bridges or other types of property.

Alteration of basic data in this manner violates some statistical principals. Records were also analyzed using a zero day separation between flood events. This resulted in including many more smaller floods and only a few of any significant size. The effect on the frequency curve was to raise the values for frequencies of less than one year and lower the values for the larger frequencies. In the majority of cases the estimate for the 100 year volume was reduced 12 percent when the allowable interval between flood events changed from 15 to 0 days.

An additional frequency analysis was made using only the annual values. It was believed that this method would

provide the most accurate estimate of the larger less frequent flood events. In general the estimates for the 100 year frequency based on the annual series were between the values obtained from the partial duration series using a 0 and 15 day separation. It was concluded that using the 15 day separation requirement did not significantly affect the results.

An example of the program output is presented on the attached two tables. Monthly distributions are provided for a range in volumes and also for a range of frequencies. The actual distribution used in a watershed analysis will depend on the volume or frequency that begins to cause significant flood damages.

The equation for the frequency line and volumes for selected frequencies are listed near the center of the first table. The maximum volume for each year is listed at the top of the second table. Following the annual series, the date and volume of all floods used in the partial duration study are listed by size.

In addition to the two attached tables the program provides a graphical plot of the frequency line and some of the values used in the analysis. This plot provides an easy visual means to determine if any of the larger volumes may be outliers.

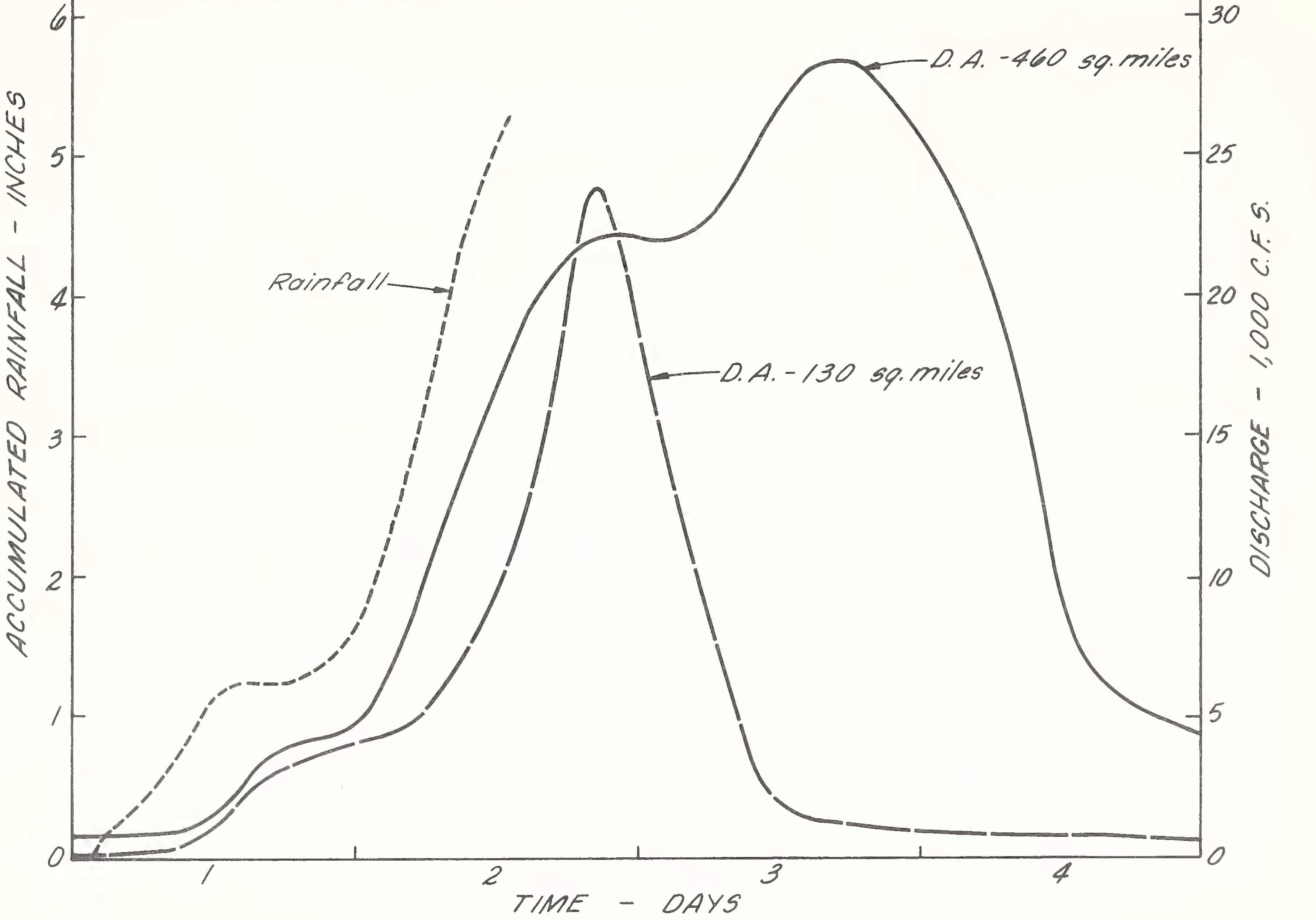
With the aid of computers this procedure provides a relatively easy way to analyze a large amount of available data. Results should not be considered as precise because of the problem of using whole calendar days. For small watersheds where a minimum base time of 2 days is used, it is possible that some of the volumes include runoff from two separate flood events. For large watersheds where the base time may be 5 or 6 days, the probability is increased that some of the volume may include more than one separate flood event. Errors of this type may be sizeable for a few individual flood events but their effect on the final frequency estimate is believed to be minor. The results provide a general estimate for a broad area and insure the hydrologist that his values are in close agreement with the measured data.

The principal reason this study was made was to check our standard procedures used in estimating runoff on a frequency basis. In many areas the measured data were in complete agreement with results obtained by use of our rational procedures. In some areas the estimates based on the

rainfall-soil cover complex method were considerably lower than the measured data. I believe this is a result of the climatic characteristics of the area and is caused by the frequent flooding during the winter months. Note the date of the flood events listed on the second table.

FIGURE 1

Example Hydrographs



STATION NO. XXXXXX BLUE RIVER
FIRST MONTH OF PERIOD OCT. LAST MONTH SEP. FIRST YEAR 1931, LAST YEAR 1966
DURATION PERIOD 4 DAYS. INTERVAL BETWEEN PERIODS 15 DAYS.
DRAINAGE AREA 461.00 SQ.MI. BASE VOLUME 0.44 INCHES.
VOLUME INTERVAL 0.61 NO. OF INTERVALS 9.

NUMBER OF FLOOD EVENTS BY MONTHS FOR 35 YEARS.

AC-IN/AC	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL	PCT/INT	ACCUM.PCT
0.44-1.05	11	9	15	14	7	7	5	2	3	3	2	8	86	54.09	54.09
1.06-1.66	5	4	6	6	4	1	1	0	0	0	4	3	34	21.38	75.47
1.67-2.27	2	5	7	3	0	2	0	0	0	0	0	0	19	11.95	87.42
2.28-2.88	4	1	0	1	2	0	0	0	0	0	0	2	10	6.29	93.71
2.89-3.49	1	1	2	1	0	0	0	0	0	0	0	0	5	3.14	96.86
3.50-4.09	0	0	1	0	0	0	0	0	0	0	0	0	1	0.63	97.48
4.10-4.70	1	0	0	0	0	0	0	0	0	0	0	0	1	0.63	98.11
4.71-5.31	1	0	0	0	1	0	0	0	0	0	0	0	2	1.26	99.37
5.32+	0	0	1	0	0	0	0	0	0	0	0	0	1	0.63	100.00
TOTALS	25	20	32	25	14	10	6	2	3	3	6	13	159		
AVG NO/YR	0.71	0.57	0.91	0.71	0.40	0.29	0.17	0.06	0.09	0.09	0.17	0.37	4.54		
PCT/MONTH	15.72	12.58	20.13	15.72	8.81	6.29	3.77	1.26	1.89	1.89	3.77	8.18	100.00		

EQUATION OF THE LEAST-SQUARE LINE IS- Q = 1.8250 + 2.3049 X LOG(RETURN PERIOD).

COMPUTED VOLUMES FOR SELECTED RETURN PERIODS-

YEARS	INCHES	YEARS	INCHES	YEARS	INCHES	YEARS	INCHES
100.0	6.4348	50.0	5.7410	25.0	5.0471	20.0	4.8237
15.0	4.5358	10.0	4.1299	7.0	3.7729	5.0	3.4361
2.0	2.5188	1.0	1.8250	0.9	1.7195	0.7	1.4680
0.5	1.1311	0.3	0.6198	0.2	0.2139	0.1	0.0

NUMBER OF FLOOD EVENTS BY MONTH BY RETURN PERIOD.

PD RETURN PERIOD	JAN	FED	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	TOTAL	PCT/INT	ACCUM.PCT
200.0 +	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0	100.00
100.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0	100.00
50.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0	100.00
25.0	1	0	1	0	1	0	0	0	0	0	0	0	3	1.89	100.00
10.0	1	0	0	0	0	0	0	0	0	0	0	0	1	0.63	98.11
5.0	1	1	2	1	0	0	0	0	0	0	0	0	5	3.14	97.48
2.0	2	0	1	1	2	0	0	0	0	0	0	1	7	4.40	94.34
1.0	3	5	7	3	0	2	0	0	0	0	0	1	21	13.21	89.94
0.5	7	9	6	9	6	5	1	1	0	0	4	3	51	32.08	76.73
0.2	10	5	15	11	5	3	5	1	3	3	2	8	71	44.65	44.65
0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0	-0.00
TOTALS	25	20	0-	25	14	10	6	2	3	3	6	13	159		

ANNUAL SERIES

1932	2.3426	1933	2.7968	1934	0.7397	1935	3.0138	1936	1.2479	1937	5.2410	1938	2.0619	1939	1.7803
1940	2.2523	1941	0.4421	1942	1.5609	1943	3.4163	1944	1.5714	1945	3.5792	1946	1.7021	1947	1.4060
1948	3.4550	1949	2.6637	1950	3.2590	1951	2.5741	1952	2.0998	1953	1.3891	1954	0.4811	1955	1.8215
1956	1.8368	1957	2.7467	1958	2.3434	1959	4.6037	1960	1.9562	1961	5.0361	1962	3.4397	1963	1.8005
1964	5.3120	1965	1.2068	1966	1.8400										

STATION NO.
XXXXXXX

DAY MON YR	Q-INCHES	YEARS	LOG(YRS)	DAY MON YR	Q-INCHES	YEARS	LOG(YRS)
9 MAR 64	5.3120	35.0000	1.5441	22 JAN 37	5.2410	17.5000	1.2430
7 MAY 61	5.0361	11.6667	1.0669	21 JAN 59	4.6037	8.7500	0.9420
4 MAR 45	3.5792	7.0000	0.8451	12 APR 48	3.4550	5.8333	0.7659
26 FEB 62	3.4397	5.0000	0.6990	19 MAR 43	3.4163	4.3750	0.6410
4 JAN 50	3.2590	3.8889	0.5898	11 MAR 35	3.0138	3.5000	0.5441
3 APR 50	2.8661	3.1818	0.5027	14 MAY 33	2.7968	2.9167	0.4649
22 MAY 57	2.7467	2.6923	0.4301	25 JAN 49	2.6637	2.5000	0.3979
14 JAN 51	2.5741	2.3333	0.3680	31 DEC 33	2.5491	2.1875	0.3399
21 JAN 33	2.4346	2.0588	0.3136	13 FEB 50	2.3837	1.9444	0.2888
19 DEC 58	2.3434	1.8421	0.2653	15 JAN 32	2.3426	1.7500	0.2430
19 APR 40	2.2523	1.6667	0.2218	16 APR 33	2.1014	1.5909	0.2016
10 MAR 52	2.0998	1.5217	0.1823	30 MAR 38	2.0619	1.4583	0.1639
19 MAR 33	2.0538	1.4000	0.1461	18 MAR 51	2.0369	1.3462	0.1291
23 JUN 60	1.9562	1.2963	0.1127	16 JUN 49	1.9449	1.2500	0.0969
14 FEB 49	1.9175	1.2069	0.0817	26 MAR 49	1.9167	1.1667	0.0669
10 FEB 66	1.8400	1.1290	0.0527	15 FEB 56	1.8368	1.0937	0.0389
17 APR 41	0.4421	0.2201	-0.6573	(SMALLEST ANNUAL FLOOD)			

USDA-SCS-LINCOLN NEBR 1970 - 88

Type IV River Basin Survey

ELECTRONIC ANALOG MODEL APPROACH FOR THE SEVIER RIVER, UTAH

Erlend Warnick, Hydraulic Engineer, Central Technical Unit, Soil Conservation Service, Hyattsville, Md.

The use of the electronic analog watershed model, to be described in this paper, was a part of the cooperative Type IV River Basin Survey of the Sevier River, Utah by the U.S. Department of Agriculture and the state of Utah. The study of the basin's water and related land resources was carried out using the water balance approach. In fact the water budgets provided the framework for determining the present water supplies and problems as well as for evaluating future needs and impacts of possible corrective projects. The question of magnitude and timing of downstream impacts due to upstream conservation projects was especially critical in this water short area. The analog computer model used in the Sevier River study was developed based on this water balance concept.

Simply stated, a water budget is an accounting of water supply to an area, the use and change in storage within that area, and the outflow from it. For a given time period and area the equation is

$$O = I - U + \Delta S$$

where
- O = volume of outflow
- I = volume of inflow
- U = sumation of all consumptive uses
- ΔS = volume change in storage (can be positive or negative)

An example of a water budget from the Sevier River study is shown in Figure 1. More will be said of the area described by this budget later.

Individual water budgets on a monthly basis were prepared for each watershed delineated in the basin. Summary water budgets were formulated for the larger hydrologic areas (Subbasins).

The major problem encountered in formulating water budgets for the Sevier River watersheds was in determining the quantity and time distribution of the water supply as it becomes available for use.

Figure 1 - AVERAGE ANNUAL WATER BUDGET SUMMARY
SUB-BASIN D, SEVIER RIVER BASIN, UTAH
(Values are in Acre-Feet)

	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec	Annual
River Inflow	2920	2540	3420	13570	25960	25660	27580	21660	18450	8650	4430	2950	157,790
Tributary Inflow	2020	2120	3000	8170	14730	11780	4270	2290	1780	1930	2030	2120	56,240
Tributary Diversions	250	240	1080	3240	5710	4740	2250	1520	1310	1260	1010	250	22,860
Other Diversions	1770	1590	2030	9310	22370	21240	23390	18550	14990	8750	5790	2720	132,500
Total Diversions	2020	1830	3110	12550	28080	25980	25640	20070	16300	10010	6800	2970	155,360
Supply to Root Zone	690	620	1020	4230	9470	8770	8660	6780	5500	3370	2290	1000	52,400
Wells				70	460	500	400	340	350	80			2,200
Supply to Root Zone				30	170	180	130	120	130	30			790
Precipitation on Irrigated Cropland	2150	2220	2510	2370	2680	1890	2750	2680	1780	2200	1920	1990	27,140
Direct Use from Ground Water	60	110	320	670	1280	2060	2550	2030	1210	600	190	80	11,160
Total Supply to Root Zone	2900	2950	3850	7300	13600	12900	14090	11610	8620	6200	4400	3070	91,490
P.C.U. on Irrigated Cropland	590	1120	2880	5490	10880	19400	22990	16610	9670	4990	1750	730	97,100
Soil Moisture Storage (Max. capacity 34,270 AC.-FT.)	9290	10910	11710	12980	15190	8630	1630	1730	1770	2610	5090	7210	
Consumptive Use Deficit							1900	5100	1390				8,390
Actual C.U. on Irrigated Cropland	590	1120	2880	5490	10880	19400	21090	11510	8280	4990	1750	730	88,710
Addition to Ground Water	230	210	170	540	510	60	-	-	300	370	170	220	2,780
Domestic Use and Water Surface Evaporation	50	120	280	440	680	820	900	820	590	300	130	30	5,160
Supply to Wetlands & Ground Water	3220	3070	3580	12650	26010	21250	15420	11090	9640	6750	5380	3950	122,010
Yield Direct to Ground Water	320	320	340	520	730	630	390	320	320	320	320	320	4,850
Precipitation on Wetlands	440	460	510	480	550	390	560	550	370	450	390	410	5,560
Wetland Consumptive Use	220	340	800	1470	2620	4040	5860	5520	3730	2040	730	340	27,710
Outflow & Change in Ground Water	3760	3510	3630	12180	24670	18230	10510	6440	6600	5480	5360	4340	104,710
River Inflow not Diverted	1150	950	1390	4260	3590	4420	4190	3110	3460	-100	-1360	230	25,290
River Outflow - Gaged	6790	6950	6980	6870	5590	4590	1220	1260	1760	4160	5240	6090	57,500
Canal Outflow	930	690	720	5300	9410	9640	10800	8490	6880	3810	1700	1130	59,500
Ground Water Outflow	850	860	930	1400	1960	1690	1050	870	840	840	860	850	13,000
Total Outflow	8570	8500	8630	13570	16960	15920	13070	10620	9480	8810	7800	8070	130,000

Since only part of the surface runoff and none of the ground water runoff in the small watersheds of this basin was measured, it was necessary to estimate the magnitude and timing of water supply by trial and error procedures. Making these trial and error solutions using desk calculators was fairly successful, but required a great deal of time and left much information lacking, especially concerning ground water movement. The use of watershed models to aid the study on the Sevier River was prompted by this need for more detailed analysis than was feasible to obtain by hand.

DEVELOPMENT OF THE SEVIER RIVER BASIN ANALOG MODEL

A modified electronic analog computer was selected as the basic computing device because it is ideally suited for solving the many time dependent differential equations encountered in hydrologic systems. Also, in dealing with problems of optimizing water use when many trials must be made with different input values, the effects on the system of changing an input parameter can be observed immediately. Slight changes in the modeled system and inputs to it can be made rapidly and conviently. Since the output is displayed graphically, the operator can visualize the results as being the dynamic responses of the prototype physical system.

The analog computer model was developed and built at the Utah Water Research Laboratory in Logan, in cooperation with the Agricultural Research Service and the Soil Conservation Service of the U.S. Department of Agriculture (1). This type of model does not directly simulate the real system being modeled. It is analogous to the prototype system in that both are described by the same mathematical relationships. This is in contrast to the direct analogy models or network analyzers where each element of the model is directly analogous to a physical element in the real system. An example is the resistance-capacitance networks used to simulate water movement in stream channels and groundwater basins.

The analog model used in the Sevier River Basin can be thought of as being made up of three sections. The first is the three component racks on the left shown in Figure 2. The components are high gain DC amplifiers, potentiometers, and relays. These are interconnected so as to make the model electronically represent the mathematical relations describing the hydrologic system being studied. The relations represented in this section of the model are those such as the water holding capacity of the soils, irrigation efficiencies,

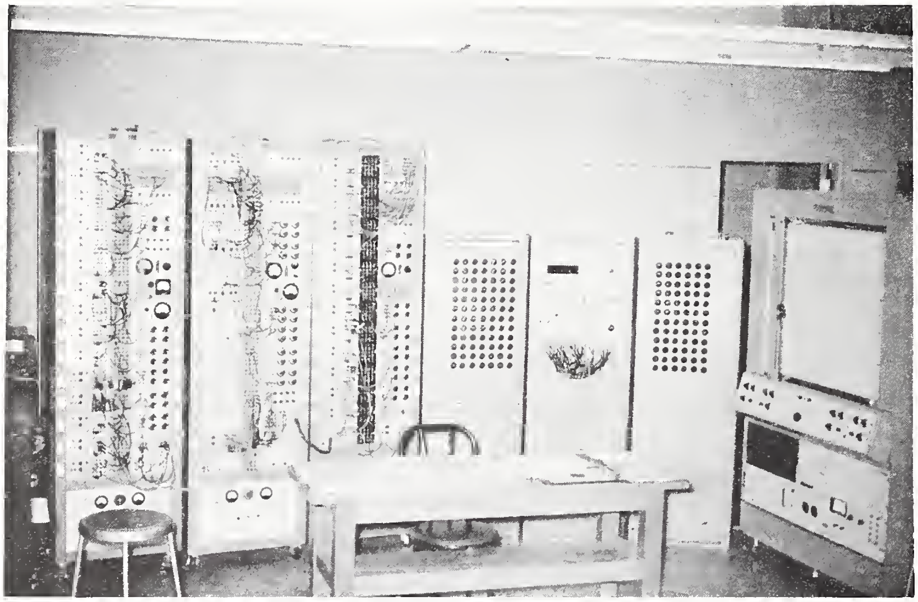


Figure 2 - Sevier River Basin Investigation Analog Model showing the amplifier racks (3), the main control panel, the input potentiometer panels, and the output plotter.

Figure 3 - SRBI Analog Model main control panel and digital voltmeter with the input potentiometer panels on each side.

routing of irrigation seepage losses, ratio of surface and groundwater runoff from adjacent mountains, amount of water in storage in the groundwater reservoir, and relation between groundwater storage and return flow to the river.

The second section of the model is the control and data input panels, the three shorter units in the center of Figure 2. Figure 3 shows a close-up of these consoles. The two outside consoles have 16 columns of 12 input potentiometers each--one potentiometer in the column for each time unit (integration period) for which the model was programmed. These provide for the input of digital types of data such as monthly precipitation, mean monthly temperature, canal diversions, river flow, tributary runoff, and crop growth stage coefficients. The crop growth stage coefficients describe the water use characteristics of different crops and are used in the Soil Conservation Service method of determining water requirements of vegetation (7).

The center console in Figure 3 is the control panel. The top unit is a digital voltmeter. The middle unit has lights that indicate each step of the operating cycle and emergency lights that indicate malfunction. The bottom unit is a central wiring board used to connect the input potentiometers and output devices with the component racks. More recently a removable wiring board has been added to make possible a more rapid changeover from one watershed model to another (4). All of the various components of the system may be interconnected from this board. The change from one watershed model to another is accomplished by merely replacing one wiring board or patch panel with another which has previously been wired to represent the desired watershed model.

The third section of the analog model is the output plotter shown on the right in Figure 2. In this type of analog model, all items of input and output data are represented by voltages. At any point within the model, the particular hydrologic process being simulated can be observed visually on the voltmeter or recorded graphically by means of the X-Y plotter.

As the model goes through an operation cycle, it progressively switches from one month's data to the next until each of the 12 months' data have been used. In each integration period (one second) of the operation cycle, 16 types of monthly data are introduced into the model, assimilated and used with respect to the relations programmed in the model, and the results plotted graphically on the X-Y plotter. Then the

switches are automatically thrown and the next month's 16 items of data are introduced with the results of the first becoming the initial conditions of the next and so on through the cycle. The operation cycle can be repeated using the same input data, allowing the operator to plot as many different items of output data as are desired. This capability of automatically stepping through successive time periods makes this model especially useful in watershed studies.

PROGRAMMING THE ANALOG MODEL

The area of the Sevier River Basin study used to illustrate the use of the electronic analog model is designated as Sub-basins A, C, and D in Figure 4. This area is mountainous with only 10 percent of it cropped in a narrow band along the river. The Sevier River flows into the upper end of Sub-basin D. Its flow as well as that of the San Pitch River (Sub-basin A) and other smaller tributaries is diverted into irrigation canals for distribution to the cropland. Return flows to the river are rediverted for use on other cropland within the area. Seepage losses from irrigation canals and any over irrigation percolates downward into the ground-water reservoirs which are inseparably connected with the surface flow of the river. There are three distinct groundwater reservoirs, which roughly correspond to the three sub-basins, with a sub-surface barrier of relatively impermeable material at the lower end of each. These barriers tend to maintain the water level in each basin. If the reservoirs were more extensively used by pumping or if irrigation seepage losses were reduced, the water tables would be lowered causing a decrease in return flow to the river.

The analog model of this area included only the valley portion of the Sub-basins and the ground water reservoirs due to a lack of data from the mountain areas. It consisted of a generalized hydrologic model of each of the three sub-basins interconnected so that the outflow, both surface and ground water, from Sub-basins A and D became the inflow to Sub-basin C. Outflow from the Sub-basin C portion of the model corresponded to the inflow into Sevier Bridge Reservoir (see Figure 4). The model of each sub-basin was made up of circuits simulating the cropland area, the wetland or phreatophyte growing area, and the groundwater reservoir with the inputs, water supply depletions, and outflows needed to represent each area.

The data used to describe the hydrologic conditions of Sub-basins A, C, and D, as programmed for this model were average

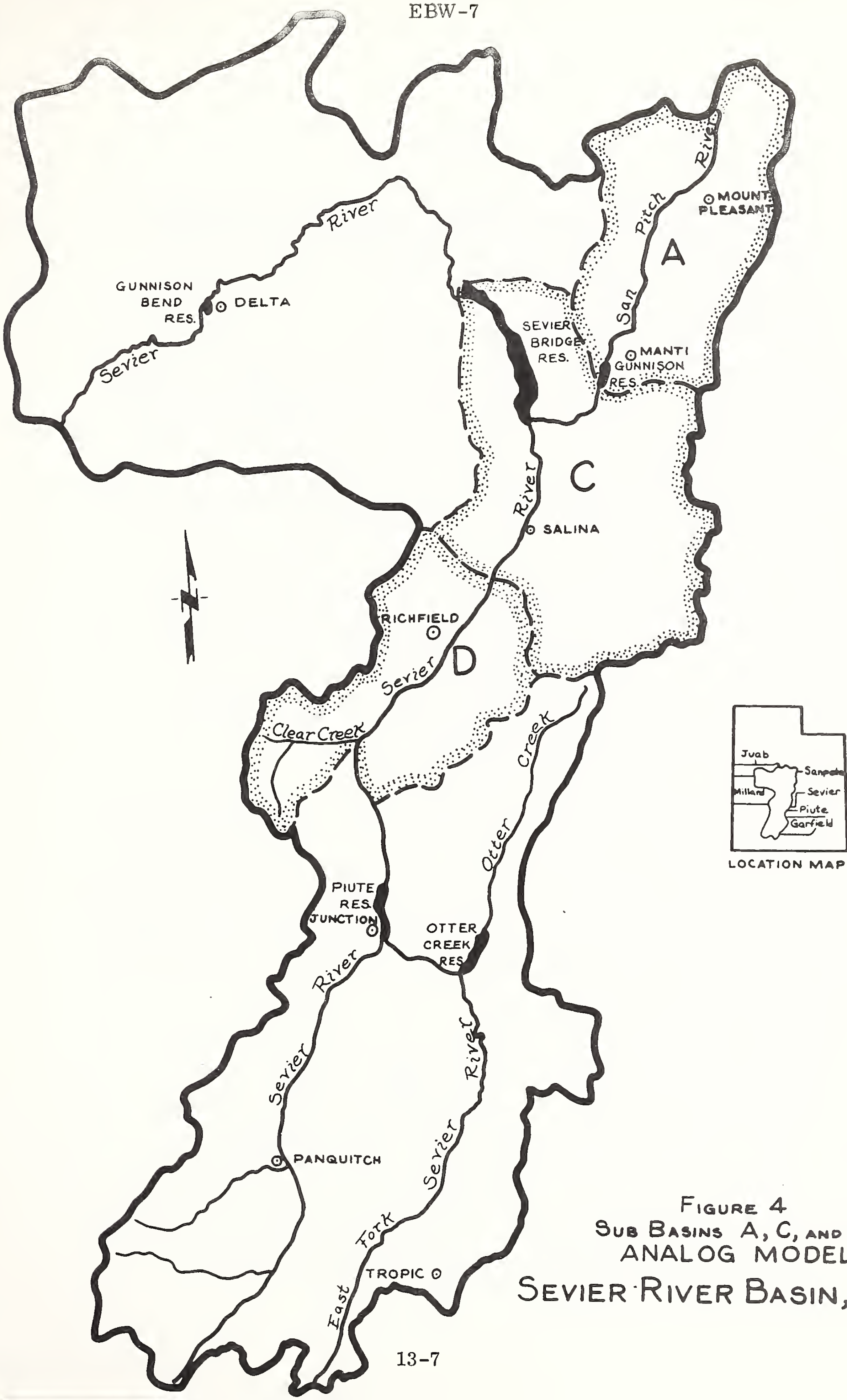


Figure 4
Sub Basins A, C, and D
ANALOG MODEL
Sevier River Basin, Utah

values for the 1931 to 1960 period. For some of the other areas modeled in the Sevier River basin, historical data for a series of years were used instead of the average, depending on the information that was desired.

TYPICAL RESULTS FROM THE ANALOG MODEL

Some of the most useful information obtained from the analog models was learned during the calibration process. Adequate information about some of the factors in the equations used to describe the hydrologic conditions of a modeled area was often lacking. In that case, the values of those factors had to be first estimated and then determined more accurately by means of trial and error during calibration of the model. That type of information for the analog model of Sub-basins A, C, and D was:

1. The percent of the tributary runoff from the mountains which enters the valley as surface flow and as groundwater flow.
2. The relation between the amount of ground water in storage and the flow from the groundwater reservoir into the river.
3. The relation between a change in groundwater storage and the resulting change in the wetland or phreatophyte consumptive use.

Figure 5 illustrates an example of one type of information obtained from the analog model and used to help evaluate the effects of conservation practices. These curves illustrate the effect on the cropland root zone water shortages or surplus brought about by increasing irrigation efficiencies. In some models, several levels of water supply were analyzed, so that a family of curves could be plotted to reflect the effect of water supply on root zone deficits. Figure 6 is an example of this type of plot and is for the area around the town of Delta on the lower end of the river.

From the curves of Figure 5, it can be observed that in Sub-basin C an irrigation efficiency of about 32 percent (that is an improvement of 2 percent) would overcome the deficit on the presently irrigated land with an average water supply. These efficiencies for Sub-basins A and D are 35 percent (7 percent improvement) and 38 percent (4 percent improvement) respectively. These efficiencies are defined as the over-all efficiency from the point of diversion to the crop root zone. They are based on the assumption of a completely uniform distribution of irrigation water among all irrigation companies within each sub-basin.

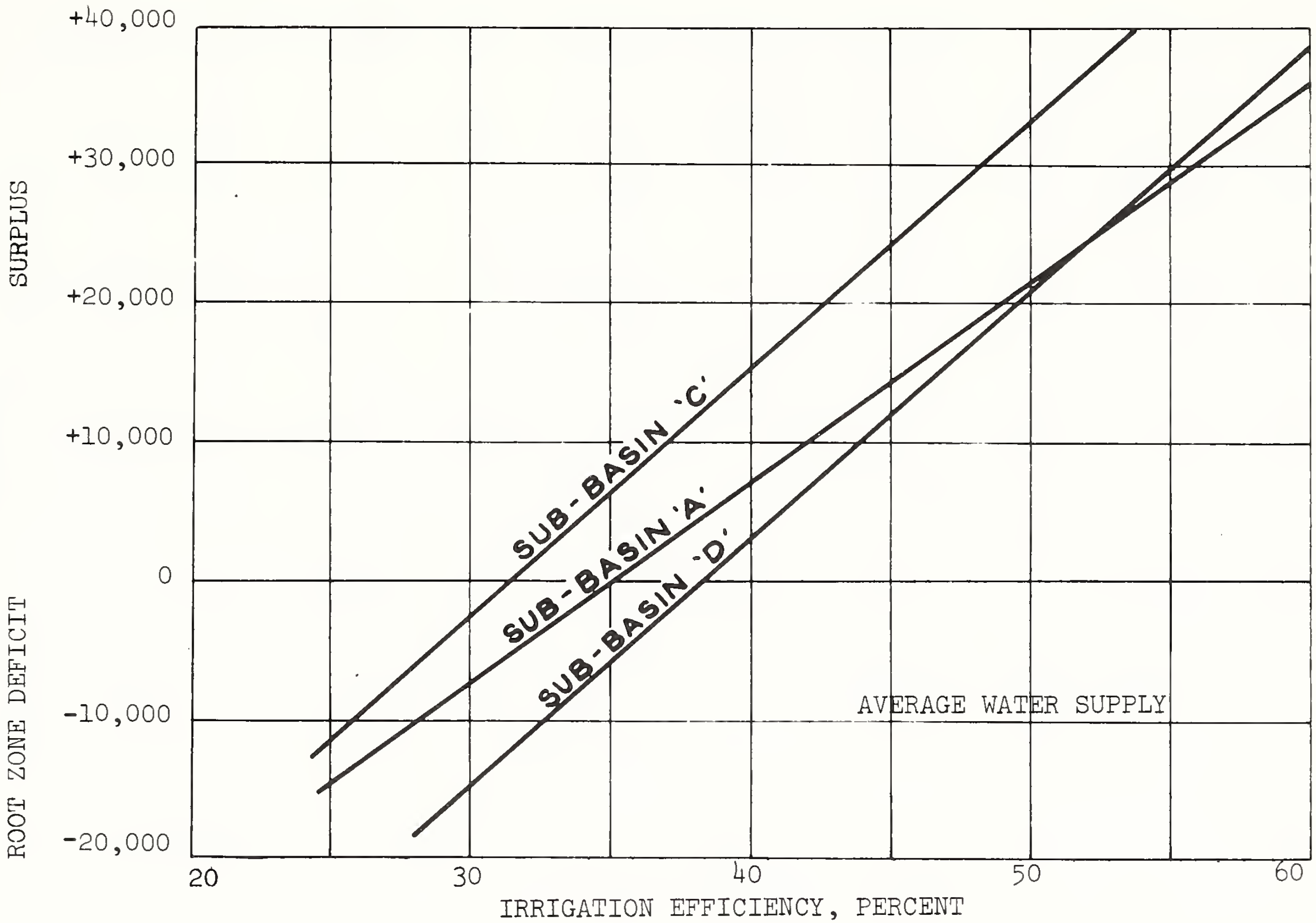


FIGURE 5. Sub-basins A, C, & D Analog Model. Root Zone deficit or surplus vs. efficiency

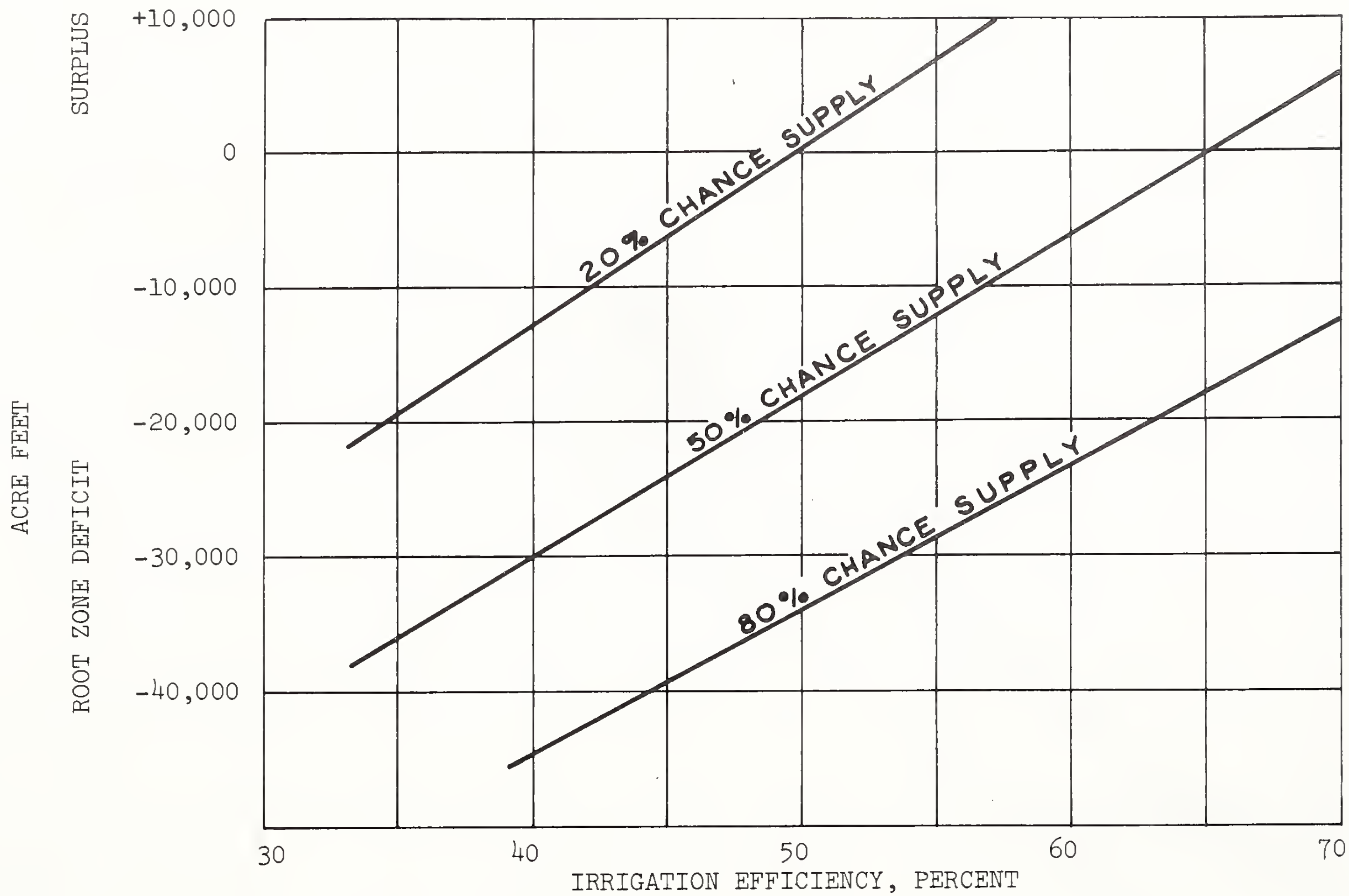


FIGURE 6. Sub-basin B analog model. Root zone deficit vs. efficiency

These curves can also be used to determine the amount of over-irrigation that would result at any specified efficiency. For example, if the irrigation efficiency in Sub-basin D were raised to 45 percent and the total diversions remained the same, 12,000 acre-feet more water than is required for the presently irrigated land would be applied to the crops. This overirrigation figure can be used to determine either the approximate amount by which the diversions could be reduced, or the increase in cropland acreage which could be supplied a full irrigation. That would be a 26,700 acre-foot diversion reduction or 7,200-acre cropland increase in this example.

Information was also obtained from the analog model concerning the effect that increasing the irrigation efficiency would have on the return flow to the river and on the wetland consumptive use. These effects were also determined for various other types of projects such as extensive groundwater development. Table 1 summarizes these effects for a case where the irrigation efficiency is increased just enough to overcome deficits with an average water supply in each sub-basin.

TABLE 1 - Downstream effects of increasing irrigation efficiency Sub-basins A, C, and D analog model.

Sub-basin	Present deficit, ac-ft	Required increase in efficiency to overcome deficit, percent	Resulting reduced wetland consumptive use, ac-ft	Resulting reduced return flow to river ac-ft
A	10,000	7	5,500	4,500
D	6,700	4	4,000	2,700
C	3,400	2	1,400	2,000
Totals	20,100		10,900	9,200

After the cropland deficit has been eliminated, any further increase in irrigation efficiency has little effect on the return flow to the river if no additional land is brought into cultivation. This is because there would be no further increase in water consumed.

Figure 7 shows one further example of information obtained. This bar graph shows the changes on inflow into Sevier Bridge Reservoir that would result from a project which

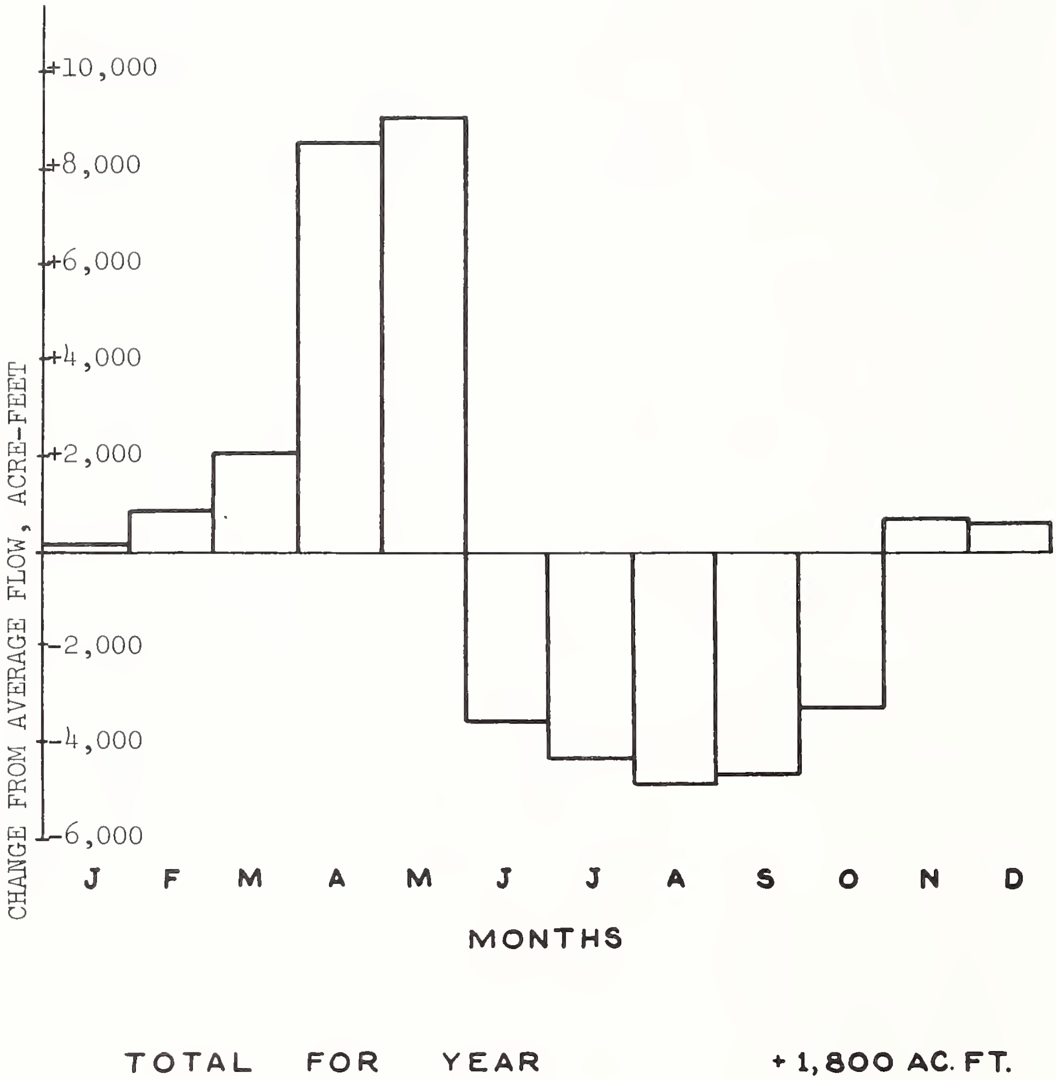


FIGURE 7. Change in river flow. Resulting from increasing irrigation efficiency and reducing diversions.

would increase the irrigation efficiency in Sub-basins A, C, and D to the 40 percent, and then reduce the diversions until there was neither overirrigation nor water shortages on the cropland. In this case, it would be possible to eliminate the cropland deficits and have a slight (1,800 acre-feet) increase in annual river flow. This increase in flow is made possible because of a resulting reduction in water use by phreatophytes which previously used the water lost through inefficient irrigation practices.

In addition to the use by the Soil Conservation Service in the Sevier River Basin (8), the analog computer watershed model has been used at Utah Water Research Laboratory for variety of hydrologic studies (2,3,5,6). Results fo these studies may be obtained from the respective reports.

SUMMARY AND CONCLUSIONS

The use of an analog computer type watershed model has provided a convenient and efficient means of gaining greater insight into the inter-relation of surface water flow, groundwater flow, and return flow from irrigation deversions in the Sevier River Basin. Because of the information gained it was possible to estimate impacts on river flow and groundwater reservoirs caused by possible future conservation projects.

Local sponsors of improvement proposals using the relations developed can plan a series of coordinated projects where the adverse downstream effects of a project in one area can be overcome by another project in the downstream area. This type of coordinated planning assisted by watershed models can maximize economic benefits and minimize costly and often irreversable mistakes in managing the limited water resources of the basin.

The analog computer watershed model described in this paper is ideally suited to the study of complex hydrologic systems which may be described at least in part by time dependent differential equations. The use of this type of watershed model in other watershed studies can produce similar benefits to those obtained in the Sevier River basin. This would especially be true where it is desirable or necessary to investigate the interaction of surface water flows and groundwater reservoirs.

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SIMULATION MODELS IN THE NORTHEAST BRANCH

Prepared for The Joint ARS-SWC Workshop on Watershed Modeling by Edwin T. Engman[1]

Introduction

Simulation models are currently being used at several locations within the Northeast Branch. This paper will present the salient features of each project with emphasis on the background, objectives, type of model, and progress to date. I have attempted to reference all work properly so that one can explore each subject in more detail if he so desires.

Computer Modeling of Nutrient Movement in Soils

Maurice H. Frere[2] and Cornelis T. deWit[3]

Agriculturalists recognize the fact that plant roots are not in intimate physical contact with all the surrounding soil particles. The uptake of water and nutrients at the root surface creates conditions that cause nutrients located between roots to move toward the roots. The uptake of nutrients can exceed, equal, or be less than the amount carried in by water as it flows towards the root. Whenever the uptake of a nutrient is not equal to the amount carried in by mass flow, the concentration at the root surface changes and the nutrient moves by diffusion in response to this gradient. Some nutrients are adsorbed by the soil, with the result that the soil buffers the concentration changes. A further complication is the presence and interaction of other nutrients. The uptake, mobility, concentration, and adsorption characteristics can be different for each nutrient ion, yet the system remains electrically neutral in all parts.

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There is little doubt that these fundamental chemical and physical relations exist, yet their practical and relative importance needs to be established. Pot experiments have provided some information on mass movement and diffusion for different plants. However, at best, these processes and interactions are detectable only within several millimeters of the root surface. Because of this physical limitation on the experiment, the authors have attempted to simulate the uptake, mass flow, diffusion, and electrical interaction between a number of ions simultaneously.

The simplest simulation approach is analogous to bookkeeping, where each account loses and gains value in a variety of ways and rates. The root is visualized as an account that takes some ions from the adjoining soil and releases others to maintain electrical neutrality. The soil is divided into a number of accounts that correspond to concentric cylinders of increasing size surrounding the root, Figure 1. The ions move into each cylinder from the adjacent cylinders according to the flow of water and the differences in the concentrations that exist. After each small increment of time in which movement occurs, each account is updated, not only for the total amount of each ion that is present but also for the distribution of each between the adsorbed and solution phases.

Before the simulation starts, the relevant parameters must be entered into the computer, accounts must be established and given an initial value, and preliminary calculations made. This model divides the soil into different sized cylinders with the width of the first cylinder defined as the length of the root hairs. Because the concentration gradients always decrease with distance from the root, the size of the cylinders can be increased by a given percent without loss of accuracy to cover a large distance with fewer accounts. The initial content of each cylinder is usually, but not necessarily, uniform.

The simulation calculations, whose arrangement is shown in Figure 2, start with the first or innermost cylinder. The amount of each ion taken up or liberated by the root is calculated. Then the amount moved between the first and second cylinders is calculated. Finally the new concentration is calculated by adding to the old concentration the net change in the amount of the ion divided by the volume of the cylinder. For the second cylinder the amount moved to or from the first cylinder has already been calculated and only the amount to or from the third cylinder needs to be calculated.

In this way the calculations progress from cylinder to cylinder out to the last one. Since the rates are calculated and then the concentrations are changed, this method of integration corresponds to the simple point-slope method of Euler. In this model it is assumed that the concentration beyond the last cylinder never changes. An alternate assumption would be that the last cylinder is midway between two roots and moves equal amounts of ions in both directions.

After a new concentration is calculated for each cylinder, the new equilibrium between the adsorbed and solution phases is calculated for each cylinder. Before returning to the uptake calculations for a new increment of time, the accumulated time increments are checked to determine if it is time to print the current status of the system and/or time to stop the simulation.

Calcium, potassium, and nitrate ions were chosen for simulation because of their importance nutritionally. Their uptake is balanced by the output of a bicarbonate ion. Figure 3 illustrates how the concentrations of these nutrients in the root hair zone changes with time. The anion concentrations were always similar to what is illustrated in Figure 3. A change in cation concentration had no effect on anion movement under the conditions studied.

It was found that the presence of a relatively small amount of exchange material reduced the depletion of potassium and increased the depletion of calcium in solution. The root removes potassium and calcium from the solution but calcium also is taken from the solution to the exchanger to maintain equilibrium.

It appears that increasing the amounts of exchange capacity has relatively little effect. Also a 2.5 fold change in the exchange constant at a moderate exchange capacity has a relatively small effect. Therefore, the presence of an exchange capacity is important but highly precise values are not needed for an adequate description.

The need of mass flow to provide an adequate supply of nitrate at the plant root is indicated in Figure 3 by the small nitrate concentration after 1 hour. To determine what effects different mass flow rates might have on the system, repeated simulations with increasing mass flow rates were conducted.

The previous discussion indicates how numerical simulation techniques can be used for analysis of what had been a physically unattainable system. Without these techniques, it would have been impossible to examine the area immediately adjacent to the root because the size scale is too small.

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MODELING WATER AND NITROGEN BEHAVIOR IN THE SOIL PLANT SYSTEM

Maurice H. Frere[4] and Marvin E. Jensen[5]

Generally water and nitrogen are the two most limiting factors of crop growth that can be controlled. This is particularly true of irrigated crops in the semi-arid west where all the water requirement during the growing season is furnished by irrigation. In these times of increasing demand for water for other uses and concern for the nitrate pollution of our environment, it becomes essential that the uses of these growth factors be optimized. Since water can leach nitrate out of the root zone, it is obvious that the system can not be optimized properly by considering the variables separately. In order to evaluate the numerous combinations of time and amount of both water and fertilizer application, a mathematical model is essential.

The main objective of the current model is to describe the behavior of water and nitrogen in the root zone from crop emergence to harvest. Plant growth is very important in this model but because the objectives are centered in the soil it is not modeled in detail. The detail is only sufficient to provide an expanding root zone, a sink for nitrogen and water, and a means of measuring the response to environmental conditions. The flow diagram of the model is given in Figure 4.

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There are three distinct phases to this model - plant growth, nitrogen, and the water cycle. The water phase has been prepared by Jensen and will not be discussed here.

Plant Growth:

The growth of the plant (as measured by accumulated dry matter) can be approximated by a normal curve. Therefore for the model, growth is described by the three-parameter equation:

$$R_t = (\text{Max}) \exp \left(- (t-to)^2/B^2 \right)$$

R_t = ideal growth rate (K_g/ha/day) on day t
Max = maximum rate obtained on day to
B = one-half the width of the growth curve at .37 Max

Growth in the field seldom achieves the ideal rate because one or more growth factors (water, temperature, light, or nitrogen) may be lacking. In this case, the Law of the Minimum is used and the value of the most limiting factor is considered to be operating.

The response of plant growth to increasing amounts of these factors is a curve with a maximum at the optimum amount. For water the best relation available is the ratio of the actual evapotranspiration to potential evapotranspiration. This ratio is an empirical function of available soil water for each crop and soil. For the other three factors, the normal curve is used as the response surface.

The rate that the root zone depth increases is considered to ideally follow the equation of a normal curve, with a maximum rate of three cm/day at 80 days and a B value of 30 days. The ideal rate is reduced by the same factor that reduces growth. The experimental areas being used to develop the model have a special restriction of a hard pan a 46 cm which prevents deeper root growth; this limitation is incorporated into the model.

Nitrogen is considered to exist in three forms in the soil - nitrate, ammonium, and soil N. The pools of ni rate and ammonium are divided into two parts - the current root zone and the remaining potential root zone. This separation is necessary to account for root growth into new areas of different nitrogen composition.

The plant uptake rate of nitrate and ammonium from the current root zone is calculated as being proportional to the growth rate and related to the concentration of nitrate and ammonium in the root zone. The logic of the uptake rate being proportional to the growth rate derived from the fact that a slowly growing plant takes up considerably less nitrogen than a vigorously growing plant.

Ammonium is converted to nitrate by way of nitrate. Under most conditions, the nitrate concentration is never high, permitting the rate of conversion to be described as proportional to the ammonium concentration. Additional amonia is converted from organic nitrogen by mineralization. Its rate of conversion is controlled by soil moisture and soil temperature. Again, a normal curve is used as the response curve for this process.

There are two different water-fertilizer management practices that must be considered. The first is rain, sprinkler irrigation, or flood irrigation with broadcast application of the fertilizer. In this case, the nitrate and ammonium pools in the root zone are increased by the amount of applied fertilizer on the day of fertilization or after a short time lag. The other management practice is furrow irrigation with the fertilizer banded between the furrow and the plants. Assuming reasonable accuracy in placement and irrigation, the simplest approach is to consider that the first irrigation after fertilization does not leach any of the fertilizer nitrate out but causes it to be uniformly distributed in the entire root zone after drainage has stopped. Subsequent irrigations will behave as in the previous case. It is possible to model this irrigation-placement interaction in much more detail to account for poor placement and/or excessive irrigations.

Model parameters that have not been evaluated independently will be adjusted so that the model can simulate the measured behavior of the 1969 field experiment. Using the same internal parameters with different initial conditions and climate, the model will be tested against data from field experiments of previous years. The model will be considered acceptable as a first approximation when the comparison of results from the model simulation compares favorably with the experimental data. Some 21 parameters and initial conditions have been selected for a sensitivity analysis of the effect of systematic variation on the model behavior. The results of this study should indicate parameters and relations that need more evaluation.

In this example, numerical simulation is being used to examine a very complicated system whose physical scale is too large to analyze in laboratory fashion. Also in this case, various optimum conditions have been simulated numerically with considerable time and dollar saving over what it would have taken to accomplish in the laboratory.

Reference:

Frere, M. H., and Jensen, M. E., "Modeling Water and Nitrogen Behavior in the Soil Plant System", prepared for the Computer Simulation Conference in Denver, June 1970.

SLEEPERS RIVER RESEARCH WATERSHED: DANVILLE, VERMONT

Sam H. Kunkle[6] and Eric Anderson[7]

The Office of Hydrology of the U.S. Weather Bureau, ESSA, and the Sleepers River Research Watershed of the Agricultural Research Service, USDA, are conducting a cooperative research project in the physics of snowmelt.[8] Present modeling aspects of this research involve the Office of Hydrology's attempt to put their river forecasting techniques on a more objective basis.

In general, the modeling approach of the Office of Hydrology has been to develop a physical model of the snow accumulation and ablation processes. When needed, the Stanford Watershed Model has been used to determine the effect of the soil and the stream channel on snowpack runoff, though other models of the land and channel phases of the hydrologic cycle could be used. The first attempt to utilize a snow subroutine in the Stanford Model was presented by Anderson and Crawford.[9] Two approaches were tested: An empirical approach using air temperature as the only index to heat exchange, and a simple energy balance method to determine the heat gained by the snowpack during periods of melt. Although the present Stanford Model (1966)[10] uses a combination, of these two approaches, the subroutine is still empirical to a great extent.

Anderson[11] develops equations for a continuous energy balance of a snowpack and discusses the assumptions involved. These

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equations form the backbone of the present Office of Hydrology model of the snow accumulation and ablation processes (Figure 5). This model is currently being tested on data from several watersheds in various geographical regions of the United States. However, to test the validity of the basic equations under all conditions and the limitations imposed by the assumptions, accurate point data on all the variables involved is necessary. The only data judged of reasonable quality was that from the Lower Meadow Lysimeter at the Central Sierra Snow Laboratory collected during the 1954 melt period. Application of the equations to these data gave results well within the accuarcy of the data.

This lack of quality data is the basis for the present data collection and instrumentation at the Sleepers River Research Watershed. For this, an extensively instrumented location has been constructed to measure all components of the energy budget and snowpack. Figure 6 and Table 1 illustrate the physical arrangement of the instruments and a listing of the instrumentation. The instrumentation phase was completed during 1967. Many problems arose during the winter of 1967-68 concerning the operation of the sensors and recorders. Most of these were related to mechanical failures. Complete data of questionable quality were collected near the end of that snow season. During the 1968-69 snow season, a complete set of data was collected from December through April. Preliminary analysis indicates the data are of reasonable quality. Final analysis was delayed pending further calibration checks of solar radiometers. These checks were recently concluded. Based on the preliminary analysis, some changes were made in the instrumentation for the 1969-70 winter. Data collection for this snow season began November 1, 1969.

Reference:

8. Johnson, M. L. and Anderson E., The Cooperative Snow Hydrology Project - ESSA Weather Bureau and ARS Sleepers River Watershed, Proceedings Eastern Snow Conference, 1968.

9. Anderson, E. A. and Crawford, N. H., "The Synthesis of Continuous Snow Melt Runoff Hydrographs on a Digital Computer," Tech. Report 36, Department of Civil Engineering, Stanford University, 1964.

10. Crawford, N. H. and Linsley, R. K., "Digital Simulation in Hydrology: Stanford Watershed Model IV," Tech. Report 39 Department of Civil Engineering, Stanford University, 1966.

11. Anderson, E. A., "Development and Testing of Snow Pack Energy Balance Equations," Water Resources Research Vol. 4, No. 1, pp 19-37, February 1968.

12. "Snow Investigations, Lysimeter Studies of Snow Melt," Research Note #25, North Pacific Division, Corps of Engineers, Portland Oregan, 1955.

WATERSHED MODELS AS A RESEARCH TOOL

Edwin T. Engman[13]

The research program of the Northeast Watershed Research Center is based on the philosophy that future water resource projects will be planned and coordinated within a river basin concept. The role that an agricultural watershed plays in this concept is extremely important in that a river basin is made up of a series of agricultural watersheds. If we were able to define the hydrology and water quality output of these agricultural watersheds today, we could synthesize the total performance of an entire river basin with a good degree of success. Our research program is designed to describe the entire water resources of complex, multiple use rural watersheds.

In recent years important advances in instrumentation, data handling, and application of digital computers have improved our understanding of the hydrologic cycle. Scientists are now able to treat the hydrologic cycle as a dynamic, sequential system having input/output and some form of interconnections within the system. This system can be modeled and interpreted by systems analysis techniques that have been developed in other fields.

At the Northeast Watershed Research Center, it is models of this type which serve as a tool for evaluating our present research, identifying areas that need specific attention, and allowing us to analyze the complexities and interactions in an entire watershed. Hydrology and water quality studies are organized within an interdisciplinary group so that each experiment is part of the solution of an overall model. Each scientist has at least a portion of his research coordinated by a process-oriented watershed model.

[13] Director, Northeast Watershed Research Center, University Park, Pennsylvania

One of the problems that was encountered in implementing this approach was narrowing down an objective that all could contribute to. We tried unsucessfully to agree on the form and objectives of such a model as it might be applied to the Mahantango Creek Research Watershed. However, we have been able to agree on an approach and techniques to use in modeling another watershed - Perkiomen Creek near Philadelphia. Here we had an opportunity to examine a watershed where an objective had been defined: *Is agricultural pollution causing a serious aquatic weed problem in Perkiomen Creek?* Immediately individual biases were overruled and we were able to come up with a research plan.

The objective we defined was extremely simple: To determine the average monthly Kedjahl Nitrogen concentration in Perkiomen Creek. As this objective is accomplished, we will define a new objective more complex than the first and directed by what has been learned in the first simulation effort. To carry out this objective the staff has been divided into two groups, one to build the nitrogen model and one to synthesize the watershed's hydrology. In assembly, the water quality model will be superimposed over the hydrology model.

RELIABILITY AND ACCURACY OF ET ESTIMATES

Leslie H. Parmele[14]

The accuracy required of an ET estimate depends in a large measure upon the use to be made of the data. Are hourly, daily data needed in detail or will simple monthly or seasonal estimates be sufficient? One also has to consider that the accuracy of a watershed model is no better than the accuarcy of the least reliable component used to arrive at the answer. A determination of ET to three significant figures may be of little value if the estimate of streamflow is only significant to two figures because of errors in the other input data. When dealing with the endless variability of ET over a large watershed, a simple climatological index may give sufficient accuracy, especially in light of the accuracy obtainable for measured hydrologic variables such as precipitation, streamflow, and soil water movement. However for detailed research investigations on small experimental catchment areas, estimates of ET must reflect

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the time variation that will be encountered, and accurate, short term estimates may be required.

One of the major problems in the development and application of complete hydrologic models in that little is known about the sensitivity and accuracy required for the estimate of the evapotranspiration component of a particular model. There is a general lack of knowledge about evapotranspiration and its influence on streamflow and other components of the hydrologic cycle; and the resulting interactions. This difficiency has resulted in part from emphasis on detailed study of the evapotranspiration process, while generally forgetting that ET is a part of a complex system where each component or subsystem in some measure influences the response of other components.

Accuarcy of evapotranspiration data was analyzed with respect to simulated watershed streamflow with the Stanford Model. In the analysis synthetic output from the Stanford Watershed Model using original data and optimized control parameters was considered as the true watershed response. Variations in the form of random fluctuations and fixed biases were introduced into the evapotranspiration component of the model. By leaving the other inputs and parameters unchanged, such an analysis shows in some degree the effect of evapotranspiration on streamflow under the regulation of the other components in the model.

There were six computational runs made with 1957 water year data:

1. No change in the input data or coefficients;

2. ET input data biased by a constant +10 percent;

3. ET input data biased by a constant -20 percent;

4. A random error of ±50 percent daily ET input values added onto daily input;

5. ET input data biased by +10 percent plus a random error of ± 50 percent daily ET; and,

6. ET input data biased by -20 percent plus a random error of ± 50 percent daily ET.

A summary of the annual values of the hydrologic components as computed by the various runs is given in Table 2. It can be seen that placing a positive bias of 10 percent on the evapotranspiration input data decreased total streamflow by 1.3 inches, and a negative bias of 20 percent resulted in an increase in streamflow of 3.0 inches. The direction of change could be expected; and since the major component of streamflow comes from groundwater flow, it is not surprising that this component would reflect the effects of the biased input. Introducing a random error of ± 50 percent of the daily input value for evapotranspiration did not significantly change the flow from that computed with a simple fixed bias. For the data used, the influence of the random error component is overshadowed by the bias in the evapotranspiration estimate as shown by the closeness of the synthetic streamflow values with random error input.

Figure 7 shows the comparison of the bias between synthetic and observed flows for four specific flow intervals. If considered by itself, the effects of a constant bias in the evapotranspiration input is significant and can reverse the error between synthetic and observed flows.

Figure 8 shows the annual hydrograph of streamflow by days and compares the synthetic output having no change in its input with that output resulting from a constant -20 percent bias in the evapotranspiration input data. It can be seen that in the beginning of the water year the effect of the biased input is almost unnoticeable. But as the year advances, the biased error has an accumulative effect apparently on the soil moisture storage. The soil moisture is not depleted at the "accepted" rate, and thus the soil is "wetter" than it would be otherwise. This results in a change in the infiltration rate since this variable is dependent upon the available water stored in the upper and lower soil profiles. The overprediction of the hydrograph peaks by as much as 200 cfs is the probable result of this changing of the infiltration rate.

It has been shown that a constant bias of as little as 20 percent in the ET input data has an accumulative effect and results in considerable error in hydrograph peaks and hydrographs recession limits. This effect would tend to increase unless some method of self-learning was used in the model (feedback) or unless a comparison was made between actual and synthetic soil moisture indices. This could be done on a monthly basis for the watershed record simulated in this paper.

While the influence of the random error on estimated streamflow was insignificant for the large watershed studied, it will be necessary to give more consideration to this form of error as watershed size decreases, especially for small experimental watersheds.

Thus, a comprehensive systems analysis into the accuracy and sensitivity of ET estimates required for hydrologic models would seem to be highly desirable before large amounts of manpower and money are spent for future watershed evapotranspiration research. More calculations should be made for various ranges using the two possible types of error (fixed bias and random), and for different watersheds to establish a range for the accuarcy required of the input data.

Table 1. A SUMMARY OF OBSERVATIONS MADE AT THE COOPERATIVE ESSA-ARS SNOW HYDROLOGY RESEARCH STATION

Observation	Values to be Recorded	Frequency
Solar Radiation		
Incoming	Integrated Hourly	Continuous - Year Round
Reflected	Integrated Hourly	Continuous - Snow Season
Total Hemispherical Radiation	Integrated Hourly	Continuous - Year Around
Net Radiation		
Ventilated radiometer	Integrated Hourly	Continuous - Snow Season
Shielded radiometer	Integrated Hourly	Continuous - Snow Season (over X-3 pan during remainder)
Wind	miles traveled/ hour at 1/2 m and 2 m above ground or snow	Continuous - Year Around
	Miles traveled/day at 2 m above ground	Daily - Snow Season (at 1 m above ground with X-3 pan during remainder)
Air Temperature	Instantaneous at end of each hour at 1/2 m and 2 m above ground or snow	Continuous - Year Around
	Max-min	Daily - Year Around

TABLE 1. CONTINUED

Observation	Values to be Recorded	Frequency
Dew-Point Temperature	Instantaneous at end of each hour at 1/2 m and 2 m above ground or snow	Continuous - Year Around
	Instananeous	Daily - Year Around
Snow Surface Temperature	Instantaneous at end of each hour from thermo-couple 1 cm below surface	Continuous - Snow Season
	Instantaneous at end of each hour from infra-red thermometer	Intermittent - Snow Season
Snow pack Temperature	Max-min at 1, 2, 3, and 4 feet above ground as long as sensor is covered by 2 inches of snow	Daily - Snow Season
Precipitation	Amount/Hour	Continuous - Year Around
	Amount/Day	Daily - Year Around
Snow Pack Water Equivalent	Instantaneous at end of hour from 12-foot diameter snow pillows	Intermittent - Snow Season (At least once per day)

TABLE 1. CONTINUED

Observation	Values to be Recorded	Frequency
Snow Pack Water Equivalent (Continued)	Instantaneous from 2-foot diameter pressure pans	Daily - Snow Season
	Instantaneous from gamma density gages	Intermittent - Snow Season (at least once per day)
Soil Temperature	Max-min at inter-face and several levels below ground	Daily - Snow Season
Snow Surface Properties	Density - CRREL tube; crystal size and classification	Daily - Snow Season
Snow Pack Properties	Density, etc. by CRREL kit from pit	Weekly - Snow Season
Snow Evaporation and Condensation	Amount from Freezer Pans set in snow	Intermittent - Snow Season
Areal Snow Cover	Visual observations on segments of watersheds W-3 and W-8	Weekly - Snow Season

TABLE 1. CONTINUED

Observation	Values to be Recorded	Frequency
Areal Albedo	Point Measurements from albedometer on segments of watersheds W-3 and W-8	Weekly - Snow Season

TABLE 2

Annual Values of Hydrolic Components for Water Year 1957

All Values in Inches

% Bias in ET	Precip.	Syn. RO	Actual RO	IM RO	OV RO	Inter. Flow	G.W. Flow	Incep.	Actual ET	Avail. ET
+10	83.6	45.4	46.7	3.8	1.4	3.3	37.2	13.7	34.2	38.0
0	83.6	46.7	46.7	3.8	1.5	3.6	38.2	13.3	31.6	34.5
-20	83.6	49.6	46.7	3.8	1.7	4.3	40.0	12.4	25.9	27.6
+10 (Random Error)	83.6	45.4	46.7	3.8	1.4	3.2	27.3	13.4	34.2	38.3
0 (Random Error)	83.6	47.5	46.7	3.8	1.5	3.7	38.7	13.0	30.4	33.3
-20 (Random Error)	83.6	49.6	46.7	3.9	1.7	4.3	40.0	12.1	25.9	27.7

FIGURE 1

A Diagrom of the Soil Around A Unit Length of Root Divided Into Concentric Cylinders

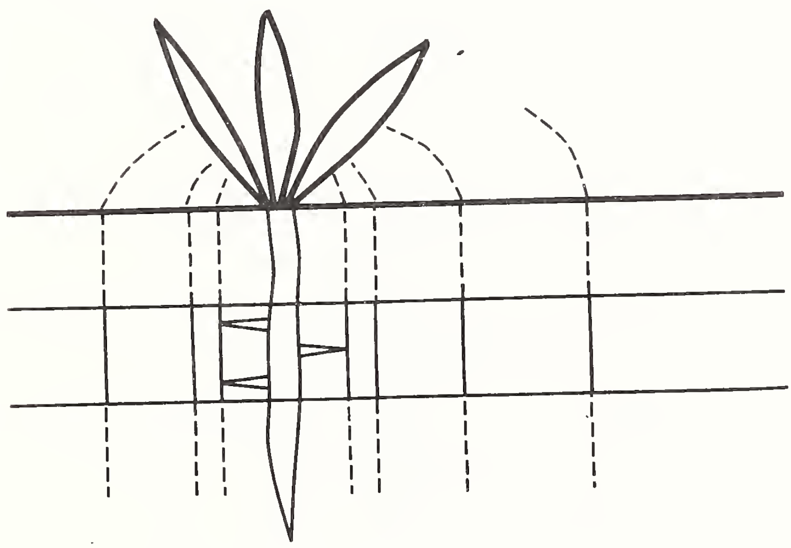


FIGURE 2

The Flow Diagram of the Calculations for Simulating Nutrient Movement through the Soil to Plant Roots

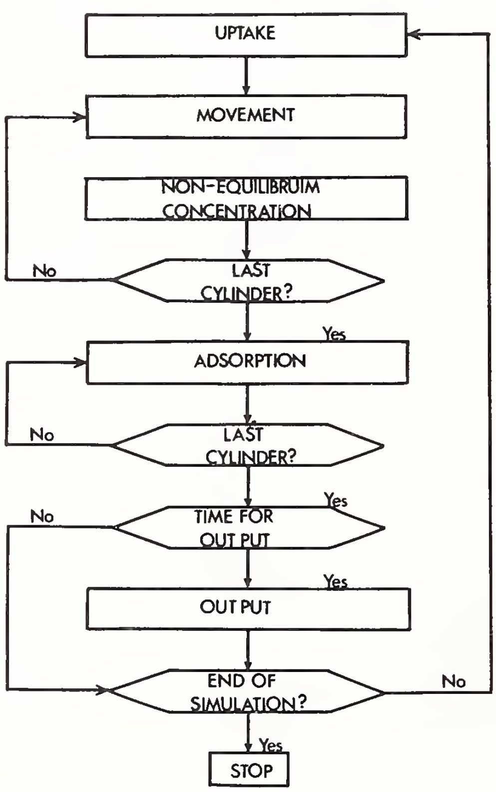
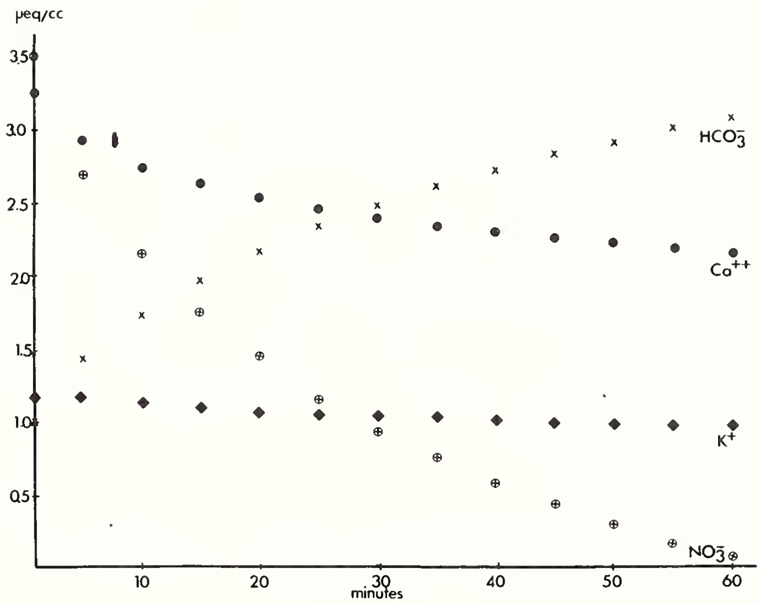


FIGURE 3

The Nutrient Concentration in the Solution of the Root Hair Zone as a Function of Time with Zero Water Flow



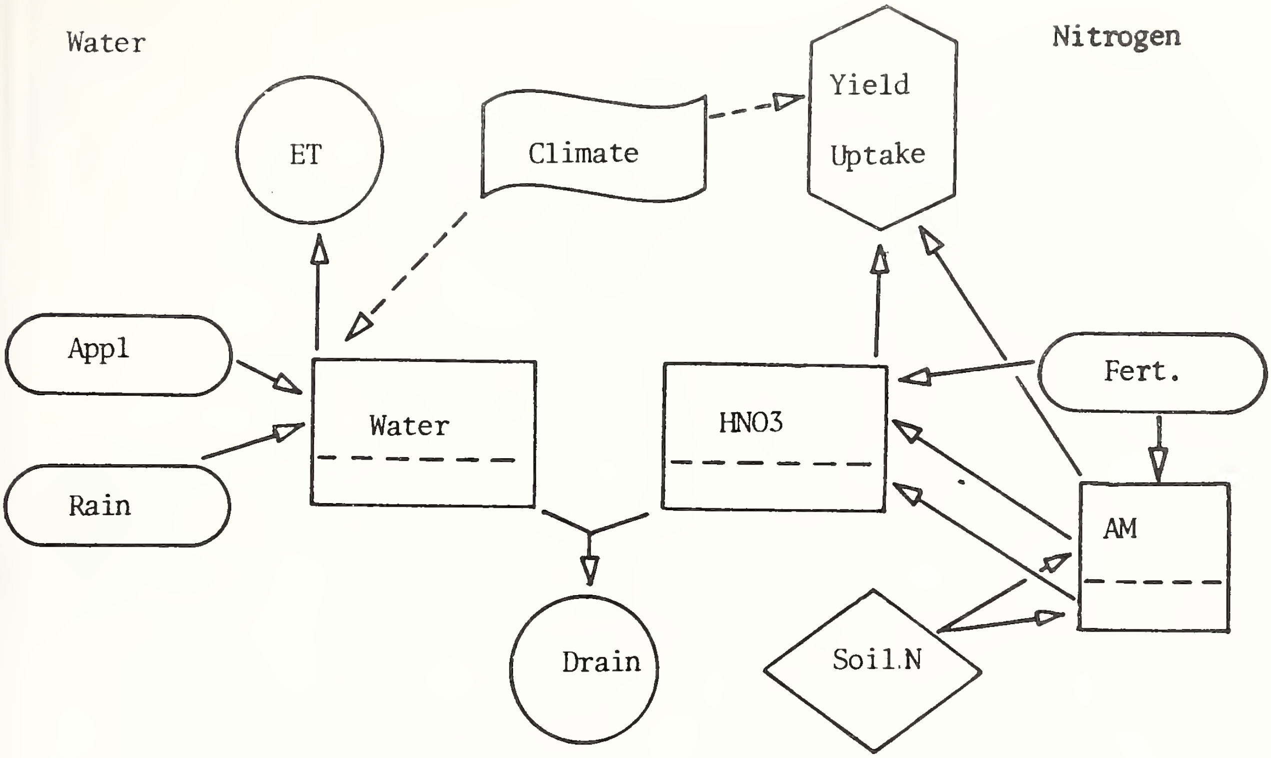


FIGURE 4. Flow Diagram of Nitrogen in the Soil-Plant System - September 1969

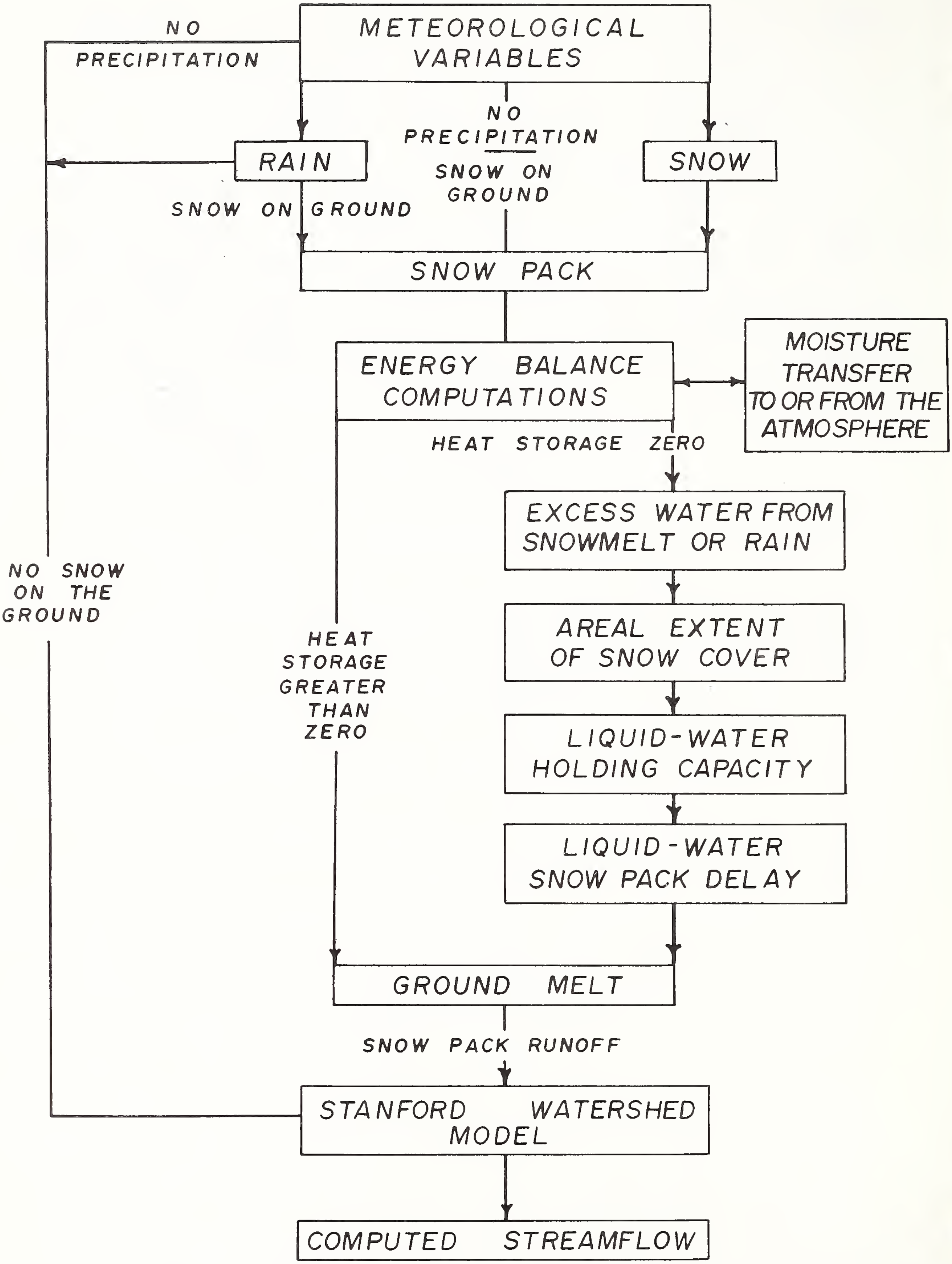


FIGURE 5. Flow Chart of Snow Melt Simulation Model

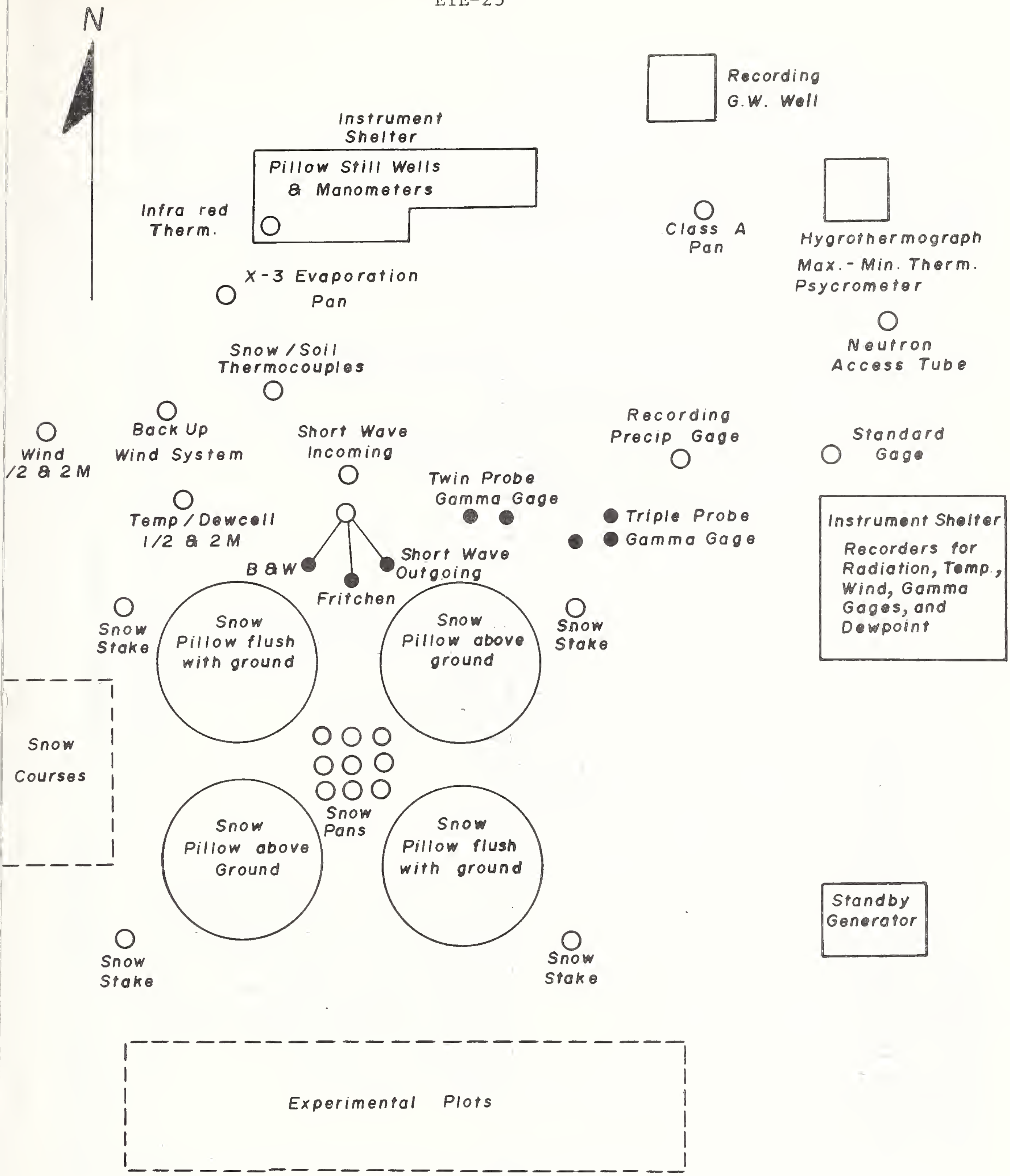


FIGURE 6. Plan View of Instumentation at the ESSA-ARS Cooperative Snow Hydrology Research Station

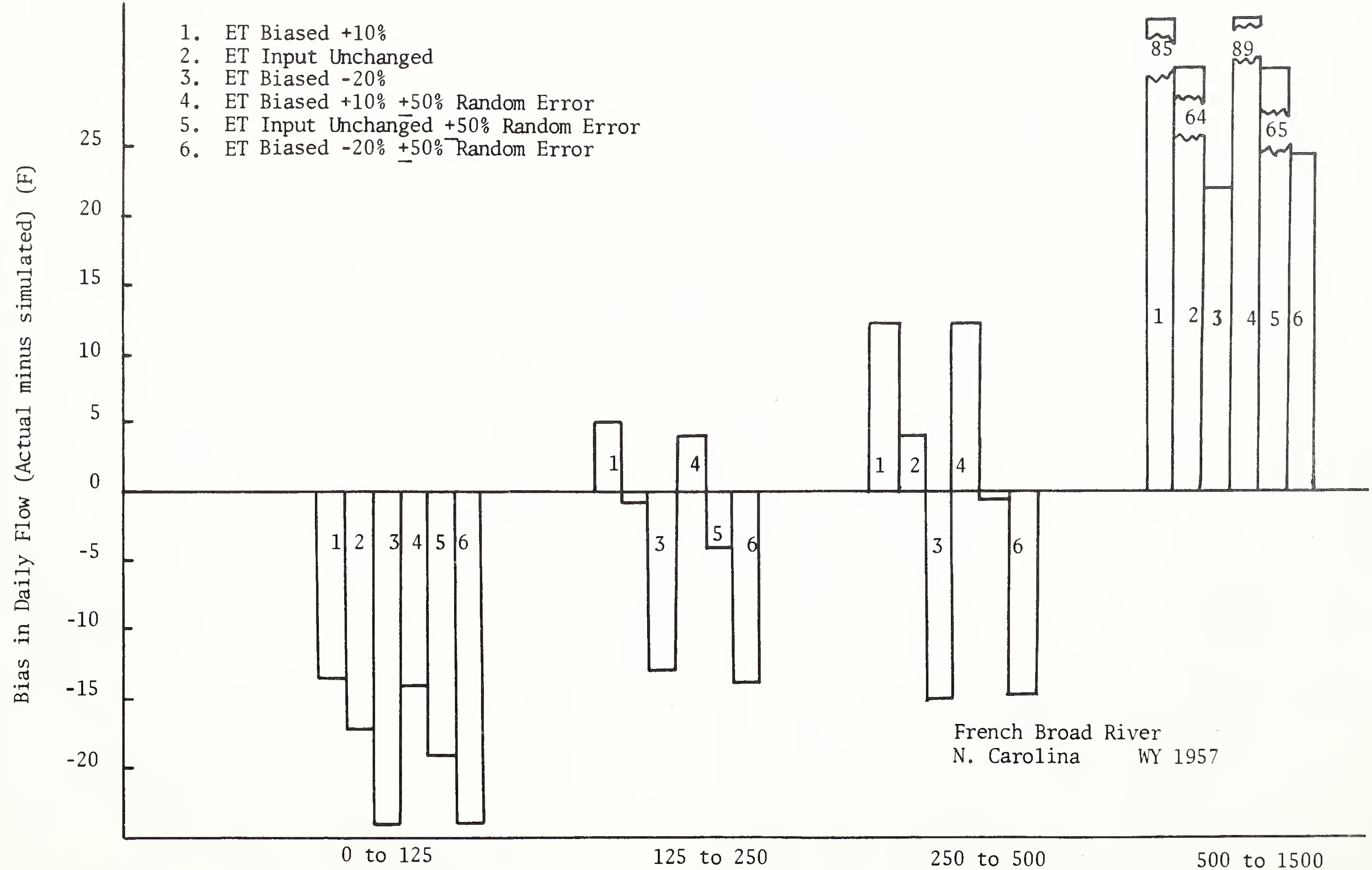


FIGURE 7. Comparison of Bias in Daily Flows for Specific Flows Intervals after ET Input Biased by Set Amount and Subjected to a Random Error

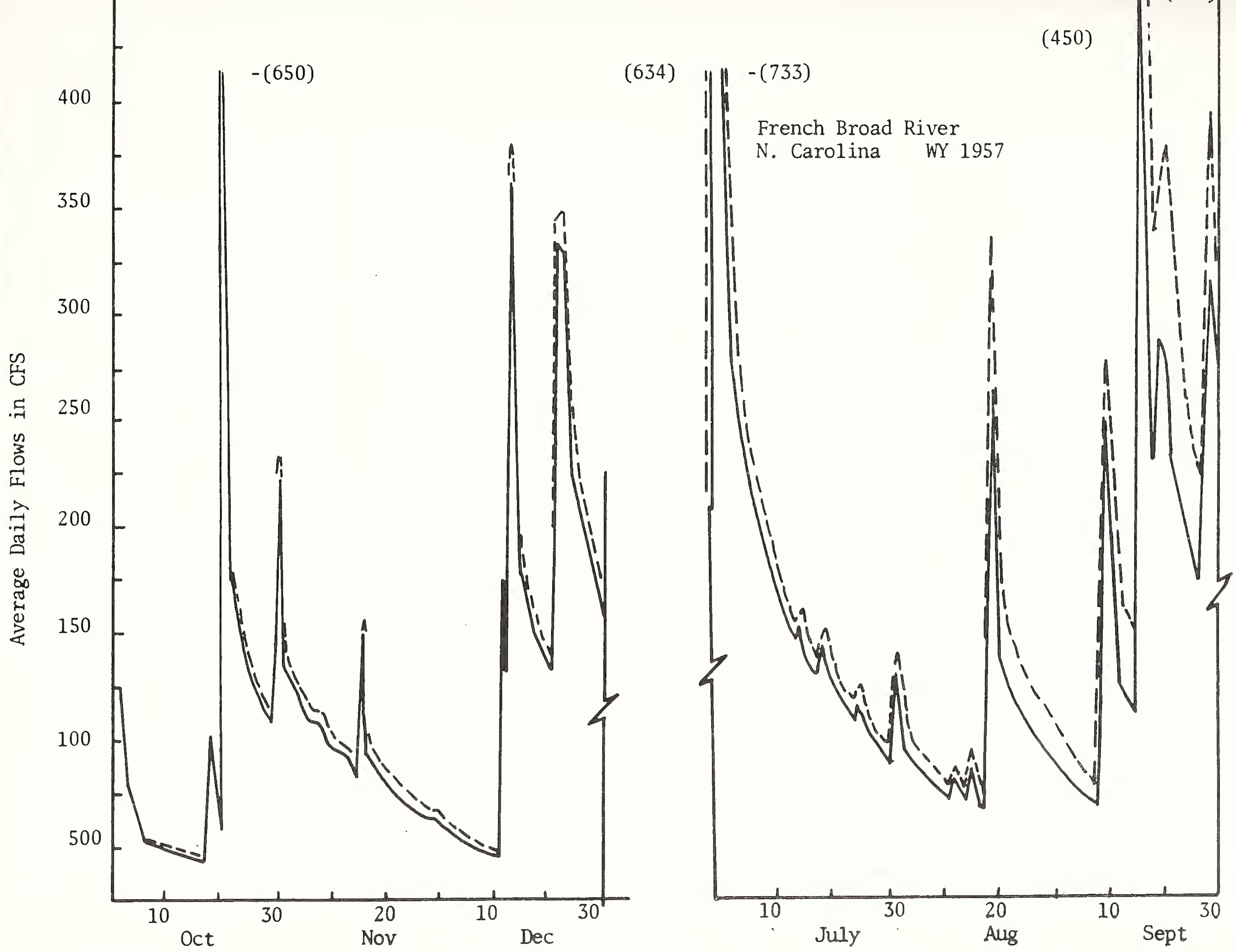


FIGURE 8. Comparison of Synthetic Output with no Change (and Output with ET Biased by -20% (-----)

CORN BELT BRANCH PROGRAM

L. L. HARROLD

The Corn Belt watershed model research program is located at the North Appalachian Experimental Watershed, Coshocton, Ohio; The North Central Watershed Research Center at Columbia, Missouri and Treynor, Iowa; The Watershed Group at Madison, Wisconsin, and the Erosion Group at Lafayette, Indiana. Work at each will be described separately.

Coshocton, Ohio

The Coshocton research station is located in the hills of the dissected Allegheny Plateau. Natural watersheds of a few acres to 4,580 acres (figs. 1, 2) were instrumented in the late 1930's. Important features of each watershed are listed in Table 1. Residual, shallow soils overlay shale or sandstone rock. The original soil classification showed (1936) Muskingum silt loam being well drained and the most prominent. Keene and Coshocton silt loams are slowly permeable and less extensive.

In 1965, a unified national soil characterization program was made of selected soils on ARS experimental watersheds (Holtan et al. 1969). This joint project of ARS and SCS provided information of value in hydrologic studies. Profiles were described and laboratory determinations of moisture retention, bulk density, total porosity, saturated conductivity, and stone count were made.

In 1968, the Soil Conservation Service began an intensive resurvey of the watershed soils within the modern classification system so that hydrologic determination could be interpreted and applied within this system. Soil data are omitted from Table 1 since final correlation has not yet been accomplished.

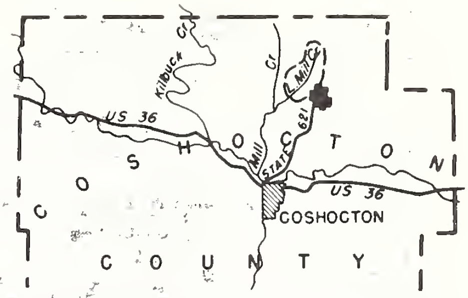
From the start, a major effort has been made to record and process hydrologic data with a high degree of accuracy. Related data to aid in understanding the hydrologic relationships of watershed flow systems were obtained, as follows:

- Climate (air temperature, precipitation, humidity, wind movement)
- Evaporation (Class A pan)
- Evapotranspiration (weighing lysimeters)
- Snow storage

U. S. DEPARTMENT OF AGRICULTURE
AGRICULTURE RESEARCH SERVICE

NORTH APPALACHIAN EXPERIMENTAL WATERSHED

COSHOCTON, OHIO



IN COOPERATION WITH OHIO AGRICULTURAL RESEARCH AND DEVELOPMENT CENTER

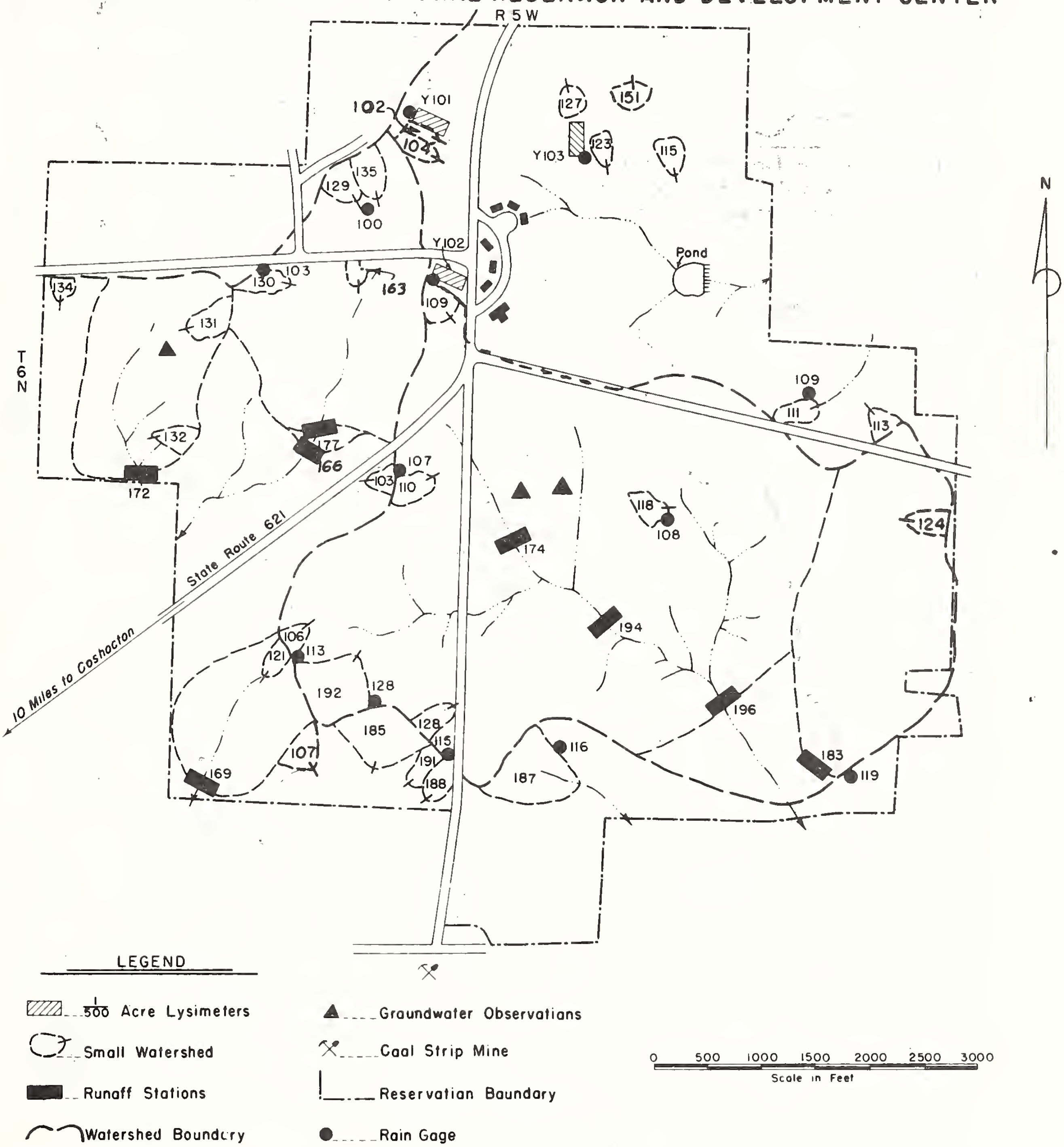


FIGURE 1. -- LOCATION MAP OF SMALL WATERSHEDS

REVISED
SEPT. 6, 1968

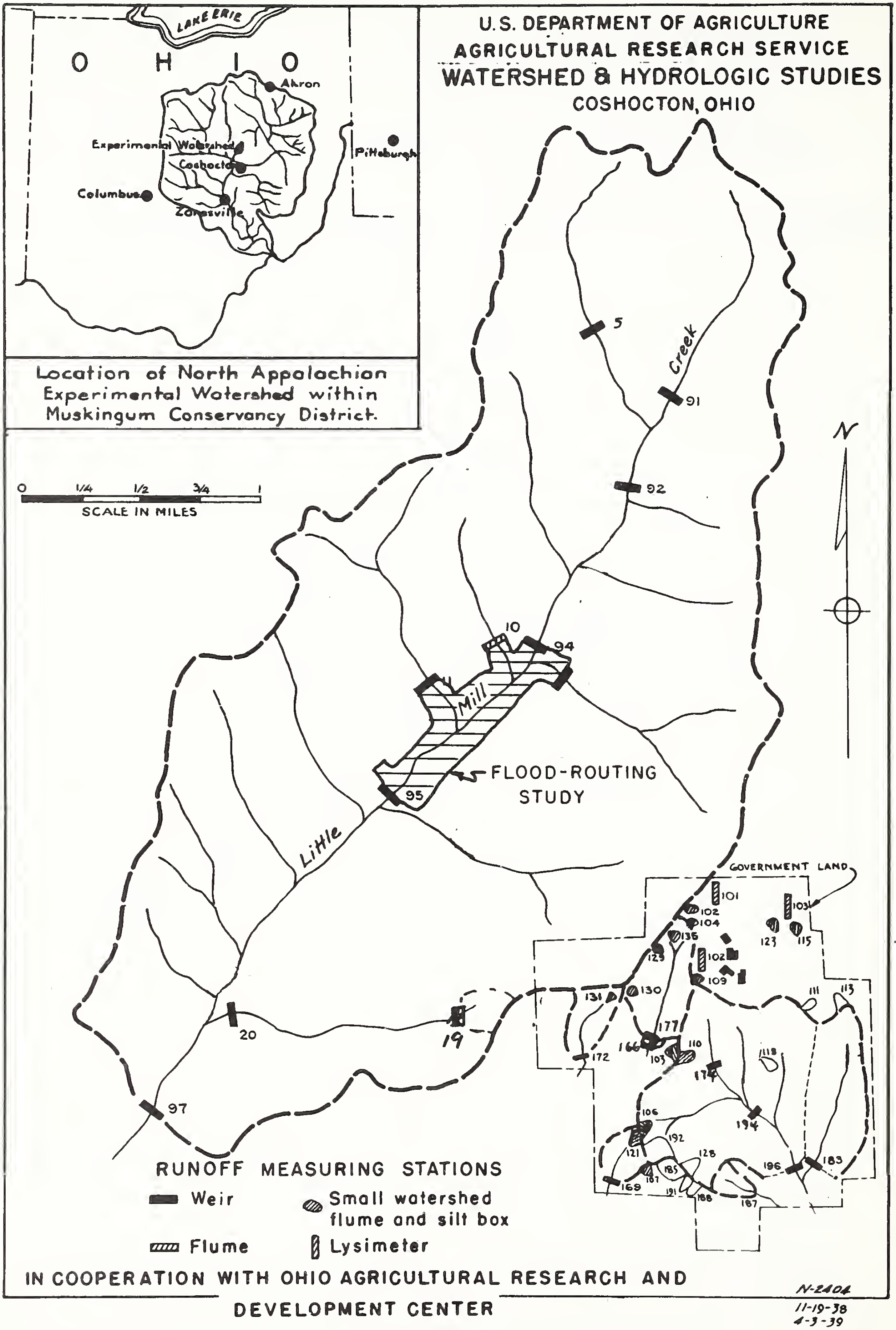


FIGURE 2.—LOCATION OF MIXED-COVER WATERSHEDS

Table 1.-Watershed characteristics, Coshocton, Ohio

Watershed No.	Area	Land use 1/	Period of record	Land slope	Average annual water yield 2/	Maximum peak flow	
						Rate	Date
	Acres			Percent	Inches	In/hr	
102	1.26	Pasture(I)	1937-	18.2	0.58	3.64	6-12-57
103	.65	CWMM(I)	1939-	11.3	2.79	4.72	7-23-40
106	1.56	MMCW(P)	1939-	14.3	2.09	7.63	8-23-44
109	1.69	WMMC(I)	1938-	12.7	1.27	4.34	5-17-41
110	1.27	CWMM(P)	1939-	13.0	2.12	4.44	7-28-50
111	1.18	MCWM(I)	1939-	7.5	2.93	3.83	6-12-57
113	1.45	MCWM(I)	1939-	9.3	2.07	3.77	6-12-57
115	1.61	WMMC(P)	1939-	6.6	2.05	4.12	6-12-57
118	1.96	MCWM(P)	1940-	9.6	2.52	3.11	6-12-57
121	1.42	MMCW(I)	1939-	15.8	1.63	7.82	8-23-44
123	1.37	WMMC(I)	1939-	5.8	2.25	5.97	6-12-57
127	1.65	WMMC(I)	1949-	11.0	3.43	3.12	6-12-57
129	2.71	Pasture (I)	1938-	18.6	.76	2.36	6-12-57
130	1.63	Meadow (I)	1938-	21.7	.88	4.06	6-12-57
131	2.21	Woods	1938-	21.6	.16	1.18	6-12-57
132	.59	Woods	1948-	12.0	1.63	2.00	6-12-57
135	2.69	Pasture (P)	1938-	15.3	.59	2.38	6-12-57
185	7.40	CWMM(S)	1939-	13.6	1.81	3.35	6-16-46
187	7.20	CWMM(S)	1941-	15.5	4.36	2.75	6-12-57
188	2.05	MMCW(I)	1939-	9.2	1.57	3.06	8-23-44
192	7.59	CWMM(P)	1939-	15.8	2.92	4.60	6-16-46
169	29.0	Mixed (I)	1940-	15.7	6.24	2.59	6-12-57
172	43.6	Reforested	1939-	22.8	11.12	2.64	6-12-57
174	53.8	Mixed (P)	1960-	15.0	7.83	1.03	4-25-61
177	75.6	Mixed (I)	1940-	15.7	7.66	3.14	6-12-57
183	74.2	Mixed (P)	1938-	14.8	10.24	2.58	6-16-46
194	187	Mixed (P)	1960-	14.5	12.02	.87	4-25-61
196	303	Mixed (P)	1937-	14.0	14.24	3.72	6-12-57
5	349	Mixed (I)	1940-	15	10.80	1.09	6-28-57
10	122	Mixed (I)	1939-	15	9.16	1.76	6-28-57
11	292	Mixed (I)	1939-	15	13.39	1.05	9-23-45
91	293	Mixed (I)	1939-	15	12.10	1.35	6-28-57
92	920	Mixed (I)	1939-	15	11.73	.78	7-5-69
94	1520	Mixed (I)	1939-	14	11.84	.92	6-28-57
95	2570	Mixed (I)	1939-	14	11.69	.61	6-28-57
97	4580	Mixed (I)	1937-	14	12.12	.72	6-28-57

1/ C = Corn, W = Wheat, M = Meadow in 4-year rotation, I = Improved, P = Poor practices, S = Contour strip cropped.

2/ Streamflow.

Frost depth
Soil water at various depths
Crops and management
Geology and ground water levels
Springflow

Although the hardware, software, and other means for solving complex mathematical models were not available at the start of the Coshocton watershed research program in 1940, the physical plant established then has developed a library of data of utmost value in developing concepts for simulation models, specifying parameters of the flow system, and optimizing their coefficients. The data are also being used to field check the value of working models for predicting runoff for the gaged watershed.

The following sections of the Coshocton report cover:

1. Published material -- a storehouse of knowledge.

2. Current studies.

<u>Storehouse of knowledge fundamental to watershed model development</u>

Years prior to the development of present-day numerical watershed model concepts and the use of high speed electronic digital computers, the Coshocton watershed research program was producing knowledge on:

a. <u>Daily evapotranspiration</u> -- the largest single factor in precipitation disposal -- as related to stage of crop growth, soil water, rooting depth, management, and pan evaporation (Harrold and Dreibelbis, 1958, 1967; Harrold et al. 1959; Mustonen and McGuinness, 1968; and Youker and Edwards, 1969).

b. <u>Soil water extraction patterns</u> related to soil depth and crop (Harrold, 1954 and Dreibelbis and Amerman, 1964).

c. <u>Storm rainfall characterization</u> input to watershed hydrology (Harrold, 1949 and McGuinness, 1962; and Harrold, 1965).

d. <u>Storm water infiltration</u> and runoff related to land use, soil saturation deficit, and storm

rainfall (Glymph and Holtan, 1969).

e. Maximum storm flow peaks related to season, land use, soil water, and watershed size (Harrold, 1949).

f. Storm flow volumes and seasonal and annual water yield (streamflow) related to watershed size (Harrold, 1957, McGuinness et al., 1961; and McGuinness and Harrold, 1962).

g. Surface flow from unit source watersheds (3 ± acres) near hill tops which represented soil and land use of portions of a mixed crop watershed (76 acres) accounted for less than a third of storm streamflow at the downstream site. Ungaged underflow at the upland unit area, returned to the surface in downhill wet spots and contributed to storm flow downstream. Storm underflow was observed to have sharp hydrograph rises and falls and was recognized as an important factor in the watershed flow system. Investigations to discover soil water flow nets and flow potentials as related to soil physical characteristics lead to the formation of the Madison group where these flow phenomena are studied in the light of physical laws of flow through porous media. The Madison report will cover this later. (Amerman, 1965 and Amerman and McGuinness, 1967).

h. Tests of watershed models. The Coshocton watershed hydrologic data have been and are being used by the U.S. Hydrograph Laboratory to develop and check segments of a watershed model based on measurable watershed parameters. This is presented elsewhere in the workshop report.

Cooperative projects were also initiated with The Ohio State University in which the Stanford Watershed Model (Crawford and Linsley, 1966) was obtained in the Fortran IV language and adapted to the University computer system. The model was checked out with Coshocton data (4,580-acre watershed) and applied for 1 year's data. The results appearing in a M.S. Thesis by Everett Balk (1968) were quite promising. Professor Ricca (C.E. Dept.

OSU) was the major advisor. The research was supported in part by a grant from the Office of Water Resources Research, USDI. Dr. E. Paul Taiganides (OSU, Agr. Engr. Dept.) was Project Director. High speed electronic digital computors, not readily available to the Coshocton staff, were used at some length. Balk's thesis reports the following:

> James, at the University of Kentucky, translated the Stanford Model III from ALGOL, used by the Stanford Computing Center, to FORTRAN IV. Some of the modifications to the model included:
>
> 1. Simplified input data.
> 2. Revised procedure for reading storage gage rainfall.
> 3. Made channel routing a function of streamflow.
> 4. Revised UZSN (nominal capacity of upper zone storage) dependent on evaporation.
> 5. The ratio LZS/LZSN (lower zone storage/nominal capacity of lower zone storage).
> 6. A print-out of the daily soil water storage.
>
> He listed and gave a short description of the basic climatological input data and watershed parameters needed for the model to reproduce a continuous hydrograph.
>
> Clarke performed a study for the Kentucky Department of Highways using the Stanford Model for small drainage basins in Kentucky. This research provided some guidance for the sensitivity study of Little Mill Creek.
>
> Balk (1968) obtained the Kentucky Version of the model and adapted it to the Ohio State University I.B.M. 7094 computer system. In

addition to explaining the model program, flow diagramming the computer program, and discussing a methodology for its use, he assembled data, determined initial basin input parameters, and applied one year of data for Little Mill Creek near Coshocton, Ohio.

Using basic inputs of hourly precipitation, daily pan evaporation, monthly pan coefficients, physical watershed parameters, initial soil moisture conditions, and initial ground-water storage conditions, the model tries to simulate the moisture balance in a watershed to produce synthesized streamflow and synthesized evapotranspiration. Three zones of activity are used by the model. The upper zone (soil surface and above) incorporates the processes of interception, transpiration, evaporation, overland flow, surface detention, and depression storage. The upper zone models the initial watershed response to rainfall and is of major importance for small storms.

The lower zone (between the water table and the land surface) activities are infiltration from the upper zone, percolation, and interflow. The lower zone models the watershed's response to major storms through its control of infiltration rates. The deep lower zone (below the water table) controls the ground-water flow to the stream and to deep storage.

Evaporation from exposed water surfaces and transpiration from plants may occur from all three zones. As precipitation falls on a watershed it will enter interception storage depending upon the watershed cover and the volume of water present in interception storage. Interception storage volume is recovered at a rate equal to the evaporation from a Class A pan. Delayed infiltration occurs in the upper zone due to moisture not directly infiltrated into the soil. The model simulates these delayed infiltration effects of surface detention and overland

flow from pervious and impervious surfaces using the equations developed by Crawford and Linsley based on the Chezy-Manning equation in the turbulent range.

The upper zone moisture storage value, updated every fifteen minutes, is depleted by infiltration to the lower zone and evapotranspiration.

From the water infiltrating from the upper zone, which consists of delayed and net infiltration, a fraction is retained in the lower zone with the remaining moisture going to ground-water flow. Moisture in the lower zone may contribute to interflow detention storage and become interflow discharge.

Ground-water flow can pursue two possible paths. One is the percolation of ground water within the basin to possibly reappear as streamflow or evapotranspiration. The other is to pass to deep ground-water storage and out of the model's moisture balance.

The model uses a modification of an empirical routing technique devised by Clark in which the time-area histogram represents an outflow hydrograph when routed through an imaginary level pool reservoir situated at the basin outlet. One exception to the above method is that the OSU Version attempts to account for inbank and flood-plain flows by the use of a routing parameter that coordinates the synthesized streamflow with the preassigned channel capacity as designed by James in his work with the model at the University of Kentucky.

This cooperative project was continued and a second M.S. Thesis resulted -- authored by Dana L. Briggs (1969) and titled "Application of the Stanford Streamflow Simulation Model to Small Agricultural Watersheds at Coshocton, Ohio". Major objectives of the Thesis were:

1. Convert the OSU Version of the program from the I.B.M. 7094 computer system to the

recently installed I.B.M. 360 "time-sharing" computer system.

2. Program an option to plot logarithmic and arithmetic streamflow hydrographs for better and quicker analysis of results.

3. Perform a sensitivity study (varying one parameter at a time) using five years of data to determine the best set of basic input parameters for Little Mill Creek (Watershed 97).

4. Apply the model to five smaller highly instrumented sub-watersheds within Watershed 97, using the results of the sensitivity study and five years of data, in an attempt to determine how small a basin can be in which the model is yet applicable.

Simulation of runoff to five watersheds (122 to 4,580 acres) and 5 years resulted in the following:

1. Annual yield deviated from the actual by values ranging from -30 to +26 percent. Agreement is better for the last 3 years than for the first 2 years. Briggs thought that the model did not reach equilibrium in the soil water balance until the third year. Synthesized peak discharge rates accounted for at least half of the error each year. An extreme case is 39 cfs recorded and 253 cfs computed.

2. The nominal capacity of the Lower Zone Storage (LZSN) of the sub-basin was decreased from 20.0 to 12.0 inches due to a general decrease of the synthesized yields for the smaller basin -- "which can most probably be attributed to a decrease in depth of value alluvium and a shallower soil mantel at the headwaters."

3. Synthesized recession flows in the winter and spring months left much to be desired -- under estimated. Although 40 computer

runs were made to increase these flows, the results did not come up to hoped-for values.

4. Due to lack of knowledge of snow melt contribution to flow, all forms of precipitation were treated as rainfall input at the time it occurred. When it was obvious that snow melt occurred on certain dates, the precipitation amount was entered on that date. Frozen soil also is a factor recognized, but its effect was unknown and not included in the model.

5. By adjusting input values, water yield values were derived with fair accuracy. In doing so, streamflow hydrographs were not well correlated. Readjustment may improve the computed hydrograph, but may decrease the accuracy of low flows and water yield.

6. The model was not hindered by the size of the watershed -- down to the lower limit of the test, 122 acres.

It is obvious that present streamflow simulation models are far from perfect. Perhaps one should consider that no claim is made that a model in its present state of development really models in a physical way any segment of the system. These models need considerable refinement if they are actually to represent "real life" conditions. The models, however, are a step in the right direction and are helpful in pointing out those segments of the hydrologic system which need additional research. Then when these segments are adequately defined so that the pieces of the flow system fit together to make up the whole; then a model is a real one and can be used to predict flows from ungaged basins.

Current studies relating to watershed model development

Current studies that will add to greater understanding of segments of watershed flow system are noted below:

a. A channel reach on Little Mill Creek (fig. 2) was instrumented in 1969 to develop information on flood routing in headwater streams. Main stream inflow and outflow and side tributary flows are gaged. Topographic maps from low level flight photographs are providing accurate information on valley storage.

b. Portions of watersheds are being identified as essentially areas of soil water recharge and other areas of soil water discharge -- the latter being areas of maximum runoff from rainfall, or as the only areas contributing runoff from some storms. A concerted effort should be made to obtain information on the location, extent, and temporal changes in these discharge areas, as they relate to soil, geology, and climate. Remote sensing may be the best means of getting this information and should be investigated.

c. Large peak rates of runoff during major flood periods are related to storm rainfall intensity and area in the Rational Formula, $Q = CIA$. In this formula, Q is the peak runoff rate in c.f.s., C is a coefficient, I is the maximum rainfall intensity in inches per hour prior to the peak and A is area in acres.

Intuitively, it seems that Q would also be influenced by the amount of storm rainfall prior to the peak rate. To check this point, data were selected for nine major flood peaks on watershed 196 (table 3). The C values as given in column 2 were computed by substituting measured values of Q, I, and A into the Rational Formula. The remaining columns of table 3 contain various rainfall amounts prior to the time of peak runoff.

A plot of C- values vs. storm rainfall is given in figure 3. These are the data from columns 2 and 3 of table 3. The correlation coefficient for the data of figure 3 is 0.94, highly significant. Correlation coefficients between C- values and the last two columns of table 3 were no better than that for data in column 3.

From these data, it seems clear that antecedent rainfall does have an effect on the C-value in the Rational Formula. If, however total storm rainfall is highly correlated with rainfall intensities, the relationship could be spurious. However, in this study there was no correlation and the relation between C and antecedent rainfall is valid.

Table 3.-Relation of antecedent rainfall to Rational Formula coefficient C, major floods, 303 acres, 1940-1969

1	2	3	4	5
Maximum rain intensity for 30 min.1/	Rational Formula Coef. C	Antecedent Σ rainfall2/		
		Storm, D_o	$D_o + D_{-1}$	$D_o + D_{-1} + D_{-2}$
		Inches	Inches	Inches
1.50	0.390	1.43	1.87	1.87
.86	.445	2.14	2.58	2.58
1.04	.443	1.40	1.53	1.53
1.04	.444	1.65	1.65	1.65
3.74	.511	2.66	2.66	2.66
2.68	.667	3.66	3.70	4.40
5.08	.715	3.05	3.40	3.40
1.62	.633	2.91	3.27	3.26
.96	.780	4.41	4.77	4.82
r value	--	.94	.93	.92

1/ Prior to time of peak.

2/ Up to time of peak. D_o = day of storm, D_{-1} = day before storm, D_{-2} = 2 days before storm.

d. Unit watershed hydrologic data analysis showed that soil, crop, and management have a gross effect on watershed flow, but there are insufficient data to make it possible to define the separate effect of each. It appears necessary to first define the effect of soil by applying a uniform crop and practice on all watersheds -- a calibration. Then it will be possible to separate out the effect of future practices and crops for application in watershed models.

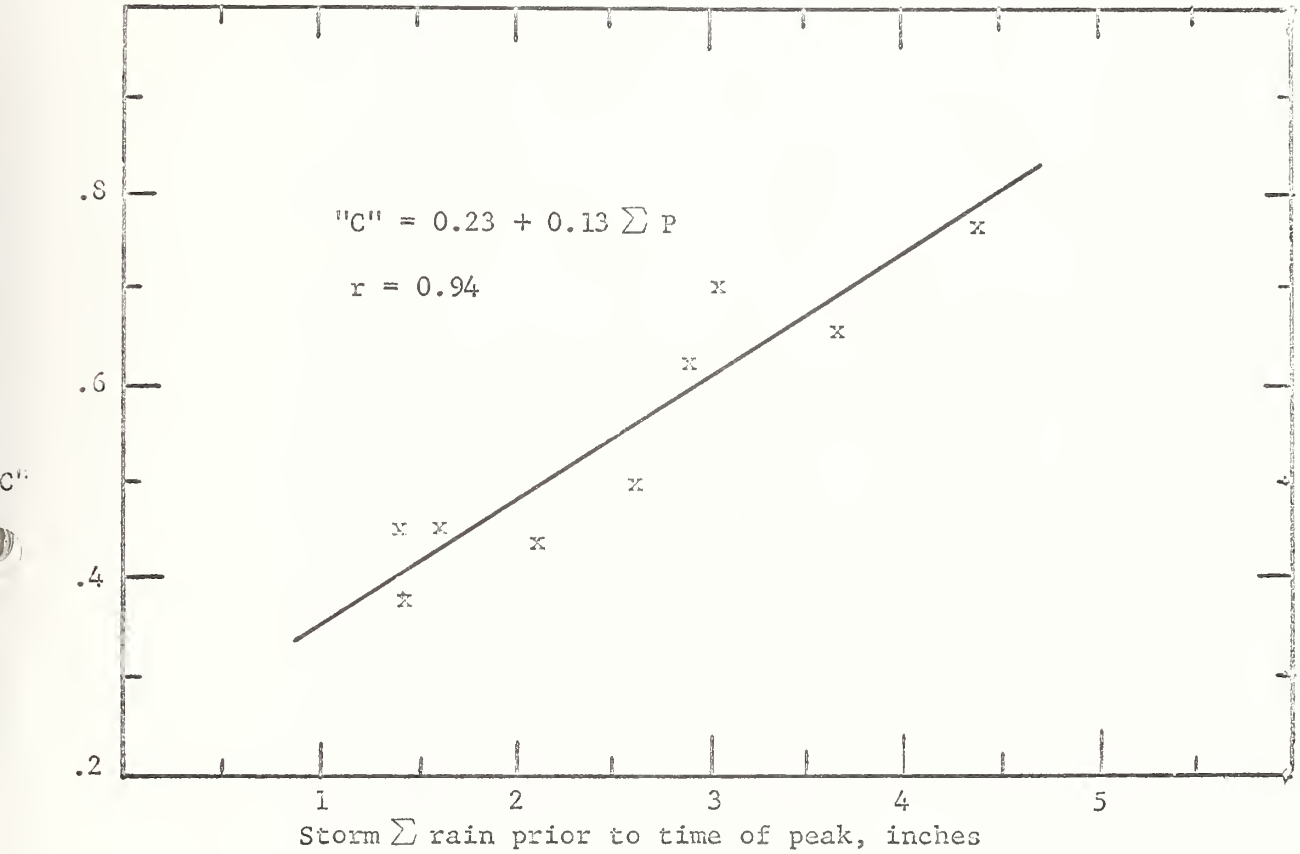


Figure 3.-Relation of Rational Formula "C" to $\sum$ rain prior to peak for high peaks, watershed No. 196, 1939-69

0.04 from 0.90, we get 0.86, which in turn is subtracted from 4.43, giving a value of 3.57 inches -- the effective antecedent rainfall. This adjusted rainfall is plotted against the "C" value of 0.667 for this storm (fig. 3).

d. Unit watershed hydrologic data analysis showed that soil, crop, and management have a gross effect on watershed flow, but there are insufficient data to make it possible to define the separate effect of each. It appears necessary to first define the effect of soil by applying a uniform crop and practice on all watersheds -- a calibration. Then it will be possible to separate out the effect of future practices and crops for application in watershed models.

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USDA-ARS-SWCRD Research at Madison, Wisconsin

C. R. Amerman

A watershed model these days looks very much like a flow chart for a computer program. Visually it consists of a collection of function boxes connected to each other by straight lines which represent the interrelationships between functions. Each box represents a theoretical or empirical formulation of one of the physical phenomena that determine the distribution of water within a watershed. These representations are themselves models. In the context of the overall watershed model, we might call them submodels.

A submodel reflects what we see as our best understanding of the phenomenon it represents. For example the submodel we choose to represent infiltration might be the Holtan equation or Philip's equation or whatever infiltration prediction process seems best at the time we build the model. An important concept regarding any well-structured watershed model is that it should be possible to remove and replace any submodel.

Consider for a moment two questions that come to mind about a watershed model: 1) does each submodel adequately represent its prototype mechanism? 2) have all watershed mechanisms been recognized, understood, and considered for inclusion in the model? Few will claim that the best or even a very good representation has been found for any submodel. Considering the difficulties of observing and quantifying the static structure (soils and rock) of a watershed, let alone the dynamic elements (water, nutrients, pollutants, and so on), it is quite possible that some watershed mechanisms are unrecognized or imperfectly understood.

The research mission of the ARS-SWC group at Madison, Wisconsin is to study subsurface water movement in the watershed context. Our objectives relate to the two questions previously asked about watershed models - what are the mechanisms of subsurface water movement, how can they be quantified or modeled, and how can they be linked to the surface runoff mechanisms to provide a unified, more

complete understanding of watershed hydrology?

Our work will hopefully broaden the scope of watershed modeling as well. Present models are designed to predict quantity and time distribution of water yield at one or more points of interest along stream channels in the watershed. With adequate subsurface modeling, including evapotranspiration withdrawals, it should eventually be possible to predict water status at any point in the watershed, be it in the root zone high on a slope or below the water table far beneath the surface. Such capabilities will be useful not only with regard to water supply, but also in studying movement of nutrients and pollutants.

The Madison facility was set up to bring soil scientists and hydrologists together for the specific purposes of 1) determining how soil physics can be used in watershed hydrology and 2) determining in what areas soil physicists can work to help advance watershed hydrology. The SWC staff consists of two soil physicists and a hydraulic engineer. We work with several scientists on the University of Wisconsin faculty - two soil physicists and a microclimatologist. We also are in close contact with agricultural and civil engineers working on various aspects of hydrology, including watershed modeling. A long range goal at the University is to bring all scientists working on water problems, under one roof. This would include the SWC staff as well as university engineers, soil physicists, soil chemists, economists and so on.

The Soils Department presently furnishes our office and laboratory space. Field experimental sites are provided by the Wisconsin Agricultural Experiment Station and by other SWC locations.

In seeking our objectives we have in mind the following concept of what a watershed is. A watershed is a three-dimensional body having depth as well as breadth. The surface of the watershed is only one of its boundaries and hydraulic conditions on it are affected by internal as well as external influences. The internal body of the watershed is comprised of soil and rock, much of

which can be classified as a porous medium in which Darcy's law applies, i.e.

$$q = K \frac{\Delta\varphi}{\Delta L}$$

where q = apparent water velocity in the direction of L
K = hydraulic conductivity

$\frac{\Delta\varphi}{\Delta L}$ = hydraulic gradient

φ = hydraulic head and may be further defined as

$$\varphi = h + z$$

where h = pressure head and is dependent upon moisture content
z = elevation head

The lateral and bottom boundaries of the watershed may be indistinct and may shift about under the influence of water-inputs through the surface. Water input may occur not only through the surface of the watershed, but also across lateral boundaries. The internal flow system, the pores and channels of the soil, is continuously connected within itself and with the external flow system. The flowing fluid consists of water and air and various impurities. Recognizing that one of these may at times cut off and trap the other in certain parts of the watershed, we nevertheless consider the water phase to be continuously connected throughout, so that we can apply Darcy's law. This concept may change as we learn more about air entrapment by water during wetting, or the breaking of water flow paths by air during drying.

The key point of the above concept is that the watershed is chiefly a porous media flow system. By combining Darcy's law with the law of conservation of mass and considering water to be incompressible, we can derive a general equation governing the movement of water within the watershed.

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left(K \frac{\partial \varphi}{\partial x}\right) + \frac{\partial}{\partial y} \left(K \frac{\partial \varphi}{\partial y}\right) + \frac{\partial}{\partial z} \left(K \frac{\partial \varphi}{\partial x}\right)$$

where θ = water content
t = time
K = hydraulic conductivity, in general a function of position, direction of flow, and of moisture content
φ = hydraulic head, a function of position and of moisture content
x,y,z = directions in the Cartesian coordinate system

This equation is described as being a nonlinear, 2nd order partial differential equation. It can be solved only if we know the conditions on the boundaries of the system and if we know the initial condition, i.e., the initial distribution of φ at every point in the system at zero time. In the case of the boundaries we must at all times know the conditions either in terms of φ or in terms of the flow rate across them.

The point of all this is that the conditions at any point inside or on the boundaries of the porous media flow system are dependent on conditions at every other point in the system. We cannot specify conditions on the surface of the watershed unless we measure them in-situ or unless we consider conditions on all other boundaries and at points within the watershed. By the same token, we cannot in general measure such flow-system-dependent parameters as infiltration rate at one point in time when the hydraulic head distribution has one configuration and expect our measurement to apply at another point in time when the head distribution has another configuration.

To understand the latter point, consider Darcy's law as presented above. Imagine a sloping field in midsummer when it has been dry for quite a while. Moisture content and soil properties vary with depth, but are reasonably uniform laterally over the field, so that lines of equal hydraulic head (equipotentials) are roughly parallel to each other and at a slight angle with the surface. If infiltration occurs flow will be vertically downward. If we run an infiltrometer at a

series of points in the field, we should get essentially the same infiltration curve at each point - we are not applying enough water over a large enough area to significantly affect points in the system even a short distance from our infiltrometer sites.

Consider now the very extreme condition of complete saturation of the soil of the same field after a long period of flooding and consider that the lower end of the field is at the bottom of the slope. Figure 1 is an approximation to the cross section of such a field. The equipotentials have an entirely different configuration than in the dry case. Therefore hydraulic gradients are completely different. In fact, infiltration may occur only along the upper half of the field. Water returns to the surface along the lower half. Furthermore infiltration rate varies considerably along the upper half, being zero at the center of the field and having a maximum value at the upper end, these two values being connected by a smooth, concave upward curve.

Even under prolonged storm conditions, it is highly unlikely that complete saturation of any sloping field will ever occur. But the equipotential configuration will lie somewhere between that of the dry condition described and that of the saturated condition of Figure 1. So the infiltration rate or any other parameter which is dependent on K and $\Delta\varphi/\Delta L$ can be expected to vary across the field and can be expected to show a different time dependency than that measured under dry ambient conditions.

Thus far we have been talking mainly in theoretical terms and it might sound as though we wish to abandon all that has been accomplished in watershed hydrology. This is not the case. We feel that with this idealized concept as background, we can perform investigations that will add to the hydrologic knowledge already accumulated. It is possible that subsurface flows and the distribution of subsurface hydraulic gradients are important factors in answering the long-standing questions regarding applying small watershed data to larger watersheds. For the immediate future, we will be investigating flow mechanisms with an

eye to making watershed models complete and to making some of the submodels more accurately represent the prototype processes.

At present, we are working on one and two-dimensional studies of porous media flow. We are using mathematical techniques in these studies and will soon be using physical models in the laboratory. Our aim in these studies is to gain greater insight into how internal water moves under various boundary conditions and cross section geometries. We also want to develop greater knowledge of what interaction occurs between above ground and subsurface water movement along the boundaries representing surfaces in contact with the atmosphere.

Another research activity is concerned with finding ways to measure the various hydraulic parameters within a watershed - how to measure K as a function of water content, how to measure directly the internal water flow rate at any point in the system, how to measure with reasonable precision the water content and pressure (positive or negative) head at any point in the system. When we know how to measure these quantities, we will want to go into the field and study real one-, two-, and three-dimensional flow systems, both for their own sake and as checks on our theoretical developments. In fact we are already in the field, using such crude measurement techniques as are available to us at this time.

As theoretical scientists we shall be striving to obtain as complete and fundamental an understanding of the subsurface watershed flow system as possible. As applied scientists we shall at the same time be striving to find which elements of the system are most necessary to the modeling process, and we shall be striving to find the simplest yet valid methods of modeling these elements.

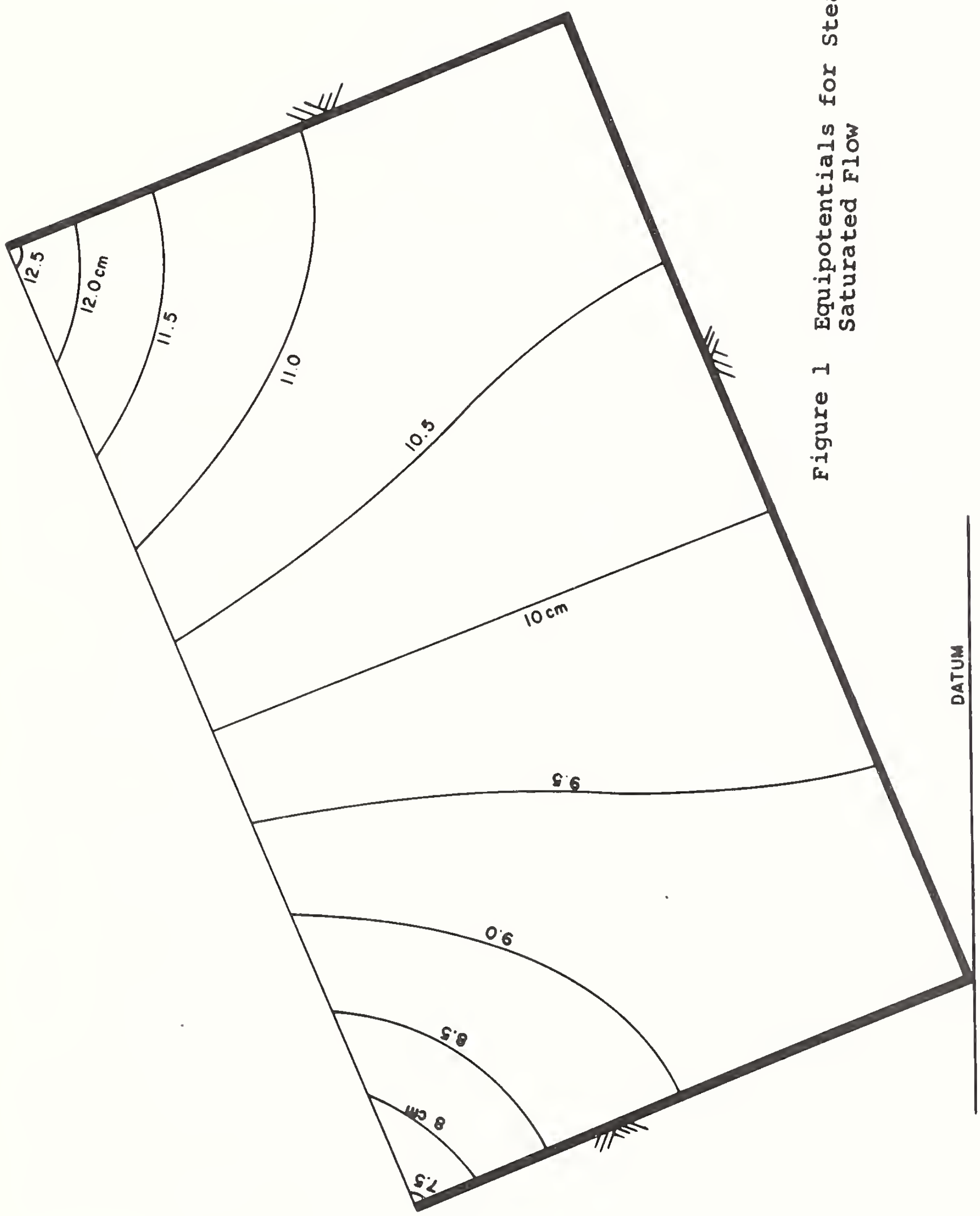

Figure 1 Equipotentials for Steady,
Saturated Flow

DATA FOR HYDROLOGIC MODELS IN THE WESTERN CORNBELT

by

K. E. SAXTON [1/]

INTRODUCTION

To confidently apply a watershed model to an ungaged watershed, it must be tested and verified by applying it to gaged watersheds. At the present time, we at the North Central Watershed Research Center, NCWRC, are collecting hydrologic and erosion data with an eye to the near future when we, or a cooperator, will use our data to guide development and verification of a watershed model. This discussion, therefore, will primarily concern characteristics of our gaged watersheds and the data which we are collecting, or have at hand, that are applicable to watershed modeling.

From my viewpoint, hydrologic models are necessary for three reasons: First, we know far more about hydrology than we are currently using in our design methods. We need better organization and ways to implement existing and new knowledge. We know or can reasonably estimate the effects of varying crops, fertility levels, channels, etc., on the performance of a watershed, but these are difficult to integrate into our present techniques. Second, there are important hydrologic variables that are now not considered, or are considered only superficially. For example, evapotranspiration plays a significant role in the hydrologic cycle; yet, we most often consider it only as an average annual geographic variable. Third, the interactions among the hydrologic variables will not be well understood and applicable until all major components in the hydrologic system can be modeled. We often estimate the antecedent soil moisture conditions from precipitation amounts during some previous period because we realize the two are related. However, we are ignoring other processes

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such as evapotranspiration, percolation, rainfall distribution, etc. Saxton (1)[2/] showed, by modeling the soil moisture component, that there is actually very little relation between soil moisture and precipitation amounts the previous 7 days. The need for defining antecedent soil moisture is itself the result of an observed interaction between soil moisture and infiltration. Hydrologic models need to be developed to enable us to utilize all of our hydrologic knowledge, involve all of the important variables, and define interactions of the variables.

The philosophy of model development is interesting when model definition, development, evaluation, accuracy, etc., are considered. Bross (2) comments "The great advances in science are those in which a useful new model is introduced," and he sites Newton's and Einstein's equations as successful examples. From this view, engineers and physical scientists are model specialists. Bross (2) also states "Progress in science is based on this constant interplay between model and data," but he concludes "...it is rather pointless to say that the model maker is a greater scientist than the data-grubber..." This need of coordination between model builders and those collecting data is heavily stressed by Smith (3) and other members of the ASCE urban hydrology committee. This raises the question: If the models are not written and the data collected by the same person or group, how can the needed coordination best be implemented? This cooperation will not "just happen" but will result only after a concerted effort.

DATA REVIEW

As we scan the data from our gaged watersheds, we need to keep organized from the viewpoint of watershed models. To accomplish this, I will use the two-phase concept, land phase and channel phase. Although these are common watershed model terms, let's review by noting that land phase encompasses the processes of precipitation, infiltration, evapotranspiration, interflow, transmission losses, depression storage, and overland flow. In essence, this phase provides a time-distributed runoff supply to the edge of a watershed's channel system. Here the channel phase processes of translation, storage, and summation govern until the runoff is delivered out of the watershed.

2/ Numbers in parentheses refer to literature cited, page 9.

We are primarily considering the deterministic portion of modeling. This is not to suggest an elimination of stochastic methods. On the contrary, I am certain any model will contain techniques of both. Our data are physical measurements over only a few years and thus will be most useful in developing and verifying deterministic aspects.

With these thoughts in mind, let's first consider the overall view of our watershed's locations and features, and then each location and its data in some detail. Figure 1 shows the location of the four sites where watersheds are operated by our research group at the NCWRC, where we have data of interest to those working on watershed models. Having several locations, we are obtaining data from watersheds of highly diversified soil, cover, geologic, and geomorphic features. Following are the characteristics of these data as they relate to watershed model application, with an occasional side glance at interpreted results. Most of these data are on computer cards or magnetic tape and therefore are readily available for computer processing.

TREYNOR, IOWA:

We have five gaged watersheds in the Missouri Valley deep loess soils area located about 20 miles east of Council Bluffs, Iowa. Compared with our other watersheds, these have the best land use control and are gaged the most intensively. We have complete land use control on four of these watersheds. The fifth, W-5, is farmed by the owners, but they are bound by a cooperative conservation effort. The watershed sizes and land uses are:

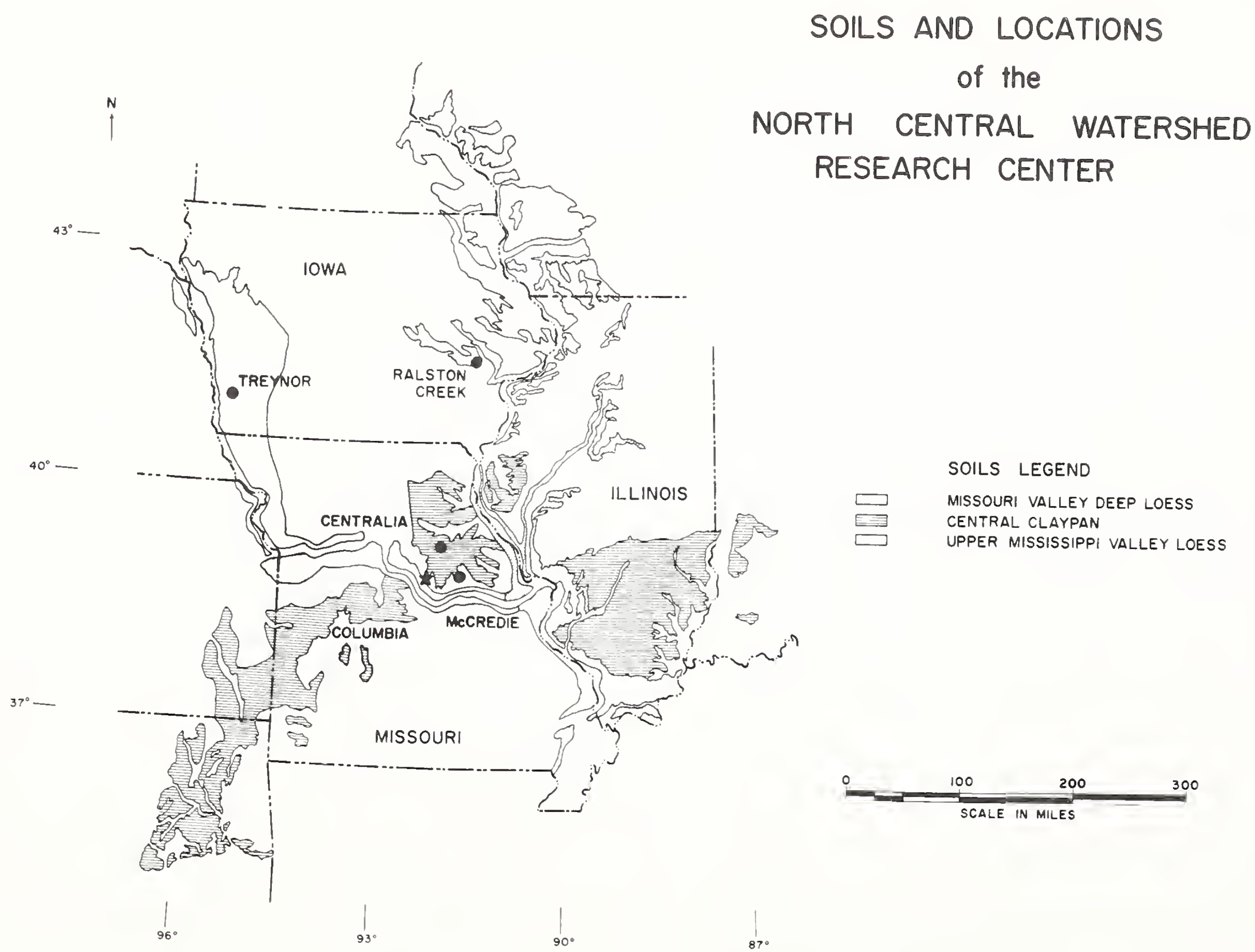

Figure 1.--Locations of watersheds being studied by the North Central Watershed Research Center.

Watershed	Area Acres	Crop	Land Treatment
W-1	74.5	Corn	Contour
W-2	82.8	Corn	Contour
W-3	107	Grass	Pastured
W-4	150	Corn	Level-Terraced
W-5	389	Mixed	Level-Terraced

The primary objective of these watersheds is to study the gullies that exist on each watershed. In addition, all major hydrologic variables and total sediment yields are being measured. Figure 2 shows the surface features and instrumentation of watershed 2--which is typical of the other watersheds. A schematic geologic section is shown in figure 3. These two figures indicate several important features. The topography is quite steep, with a well-developed but relatively small channel system. The loess mantle is relatively permeable and has a high water-holding capacity. The glacial till is quite impermeable and forms a hydrologic boundary. Water apparently percolates nearly vertically until it reaches the saturated zone, then moves horizontally, to reappear as base flow. Only a small amount is expected to percolate through the glacial till.

Each major variable within the hydrologic cycle is being measured on these watersheds. These include precipitation, streamflow, ground water, soil moisture, and evapotranspiration. These measurements are either continuous or at intervals that will allow a detailed representation throughout the year. This complete set of measurements over time is giving insight into the interrelations of the processes that govern the hydrologic cycle--for example, the relations of soil moisture, ET, infiltration, and percolation. In this way, these data will be valuable in defining the principles to be used in a watershed model. For a model to perform satisfactorily through varied conditions, it must give adequate representation to each of the components that must be considered. By having measured these components, their representation can be evaluated in addition to the usual model outputs.

A characteristic hydrograph from these watersheds is shown in figure 4. Note the quick response and sharp recessions, which reflect the steep slopes and lack of channel storage. The ϕ-indexes, average infiltration rates expressed in the same units as rainfall, show the effect of antecedent

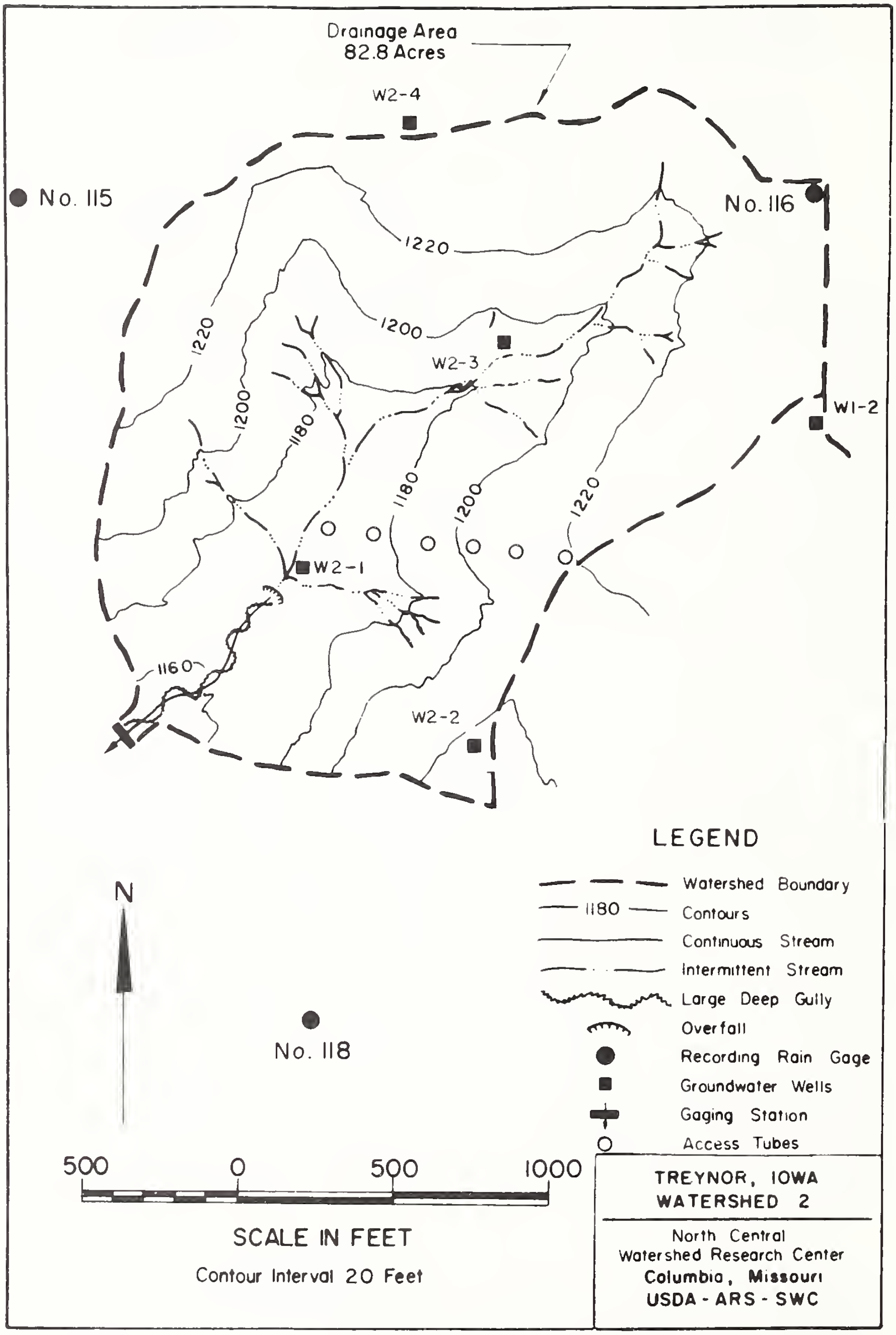

Figure 2.--Topography and instrumentation of Treynor Watershed 2.

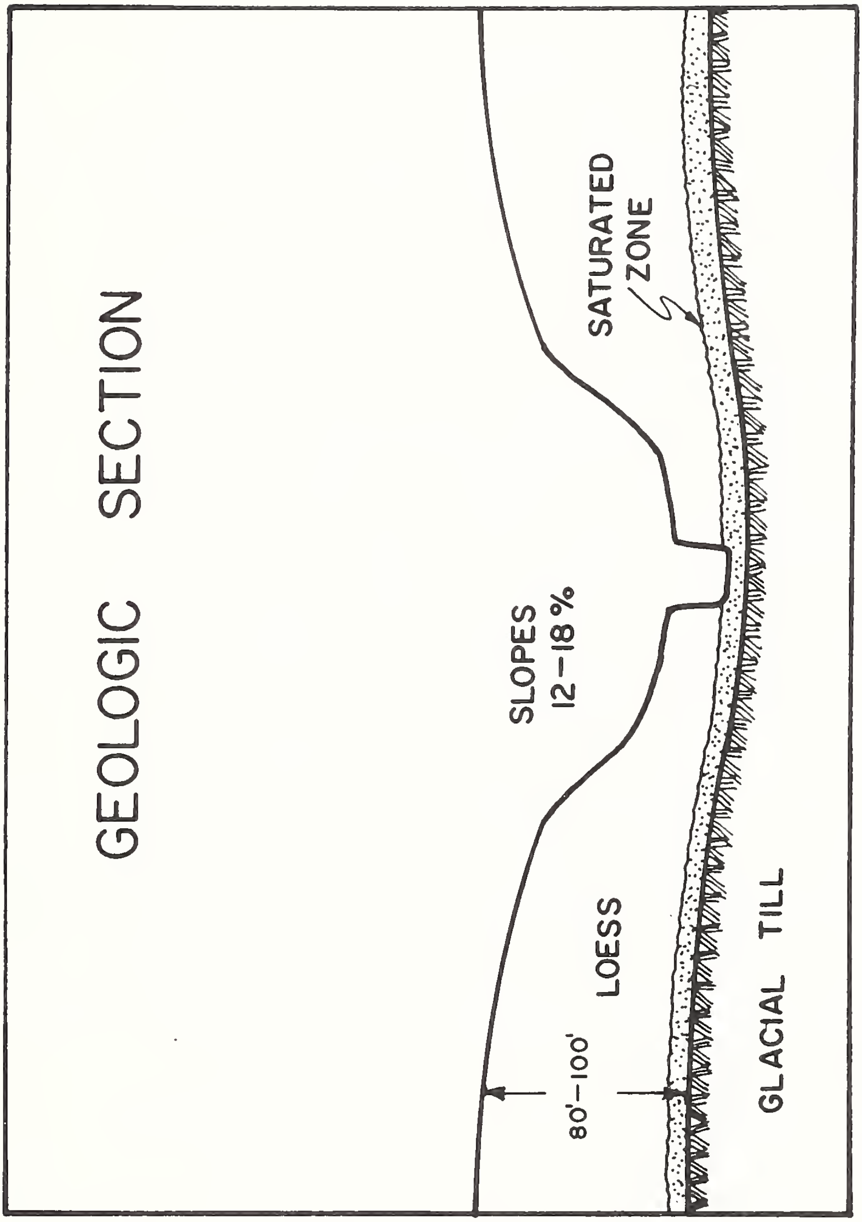

Figure 3.--Schematic geologic cross section of loessial watersheds.

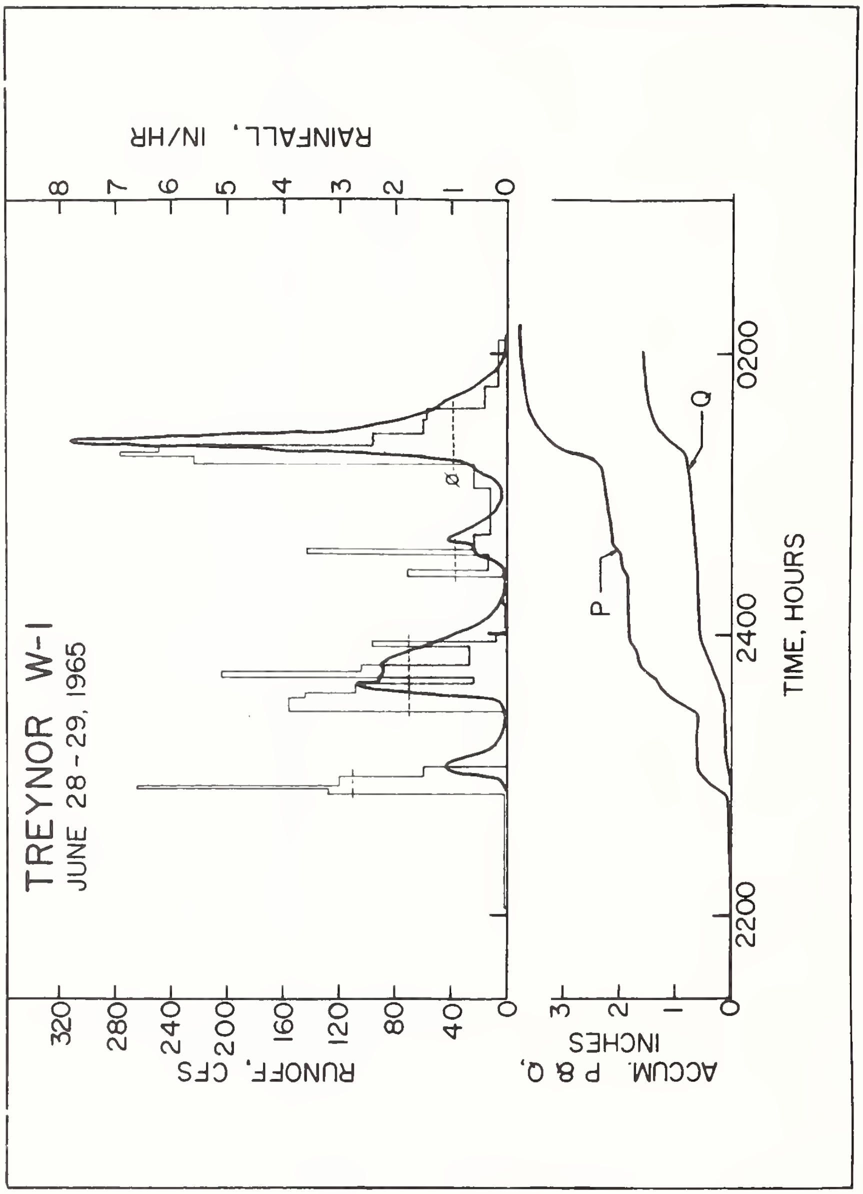

Figure 4.--Example hydrograph from the steep, loessial watersheds.

rainfall. This example shows the watersheds' reaction to two variables, surface features and soil moisture, that must be modeled.

In figure 5 are shown the results of observations within the soil and geologic profile. It is quite apparent that each zone operates in response to the precipitation input and to each other. This certainly expresses the subsurface system as a continuum and a system to be recognized in any watershed model encompassing base flow.

The long-term performance of these watersheds is indicated in figure 6 by the average annual water yields and soil losses measured from 1964 to 1968. Annual precipitation has averaged about 34 inches, compared with a normal of 28 inches, and has been quite uniform among the watersheds. Some of the significant findings shown in figure 6 are: (1) Level-terraced W-4 has similar water yields to corn watersheds W-1 and W-2, but only a small amount is surface flow. This results in drastic reduction of sediment yields but not of streamflow. (2) The grass watershed, W-3, reduces both water and sediment yields. (3) The larger watershed, W-5, confirms the findings from the other watersheds. A more detailed explanation can be found in papers by Piest and Spomer (4) and Saxton and Spomer (5).

The data just discussed are basic to hydrologic models. However, a logical extension would be to model erosion, sediment transport, and nutrient movement. These data are also being obtained. Both gully and surface erosion are measured by frequent suspended-sediment samples during each storm. Nitrogen and phosphorus have been applied at two levels and their movement in the soil and water is being monitored. Although modeling of these variables may represent a sophistication not yet feasible, the usefulness is apparent and we should make this a projected goal.

In summary of our data at Treynor, the watersheds are on a relatively uncomplicated and definable geologic setting, land use is varied and controlled, and several years of data are already at hand. For application to watershed models, these data primarily represent the land phase, since channel processes are not dominant in hydrograph development. The land use and surface soils play a dominant role in surface runoff, but a significant portion of the water yield is controlled by the deeper profile through percolation and base flow.

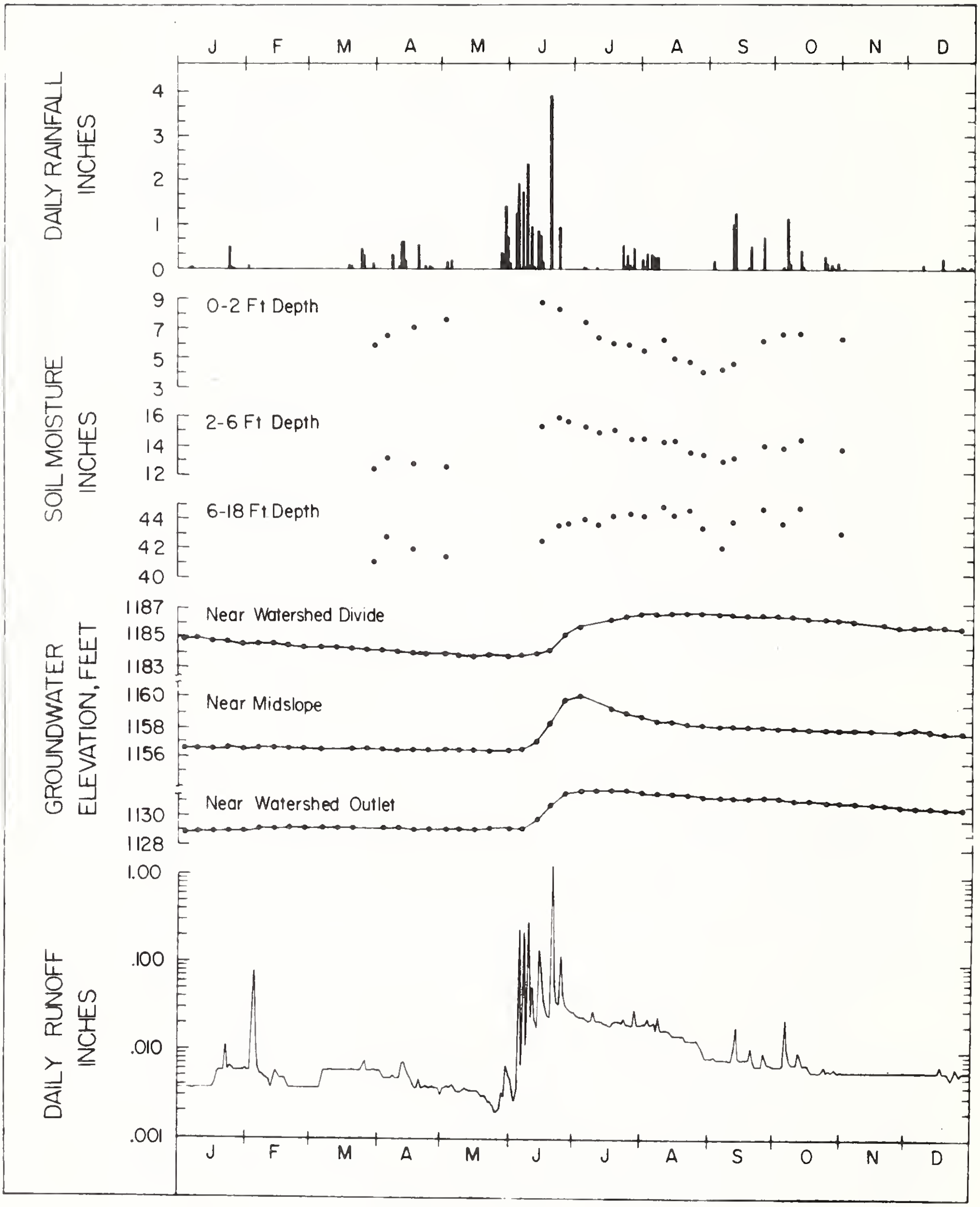

Figure 5.--Example of subsurface response on loessial watersheds, Treynor W-3, 1967.

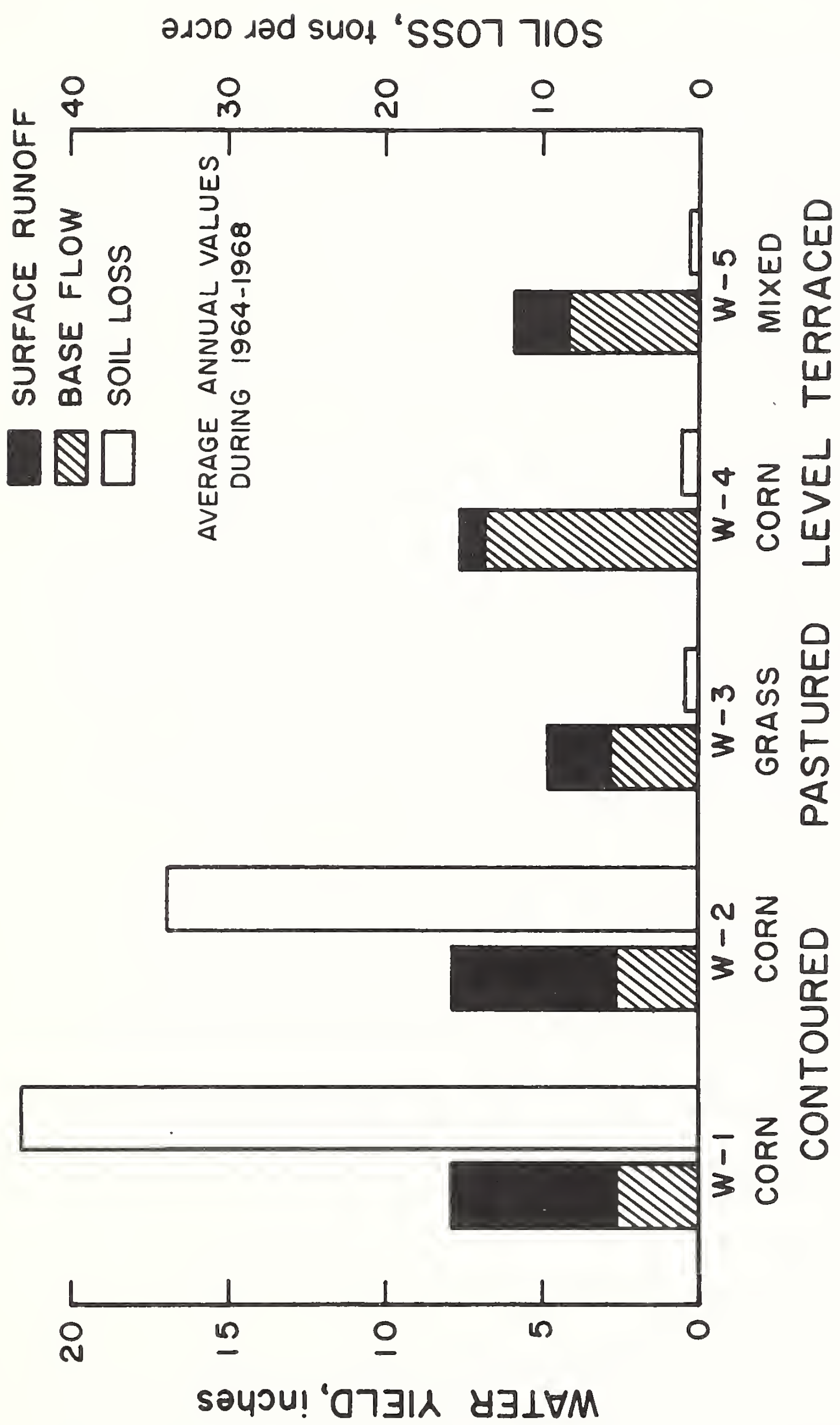

Figure 6.--Observed water and sediment yields from the Treynor watersheds.

RALSTON CREEK, IOWA:

One of the longest precipitation-runoff records on small agricultural watersheds is available from the Ralston Creek watershed (3.01 square miles) near Iowa City, Iowa. Records have been obtained since 1924. Complimentary data are available from Rapid Creek watershed (25 square miles, 1937 to present) and South Branch watershed (3.01 square miles, 1963 to present). The location of these watersheds is shown in figure 7. Their soils and geology are similar to the Treynor location, but the loess cap is only 5 to 10 feet thick and the subsurface flow system is not as definable.

The two Ralston Creek watersheds are now becoming urbanized. As indicated in figure 8, South Branch Ralston Creek is now about 25 to 30 percent urbanized and is being developed at a faster rate than Ralston Creek. In a cooperative effort by ARS, USGS, and State University of Iowa, hydrologic data are being collected on these three watersheds and the urbanization recorded by aerial flights at 2- to 3-year intervals. Other pertinent data, such as storm sewer construction, are also being obtained. These data will reflect both the land and channel phases. Particularly large changes will be expected in the channel phase as the city storm sewer system replaces the natural agricultural drainage channels.

If the watersheds urbanize as expected, data will be obtained that will allow testing of models over a long period of record on an agricultural watershed and then on simultaneous conditions of urban and agricultural. In contrast to the detailed data on the Treynor watersheds, the Ralston Creek data are only the input and the outflow of the watersheds and thus will be useful for the usual model testing rather than component evaluation. However, this testing will give much-needed insight into the effects of urbanization and allow more adequate modeling of watersheds in this land use.

MC CREDIE, MISSOURI:

The central claypan soil areas of Missouri and Illinois have features that contrast sharply with the loessal areas. Their land form is quite flat, with most slopes ranging from 2 to 5 percent. The soils have developed on a shallow

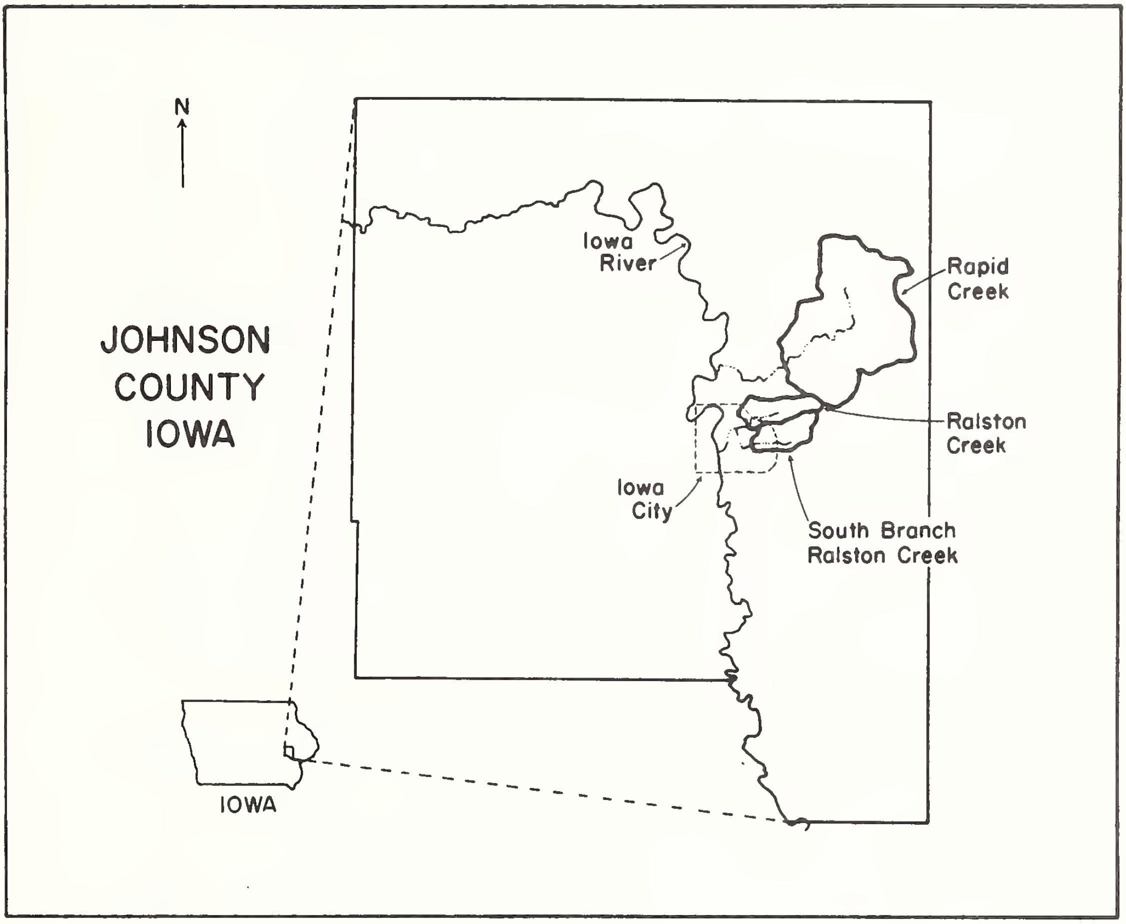

Figure 7.--Locations of the Ralston Creek Watersheds.

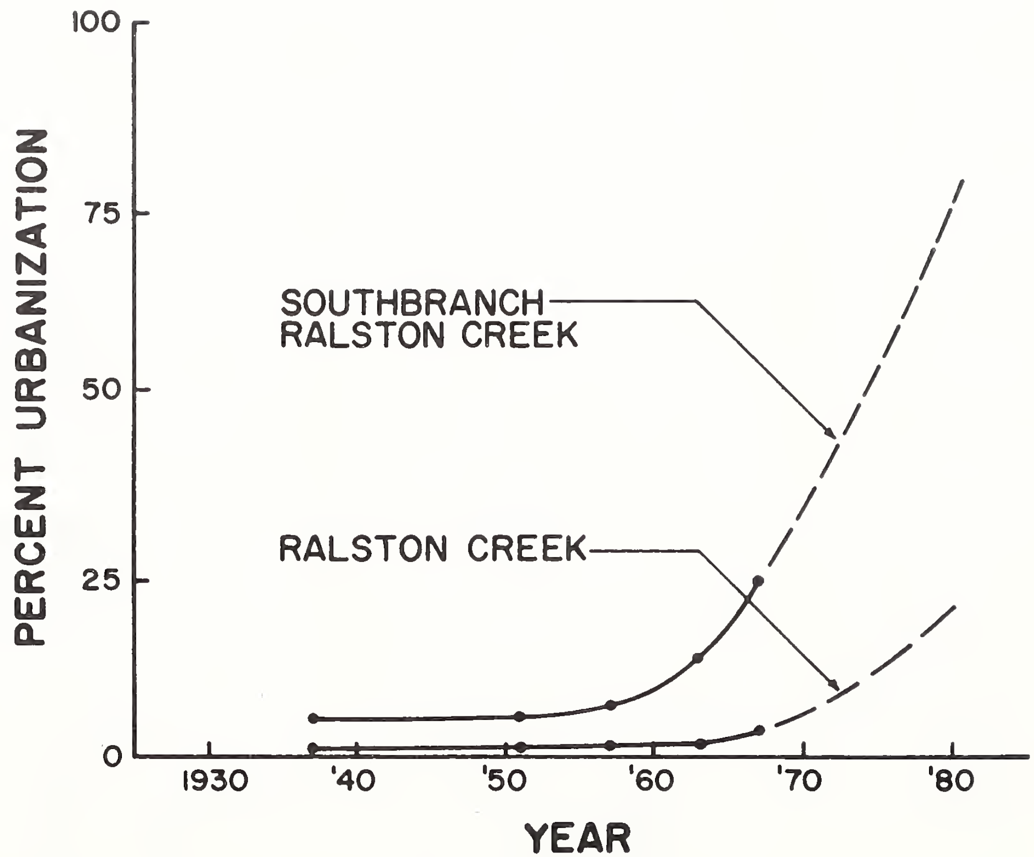

Figure 8.--Rate of urbanization on the Ralston Creek Watersheds.

(4-to 6-foot) loess cap which lies on a relatively thick, impermeable blanket of glacial till. A schematic diagram of this system is shown in figure 9. The soil profiles have a well-developed clay accumulation zone 12 to 18 inches below the surface, usually 6 to 12 inches thick. When moist or wet, this zone becomes nearly impermeable and thus severely limits the depth of profile available for water storage. As a result, the upper part of these soils will occasionally become essentially saturated and almost all precipitation will run off.

There is no ground water outflow from these watersheds due to the flat topography and the limited percolation through the glacial till. During some wet periods, trickle flows occur for several days after rainfall. This is apparently the result of soil drain-out, or interflow.

The MCredie watershed is located in the claypan area 25 miles east of Columbia, Missouri. Continuous P-Q records are available from 1941 to the present. Surface features are shown in figure 10. Land use has been mixed but is recorded. Few topographic modifications have been made. A complete review of this watershed, 25 years of data, and hydrologic analyses are given by Saxton and Whitaker (6).

The drainage system on this watershed consists of shallow, grassed channels. Thus, in terms of model use, this watershed again represents the land phase. However, the hydrologic response is quite different from the loessal watersheds due to soil, crop, and topography differences.

CENTRALIA, MISSOURI:

A new set of watersheds has been selected for research in the claypan areas. These are near Centralia, Missouri, about 25 miles north of Columbia, Missouri. Figure 11 shows the location of the four largest watersheds. A smaller watershed, 50 to 100 acres, will be selected within watershed 9 for intensive instrumentation. Rain gages are now in place and stream gaging will begin in 1970.

These watersheds have the flat topography and claypan soils that typify this area. In contrast to the McCredie data, these larger watersheds have well-defined channels; thus, both the land and channel phases will perform significant roles in the observed data. The smaller area will

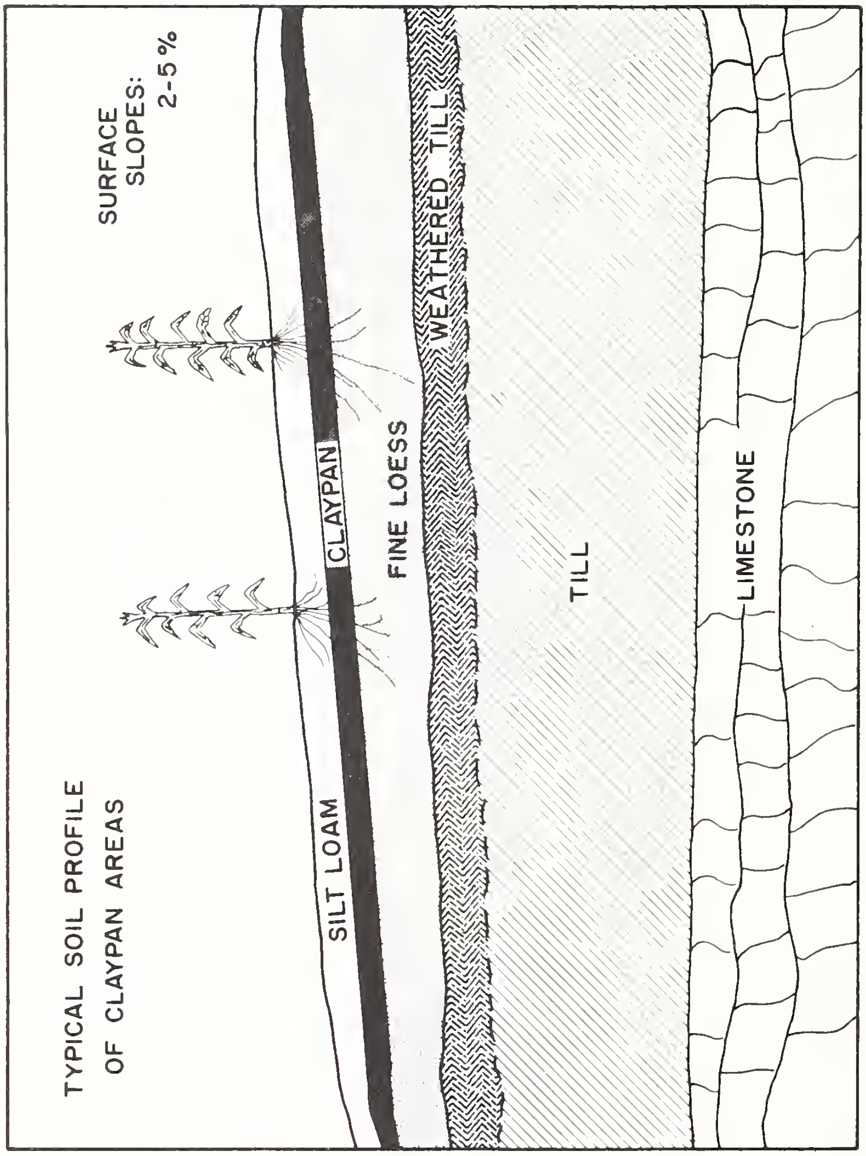

Figure 9.--Claypan soil schematic profile typical of northeast Missouri.

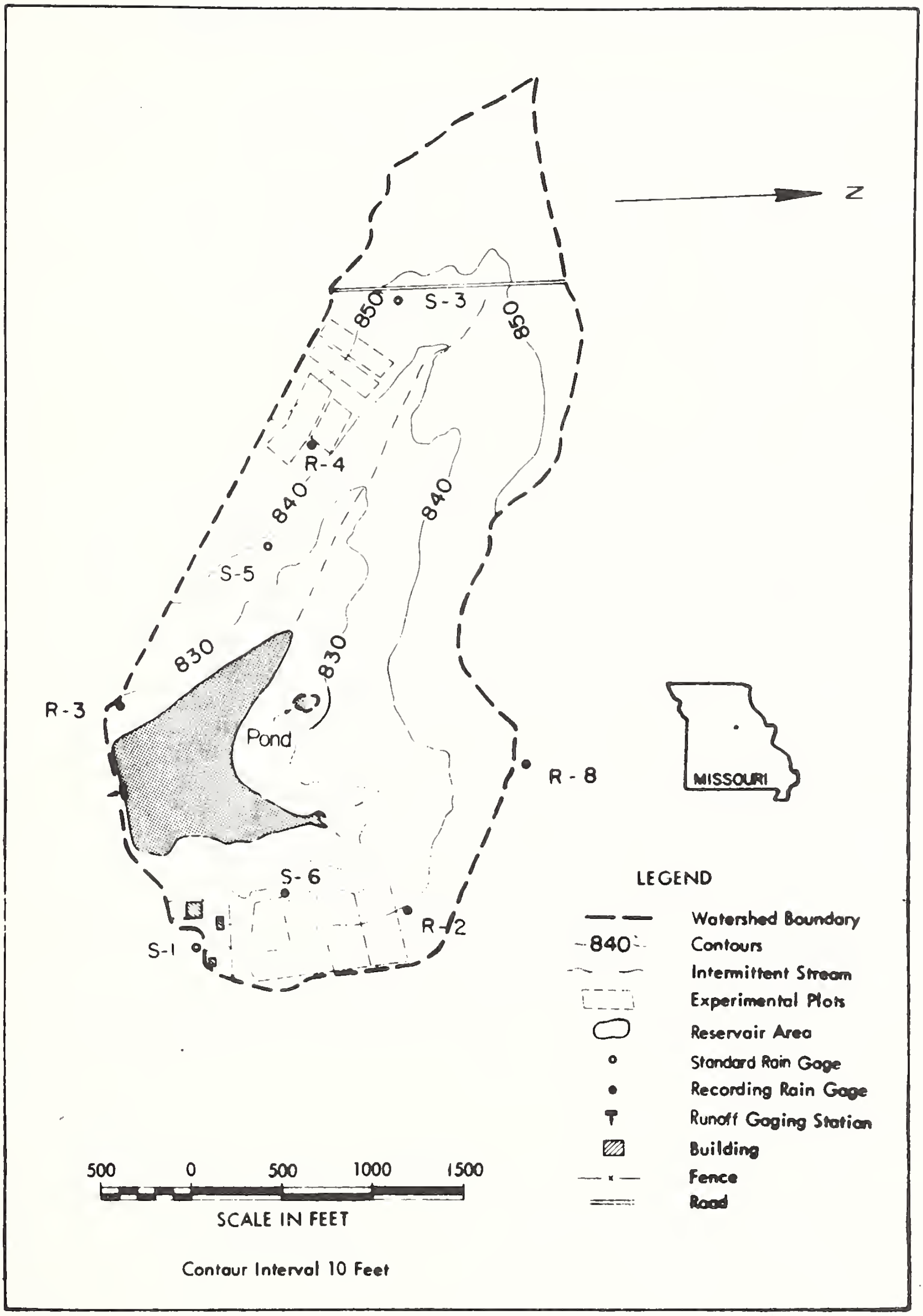

Figure 10.--Topography and instrumentation of the McCredie, Missouri watershed.

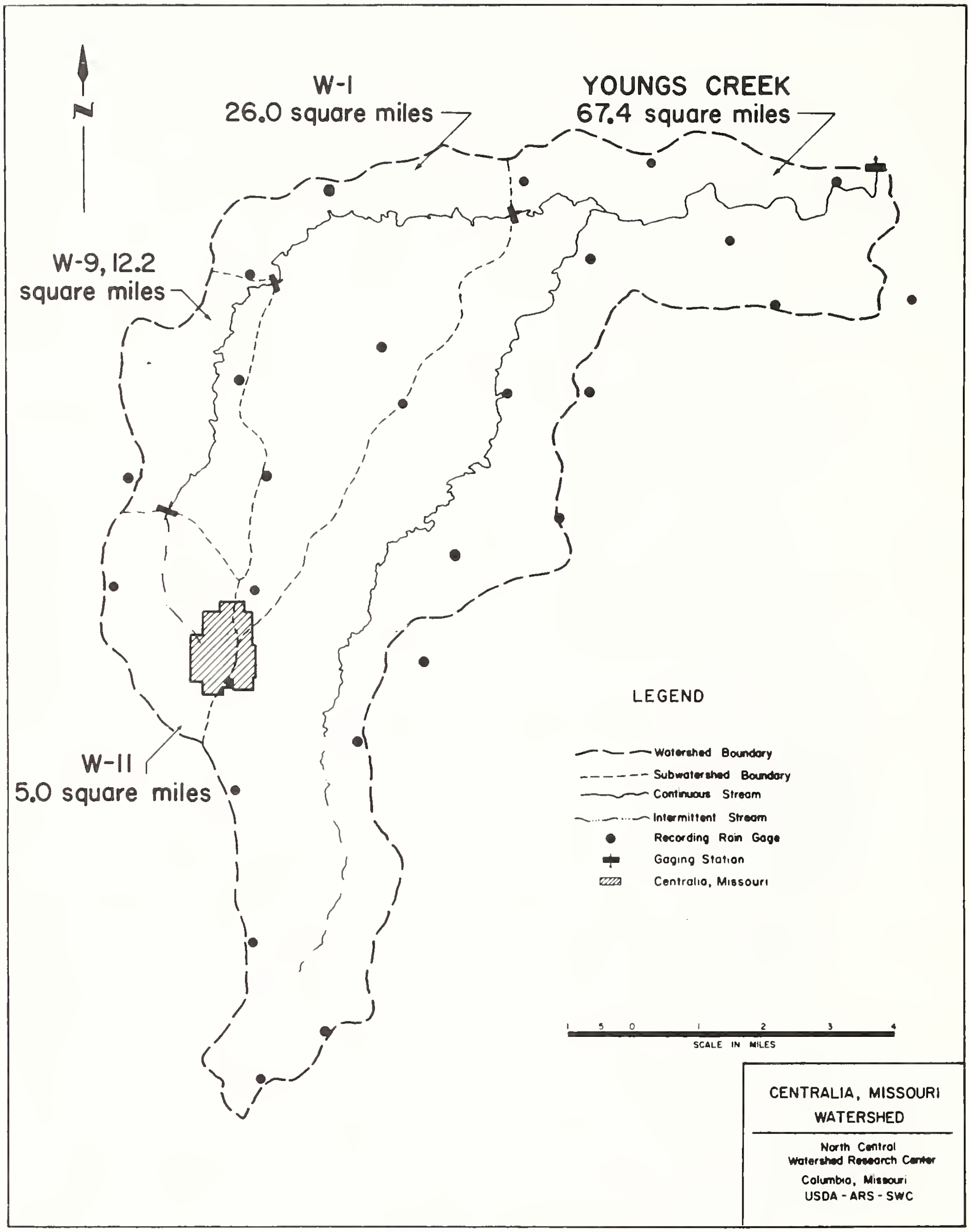

Figure 11.--Centralia watersheds, 25 miles northeast of Columbia, Missouri.

provide data on the land phase. Detailed channel information will be obtained to evaluate the channel phase.

A model for the subsurface flow system on these claypan soils is now being developed by ARS cooperaters at the University of Wisconsin. The small watershed within this group will be instrumented to verify this model system. Although only a component of the total hydrologic model, this subsurface flow system may prove to be more important than usually considered due to interactions with processes such as infiltration and evapotranspiration.

SUMMARY

We are collecting hydrologic, erosion, and pollution data which will be valuable for developing and testing of watershed models. Our data are from quite diverse physiographic watersheds, ranging from the steep loessal watersheds to the flat claypan areas. Land use is varied from continuous corn, level terraces, grass, urban, etc. Since much of our data are from small watersheds, they will contribute mostly to understanding the land phase, although the Ralston Creek and Centralia data will involve channel evaluations.

Our current analyses are concerned with land use effects and evaluating components within the hydrologic system--e.g., the methods of predicting evapotranspiration, soil moisture distributions, percolation, etc. These will provide important methods for developing the components of a watershed model. We hope to soon begin using our data and results in developing a watershed model or cooperating with someone who would like to test and improve an existing model.

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SWC RESEARCH PROGRAM

in Erosion-Sedimentation Process

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Lafayette, Indiana

The Erosion Group at Lafayette has made significant progress in mathematical modeling of the erosion-sedimentation process as it applies to field-size areas. Slightly more than a decade ago, they developed the Universal Erosion Equation, which is now widely used in 37 states to help guide selection of conservation practices on specific fields and in specific problem areas. The equation expresses average soil loss from a given area as a function of soil, topographic, weather, and management parameters. The functional relationships were empirically determined from several decades of field and laboratory studies throughout the country. This equation and sources for locational evaluation of its factors were reported by Wischmeier and Smith (1965). It has been useful in estimating sediment loads from upland segments of watersheds and in selecting practices for soil and water conservation.

Currently the erosion group, in a Branch-wide team effort, is endeavoring to develop a comprehensive rational model of the erosion-sedimentation process that will better lend itself to computer analyses of whole systems. The general framework for such a model was proposed by Meyer and Wischmeier (1970).

This approach to mathematical simulation of the process considers (a) soil detachment by rainfall, (b) transport by rainfall, (c) detachment by runoff, and (d) transport by runoff as separate but interrelated phases of the process of soil erosion by water. Each phase then may be described by fundamental hydraulic, hydrologic, meteorologic, and other physical relationships plus parameters describing the soil properties that influence erosion.

The four subprocesses of the erosion process are evaluated for each slope length increment and used as illustrated in figure 1. On each increment, the soil available for transport is the material detached on that increment by rainfall and runoff ($D_R + D_F$) plus the material carried to it from the increment upslope. This sum is compared with the sum

of the rainfall and runoff transport capacities at the end of the increment ($T_R + T_F$). If the detached soil is less than the transport capacity, available soil is the limiting factor on the slope increment, and the sediment carried to the next increment equals the amount of available material. However, if the total transport capacity is less than the soil available for erosion, transportation is the limiting factor and the sediment load equals the transport capacity. In this manner, soil and water are routed from the top to the bottom of a slope, based on conservation of mass. Thus, it is possible to simulate soil erosion as a dynamic process and to describe soil movement at all locations along a slope at any time.

This approach provides the framework for a comprehensive, rational description of the soil erosion process that has not been available heretofore. However, the inputs for this "model" must be expressed quantitatively, and they should be in terms of generally available information. Therefore, to be useful for this purpose, both theoretical and experimental research must provide results that are suitable for formulating mathematical expressions containing readily evaluated parameters. Such research is now in progress by the erosion research personnel at Lafayette, Indiana, Morris, Minnesota and Ames, Iowa.

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SOIL EROSION PROCESS